ECE 463 - Modern Control

ECE 461/661 Controls Systems

Jake Glower - Lecture #40

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Feedback

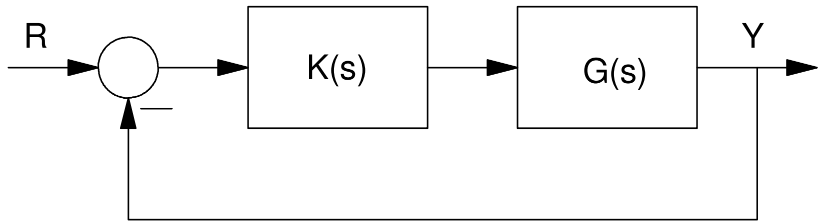
Feedback is very useful

- Instead of specifying the input, you specify the desired output
- It can improve the response of a system
- It can turn an unstable system into a stable system (closed-loop)

ECE 461: Classic Control

One solution: Use output feedback

- G(s) is the plant
- K(s) is a pre-filter, making a poorly behaving system behave like a well-behaved system



Tools for finding K(s)

- Root Locus
- Nichols Charts
- Bode Plots

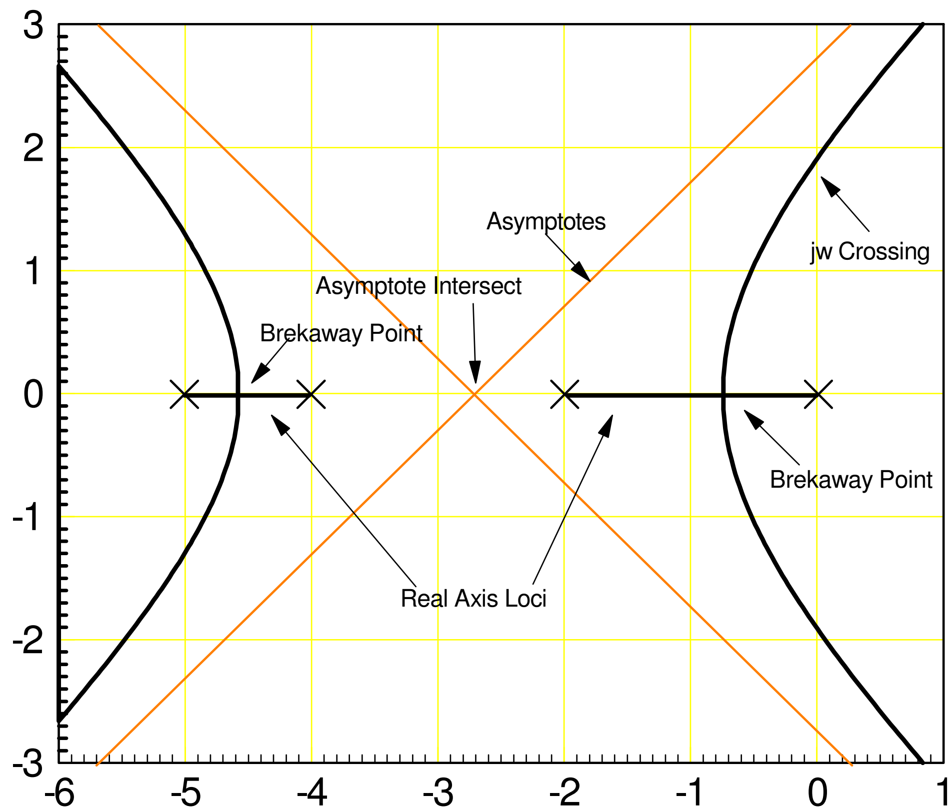
Sometimes, output feedback works very well

Open-loop system is stable

- Heat equation
- DC servo motors
- Cruise controls on cars

One unstable pole

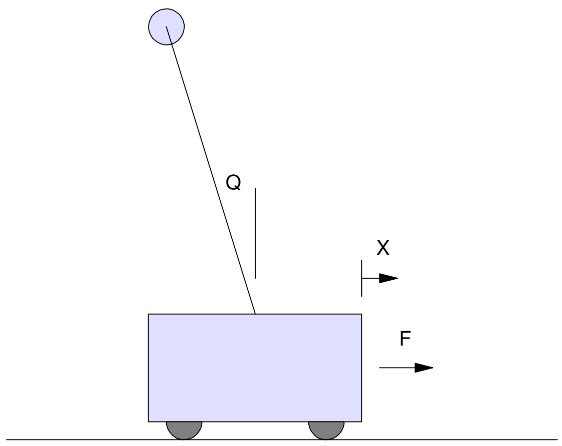
- Single-link robotic arm



Sometimes, these methods fail

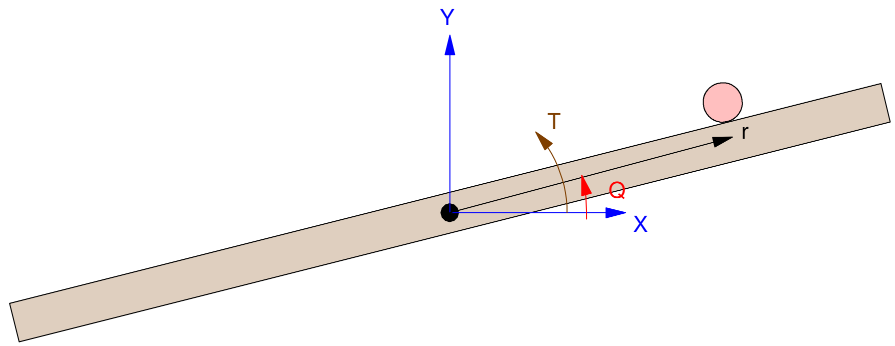
Cart and Pendulum System

$$x = \left(\frac{2(s+3)(s-3)}{s^2(s+4)(s-4)} \right) F$$



Ball and Beam System

$$r = \left(\frac{10}{(s+3)(s-3)(s+j3)(s-j3)} \right) T$$



Rockets and the 1950s

https://website101.com/wp-content/uploads/2012/08/rocket-explosion.jpg

In the 1950s, we were trying to build rockets

- Similar to a cart and pendulum system
- Open-loop unstable

The approach was to cancel the unstable pole

- Results in a system with stable poles

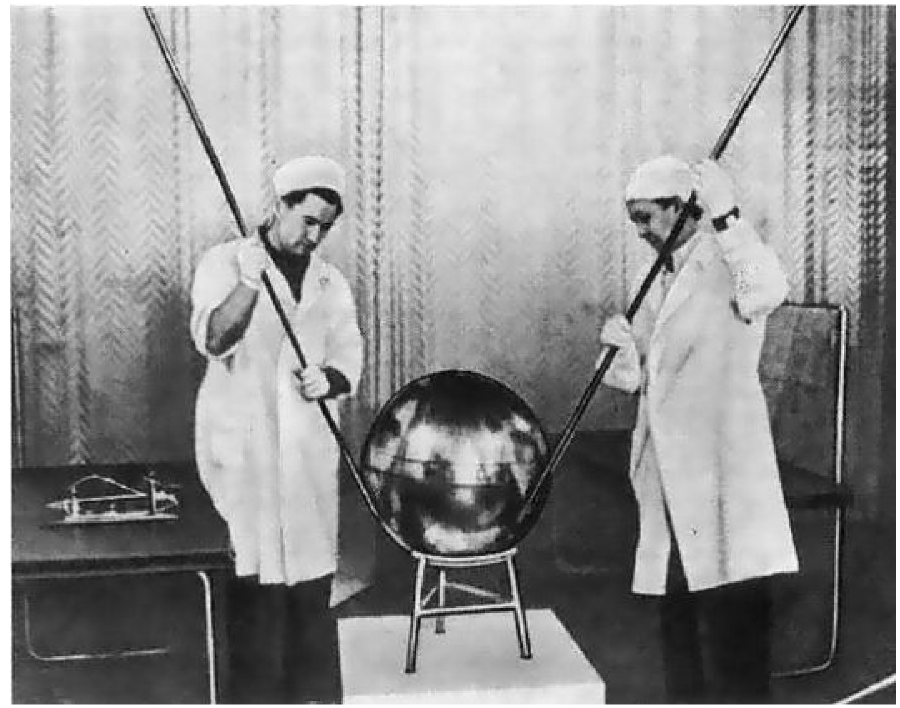
Problem:

- The rockets were always unstable
- Why do two systems with identical frequency responses have different step responses?

Sputnik

https://theinvisibleagent.files.wordpress.com/2011/09/sputnik3.jpg

While NASA was struggling to get a rocket into space, the Soviet Union placed a satellite into orbit in 1957



This was a major shock to the U.S.

- Russia was ahead of the U.S.

This led to researchers looking at Soviet technical journals to learn how the Soviets stabilize their rockets

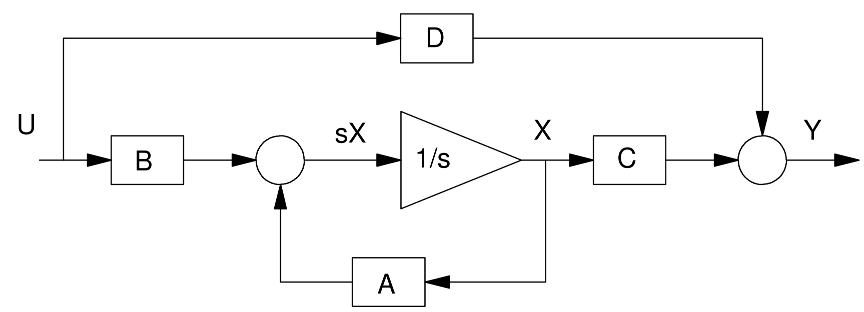
- The techniques used is termed "Modern Control"

Modern Control (ECE 463/663)

A matrix approach to designing feedback control systems

Let X define the energy in the system

- X = [position, velocity, ...]
- potential energy, kinetic energy, ...



If you specify how the energy moves around, you've specified the dynamics

$$sX = AX + BU$$

$$Y = CD + DU$$

Modern Control vs. Classical Control

The transfer function from U to Y is

$$Y = \left(C(sI - A)^{-1}B + D \right) U$$

$$G(s) = C(sI - A)^{-1}B + D$$

The eigenvalues of A are all important

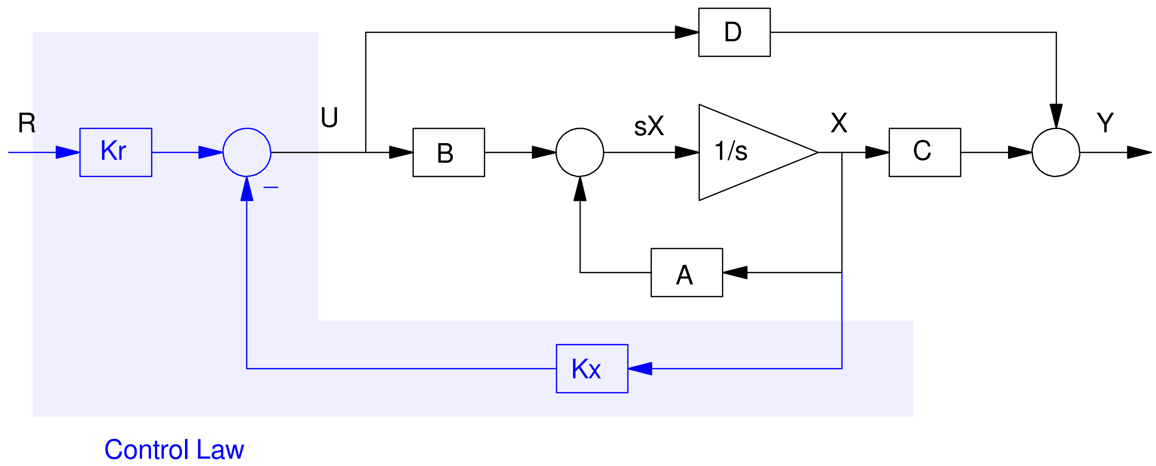
- Eigenvalues = Poles
- Negative definite = stable
- Positive real = unstable
- Complex = oscillatory

If you can shift the eigenvalues of A, you can change how the system behaves

Full State Feedback

A is an NxN matrix

- It has N eigenvalues
- It has N poles



If you define U as

$$U = -K_x X + K_r R$$

the dynamics become

$$sX = (A - BK_x)X + BK_r R$$

$$Y = CX + DU$$

With Kx, you have N degrees of freedom to place the N closed-loop eigenvalues.

How do you determine the system dynamics?

- Week 1-5
- LaGrangian formulation

Define the energy in the system

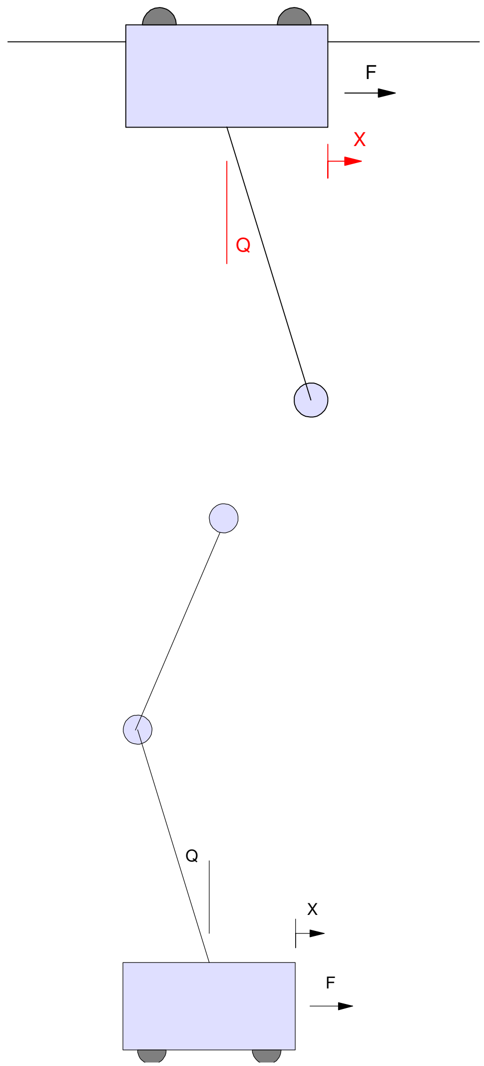
$$L = KE - PE$$

Define how the energy moves about the system

$$F = \frac{d}{dt}\left(\frac{\partial L}{\partial x'}\right) - \left(\frac{\partial L}{\partial x}\right)$$

Linearize to determine the state-space model

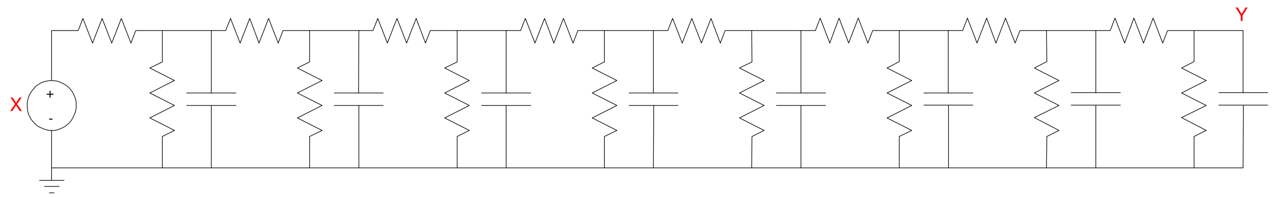
Works for both linear and nonlinear systems



Systems we look at in ECE 463

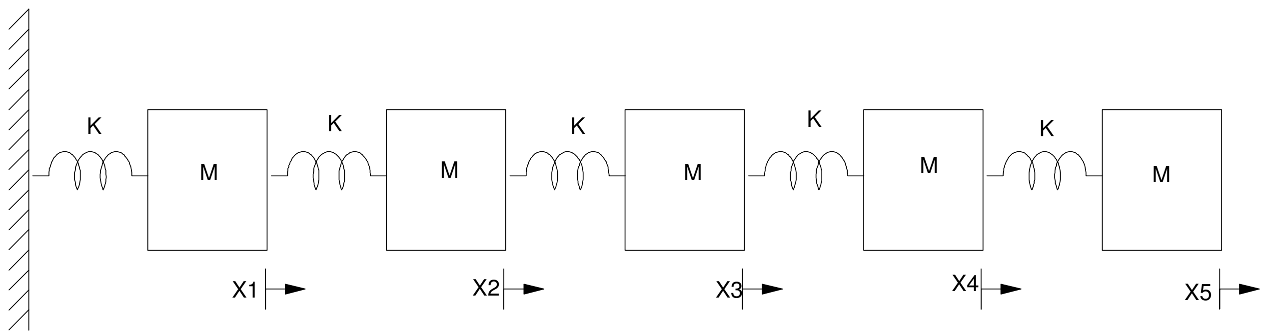
Heat Equation

- N real poles



Mass-Spring System

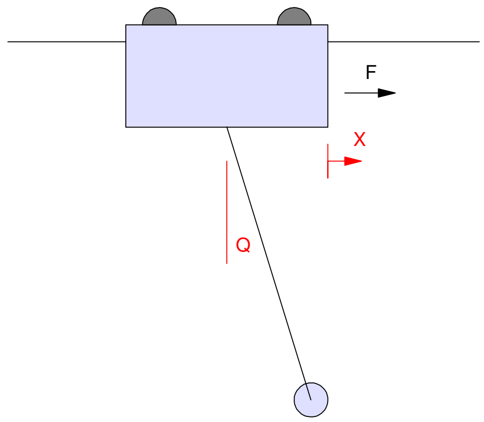
- N complex poles



Systems we look at in ECE 463

Gantry System

$$x = \left(\frac{2(s+j3)(s-j3)}{s^2(s+j4)(s-j4)} \right) F$$



Cart and Pendulum

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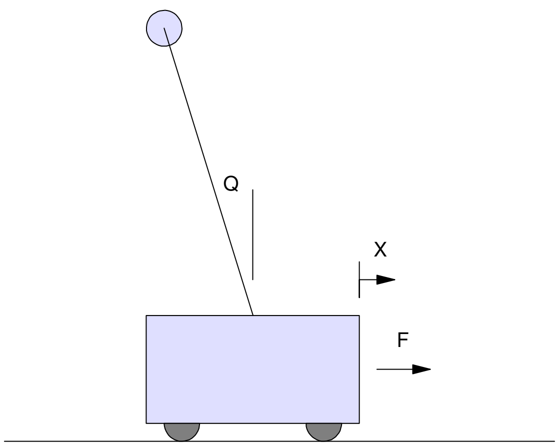
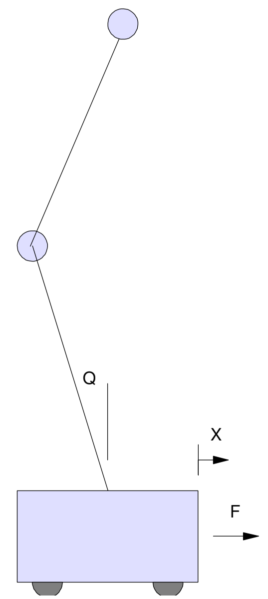


Systems we look at in ECE 463

Double Pendulum

$$x = \left(\frac{(s+3.5\pm j1.5)(s-3.5\pm j1.5)}{s^2(s+4.5\pm j1.5)(s-4.5\pm j1.5)} \right) F$$



Ball and Beam

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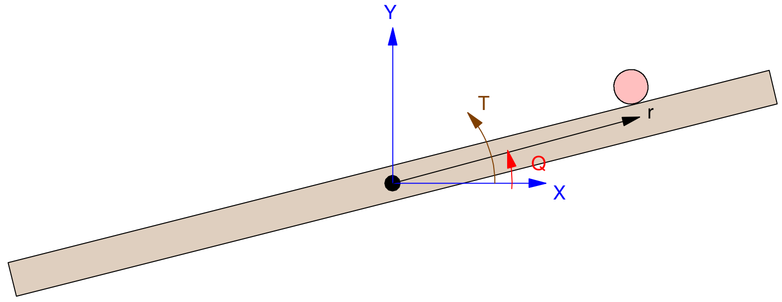


Problem: How do you determine Kx?

Pole Placement (weeks 6 - 9)

- Bass Gura
- Kx contains N terms
- (A - B Kx) has N eigenvalues
- N equations and N unknowns

Good:

- Complete freedom
- Can place poles anywhere

Bad:

- Complete freedom
- Where *should* you place the poles

Optimal Control (weeks 10-14)

- Based upon Calculus of Variations
- Define a cost function
- $J = f(\text{energy}) + g(\text{input}^2)$
- Determine Kx to minimize J

Good:

- Places all poles in their optimal spot

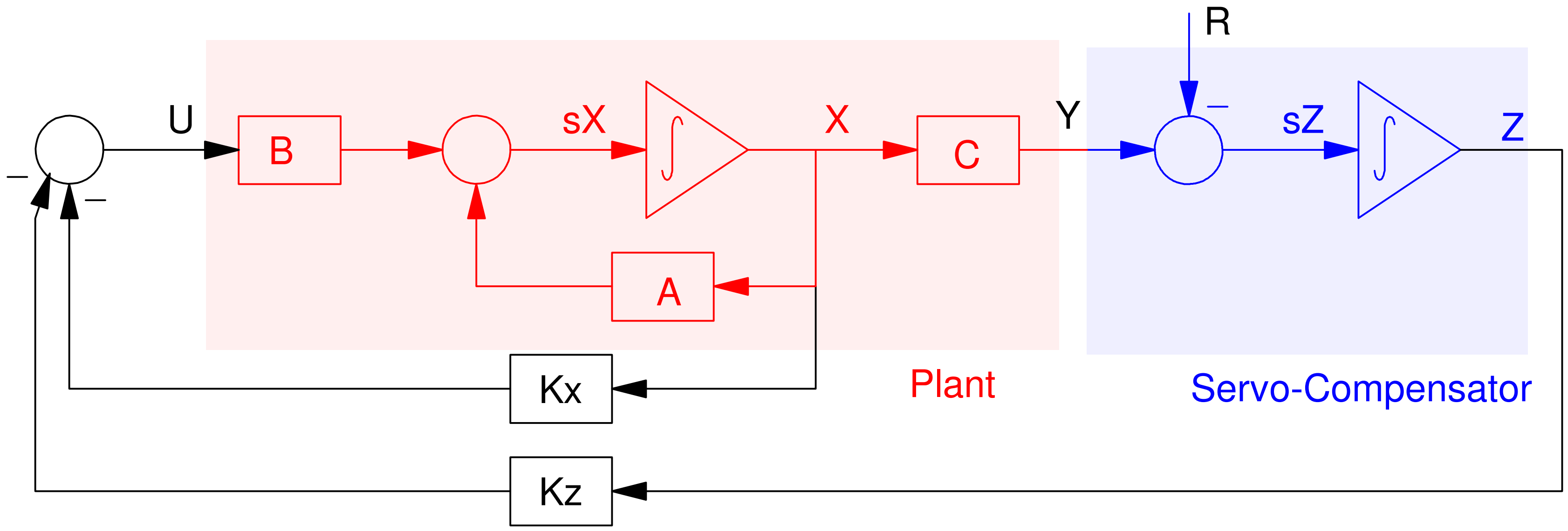
Bad:

- Cost function is arbitrary
- I'd prefer "good control" over "optimal control"
- It's a tool to find Kx

Problem: How to you track a constant set point?

- ECE 461: Make it a type-1 system
- ECE 463: Add a servo compensator

Also allows you to track multiple sine waves

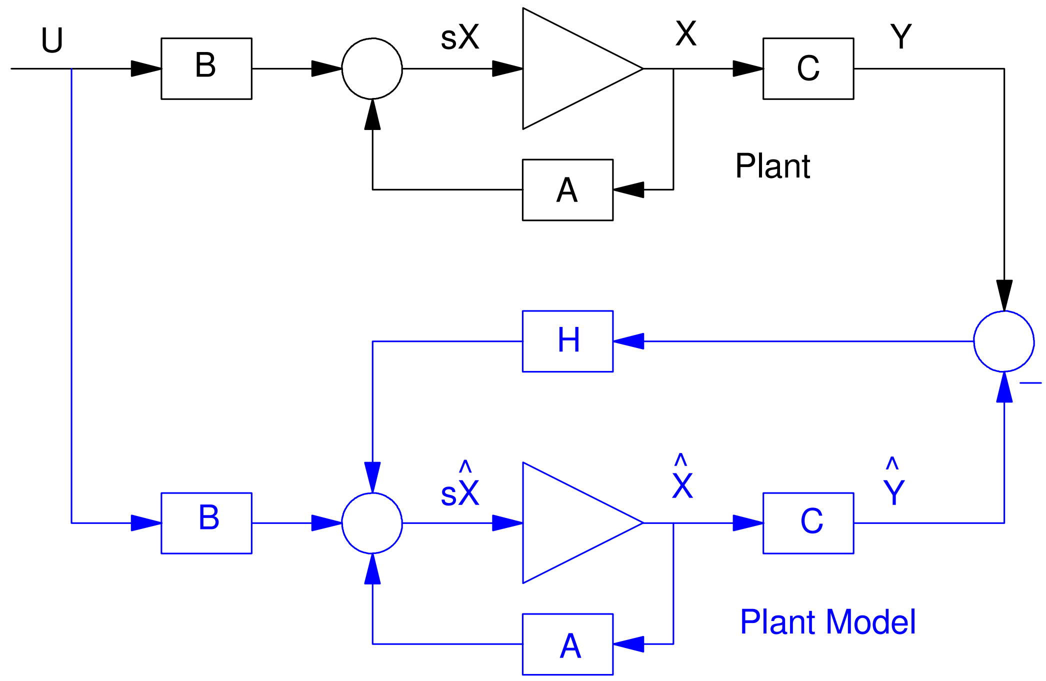


Problem: What if you can't measure *all* of the states?

- Kx contains N terms
- You need to know all N states in X

If you can't measure the states, estimate them from the input and output

- Full-Order Observer



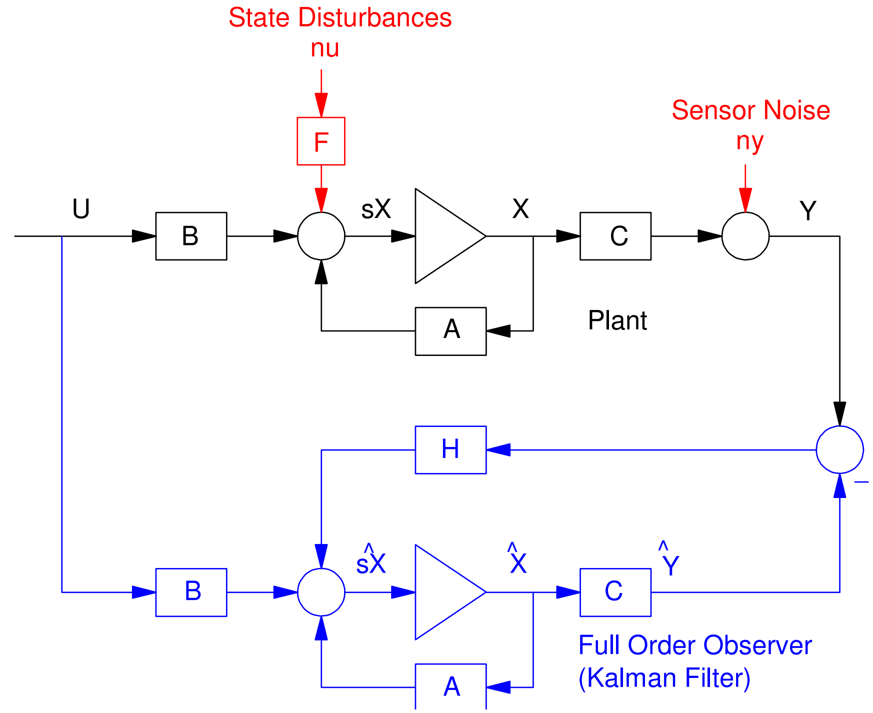
Problem: How do you choose H?

Option #1: Pole Placement

- Bass Gura

Option #2: Optimal Control

- Produces a Kalman Filter
- Optimal H given sensor noise and input noise



Modern Control

For some systems, Modern Control works really well

- Rockets
- Systems with a small number of states

For some systems, Modern Control works poorly

- Systems with a large number of states
- Heat equation
- Wave equation

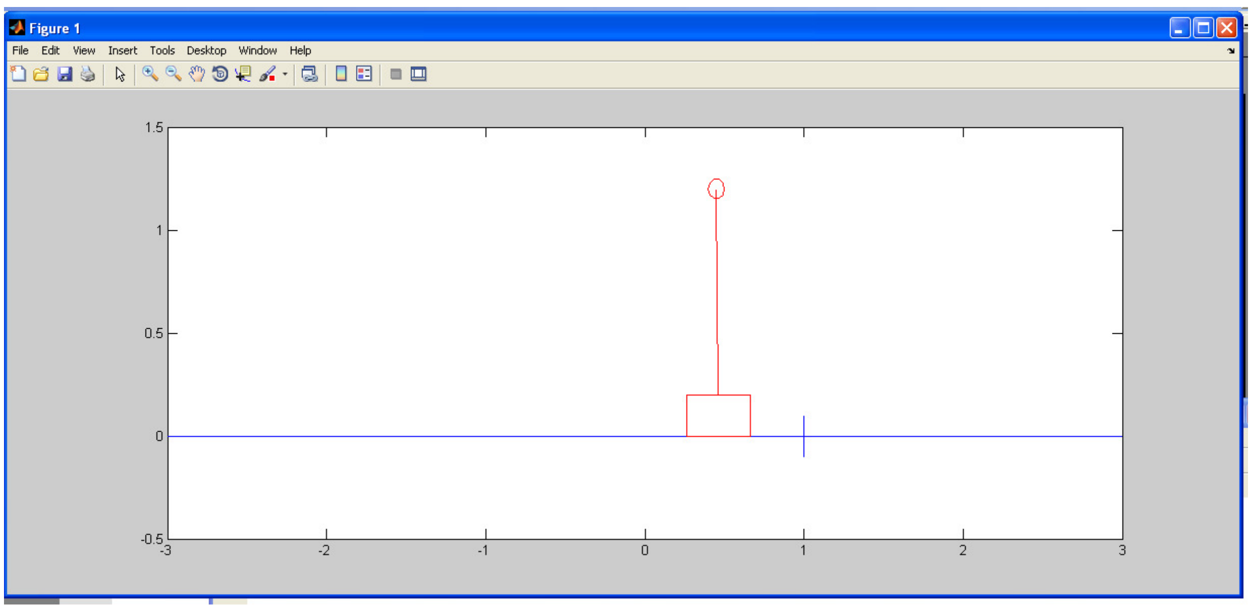
ECE 463: Modern Control

Fun class were we use Matlab extensively

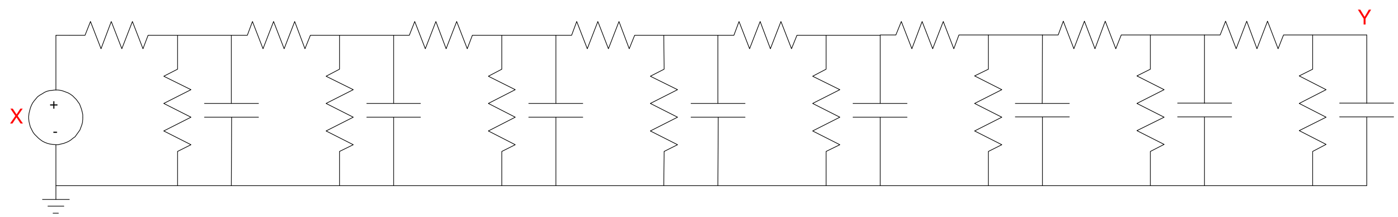
- Simulating nonlinear dynamic systems
- Manipulating NxN matrices

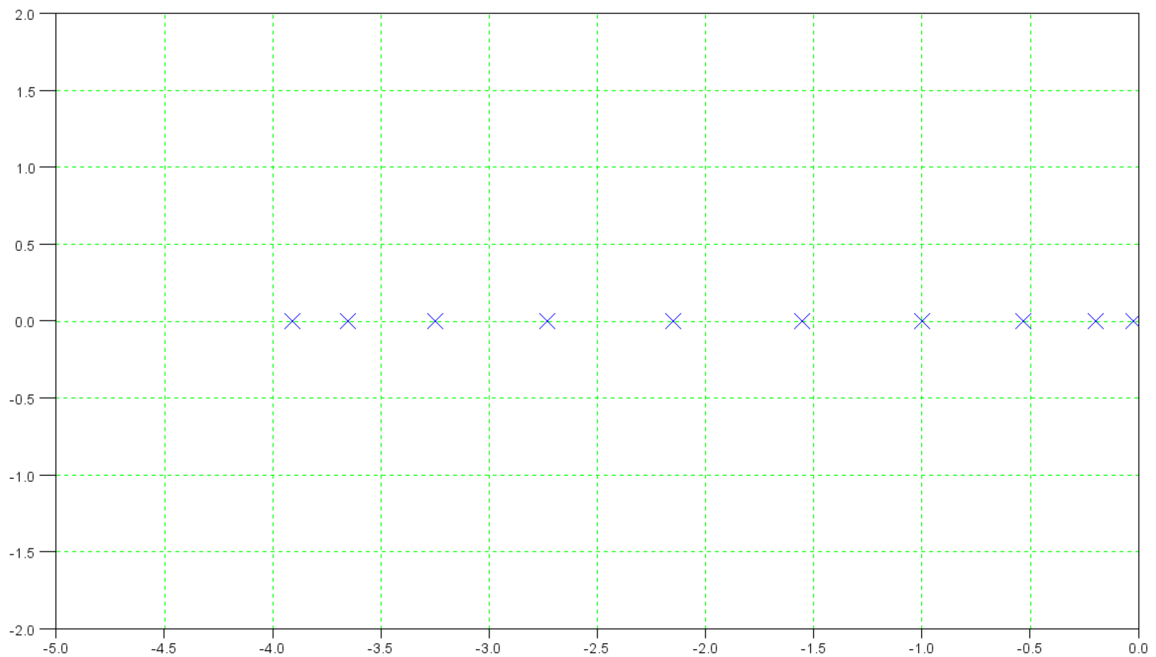
Heavy use of matrix algebra

- We cover what you'll use in this class
- Math 129 is the main class we use

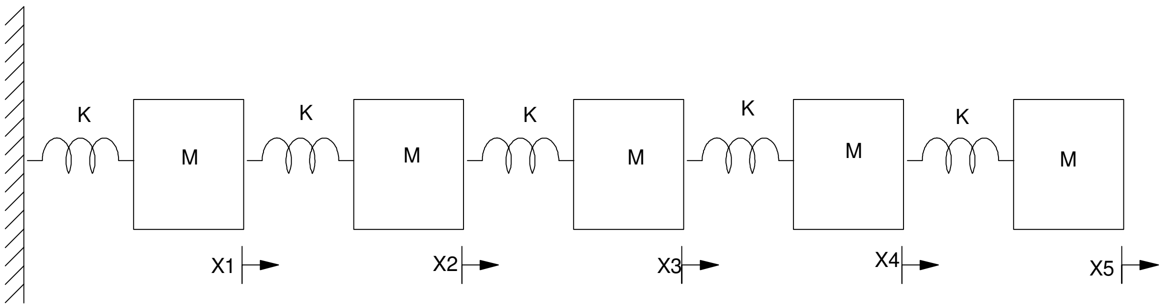


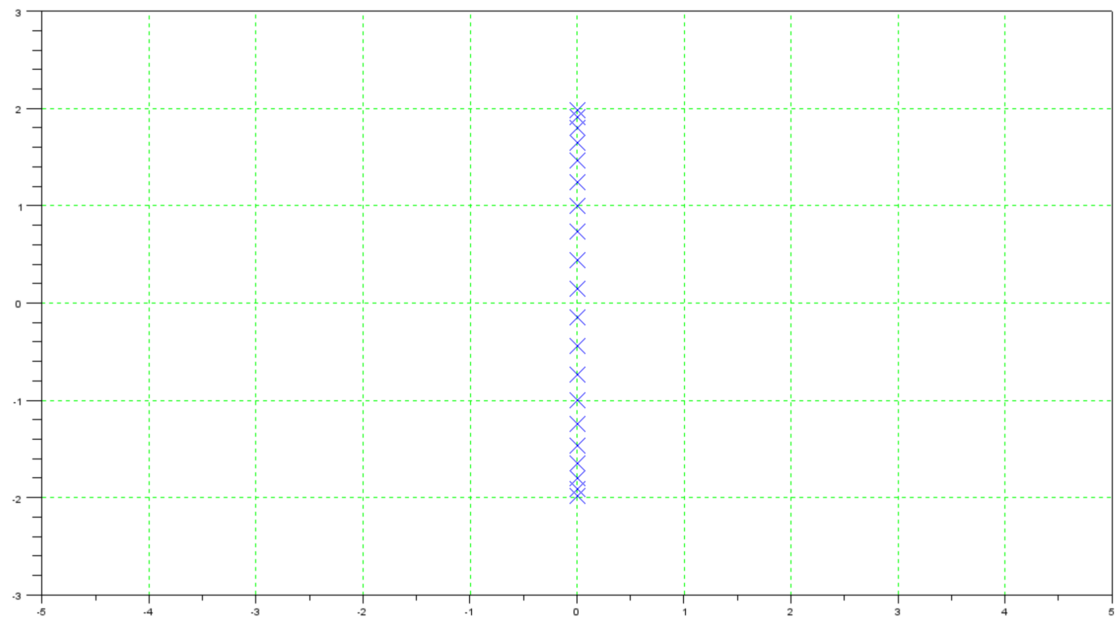
Heat Equation



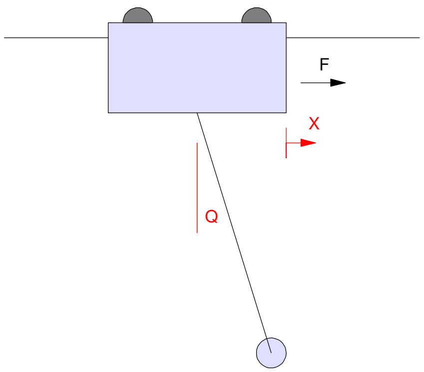


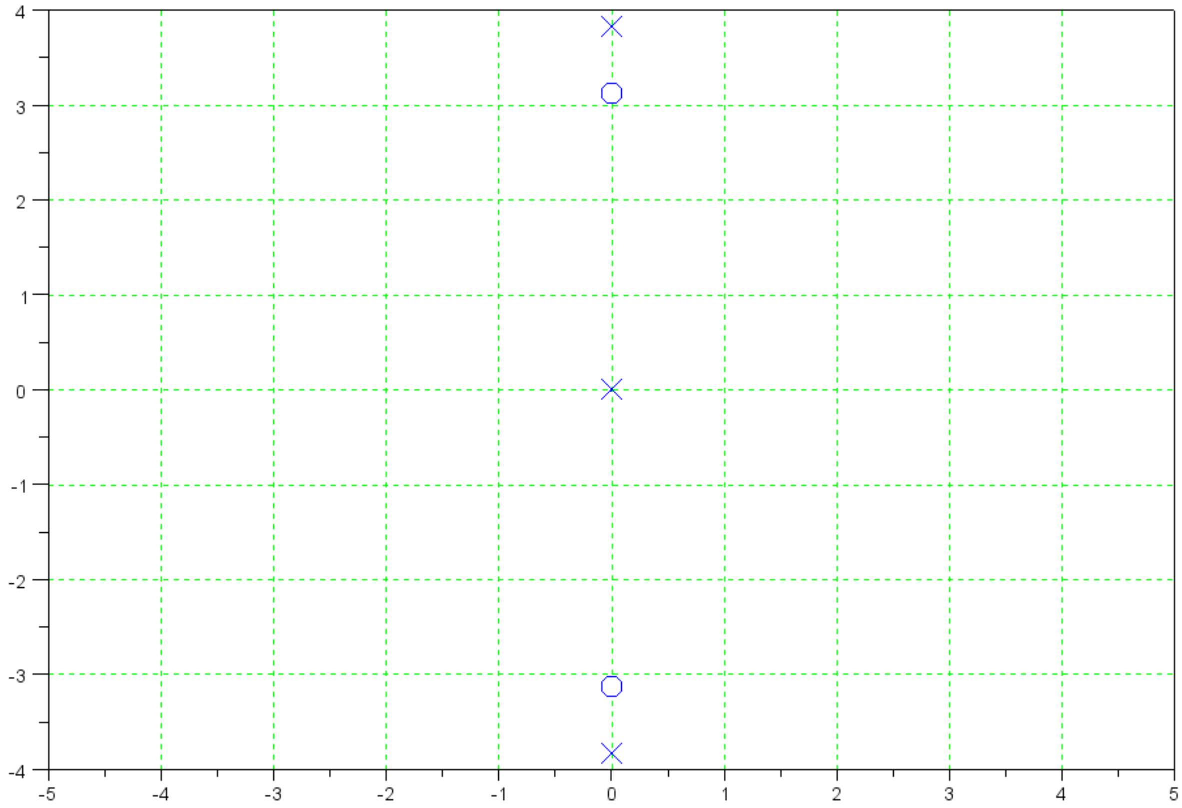
Wave Equation



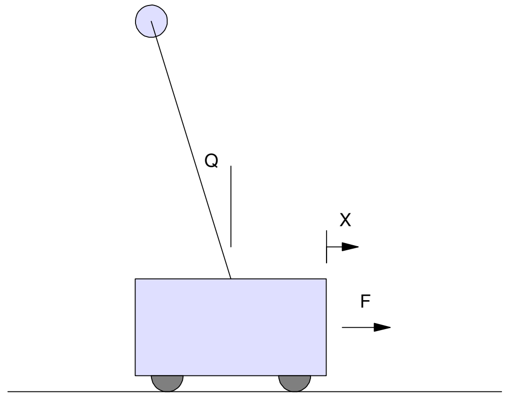


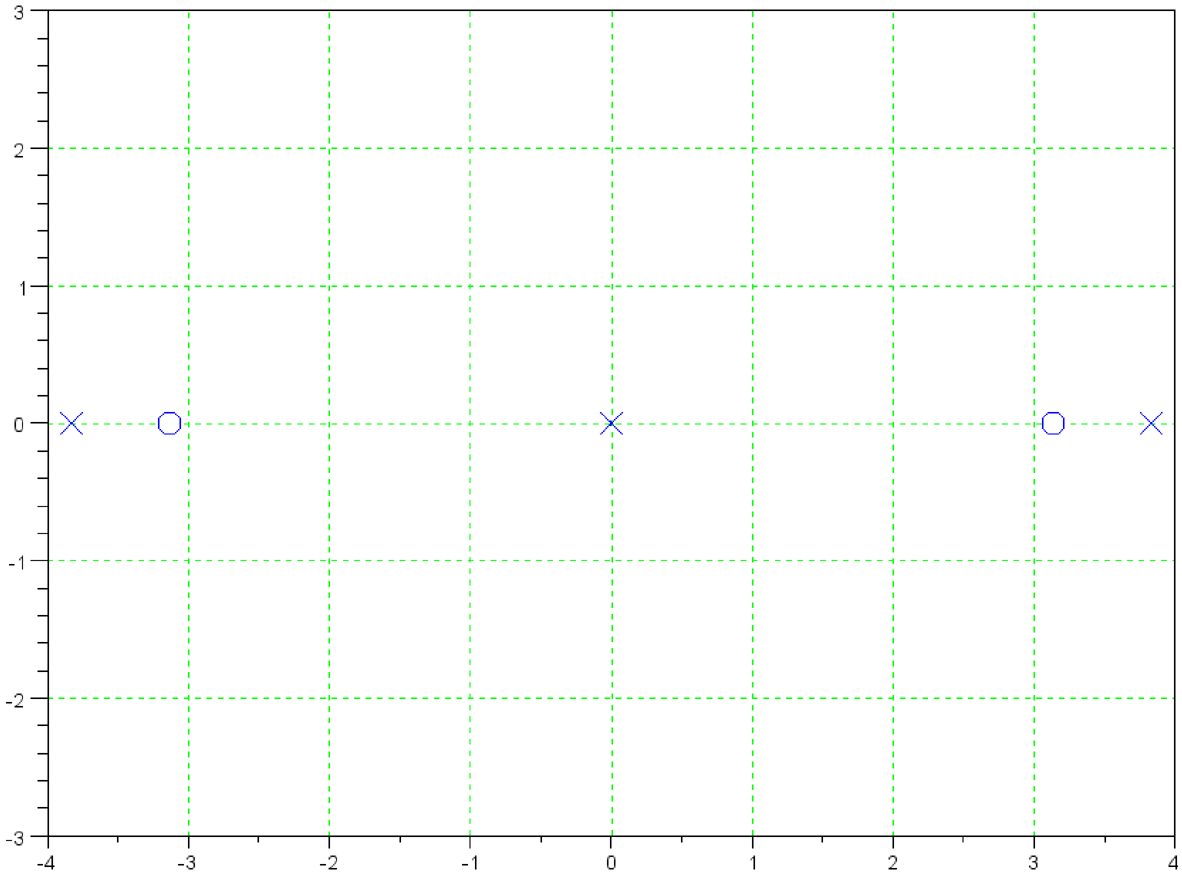
Gantry System



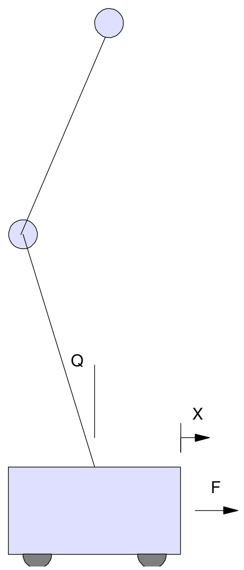


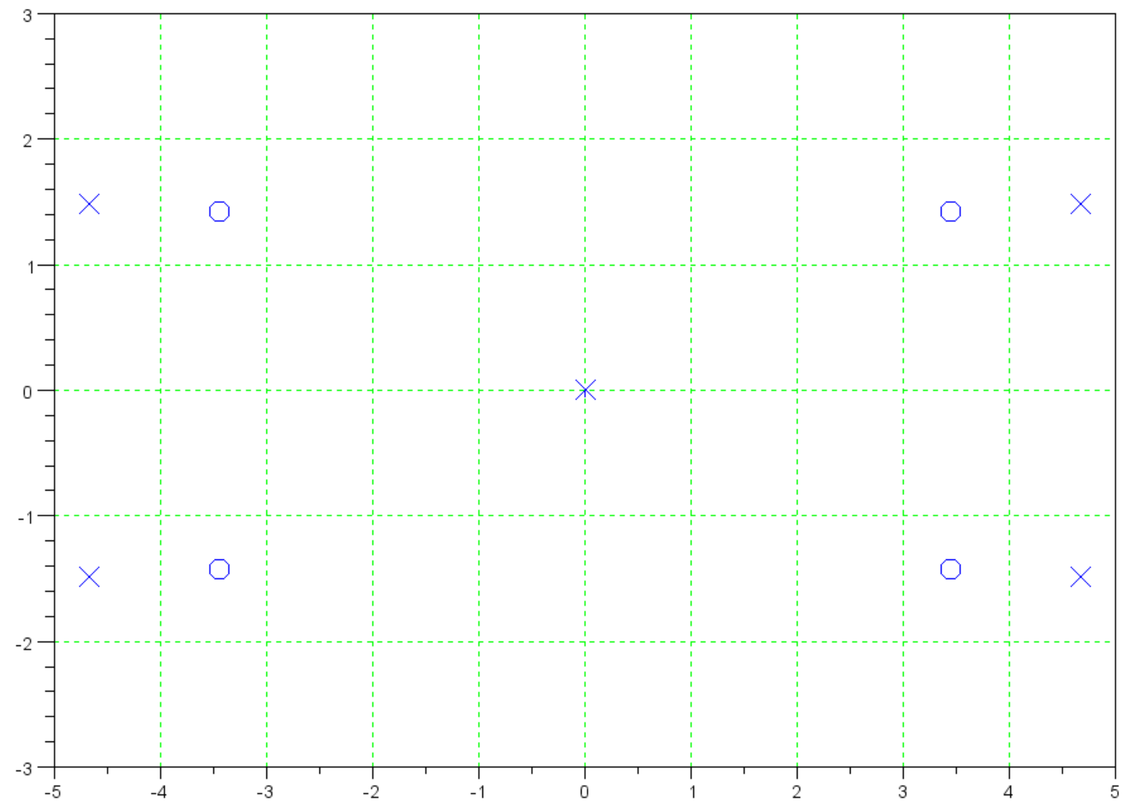
Inverted Pendulum





Double Pendulum





Ball & Beam