

ECE 463/663 - Homework #09

LQR Observers, Kalman Filters, LQG/LTR

Observers & Kalman Filters

Cart and Pendulum (HW #4): The dynamics for a cart and pendulum system with sensor and input noise is as follows

$$s \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -29.4 & 0 & 0 \\ 0 & 26.133 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -0.667 \end{bmatrix} (F + \eta_u)$$

$$y_1 = x + n_x$$

$$y_2 = \theta + n_\theta$$

where there is Gaussian noise at the input and output

$$n_u \sim \mathcal{N}(0, 0.2^2) \quad \text{mean zero, standard deviation } 0.2$$

$$n_x \sim \mathcal{N}(0, 0.05^2) \quad \text{mean zero, standard deviation } 0.1$$

$$n_\theta \sim \mathcal{N}(0, 0.01^2) \quad \text{mean zero, standard deviation } 0.01$$

1) Use a servo-compensator to force the DC gain to one (i.e. use the servo compensator from homework set #10).

Plot the step response

- Without noise (same as homework set #10)
- With noise

2) Design a full-order observer using pole-placement to place the observer poles at $\{-3, -4, -5, -6\}$

- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.

3) Design a Kalman filter (i.e. a full-order observer with a specific Q and R)

- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.

LQG / LTR

For the cart and pendulum system of homework set #4:

$$s \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -29.4 & 0 & 0 \\ 0 & 26.133 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -0.667 \end{bmatrix} F$$

Design a control law so that the cart and pendulum system behaves like the following reference model:

$$y_m = \left(\frac{0.5}{s^2 + s + 0.5} \right) R$$

4) Give a block diagram for your controller

5) Plot the step response of the model and the linearized plant for your control law for

- $Q = 100 \text{ e}^2$
- $Q = 1,000 \text{ e}^2$
- $Q = 10,000 \text{ e}^2$