

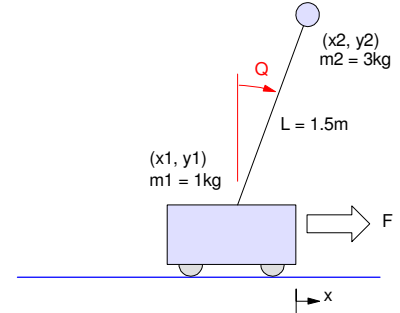
ECE 463/663 - Homework #5

Pole Placement.

Pole Placement (Inverted Pendulum)

Problem 1) (30pt) Use the dynamics of a Cart and Pendulum System from homework set #4:

$$s \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -29.4 & 0 & 0 \\ 0 & 26.133 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -0.667 \end{bmatrix} F$$



Design a feedback control law of the form

$$U = K_r * R - K_x * X$$

so that the closed-loop system has

- A 2% settling time of 10 seconds, and
- 5% overshoot for a step input

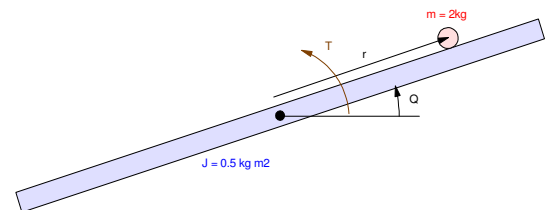
(Check the step response of the linear system in Matlab)

Check the step response of the nonlinear system

Pole Placement (Ball & Beam)

Problem 2) (30pt) Use the dynamics for the Ball and Beam system from homework set #4.

$$s \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.84 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix} T$$



Design a feedback control law of the form

$$U = K_r * R - K_x * X$$

so that the closed-loop system has

- A 2% settling time of 10 seconds, and
- 5% overshoot for a step input

Check the step response of the linear system in Matlab

(Check the step response of the nonlinear system)

Servo Compensators

Problem 3) (10pt) Using your control law from problem #2, find the step response on the nonlinear simulation if the mass of the ball is changed from 2.0kg to 2.2kg.

Problem 4) (30pt) For the ball and beam system, design a feedback control law of the form

$$U = -K_Z Z - K_X X$$

$$dZ = Y - \text{Ref}$$

where Y is the position of the ball. Pick the feedback gains so that the closed-loop system has

- A 2% settling time of 10 seconds,
- 5% overshoot for a step input, and
- A servo compensator designed to track a constant set point / reject a constant disturbance

Check the step response of the linear system in Matlab

Check the step response of the nonlinear system

- When the mass of the ball is 2.0kg (nominal)
- When the mass of the ball is 2.2kg (adding a constant disturbance)

