Lag Compensators

ECE 461/661 Controls Systems

Jake Glower - Lecture #38

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Purpose of Lag Compensators:

Lag compensators serve to

- Increase the DC gain,
- Without hurting the overall response too much

Essentially, a lag compensator is a band-aid

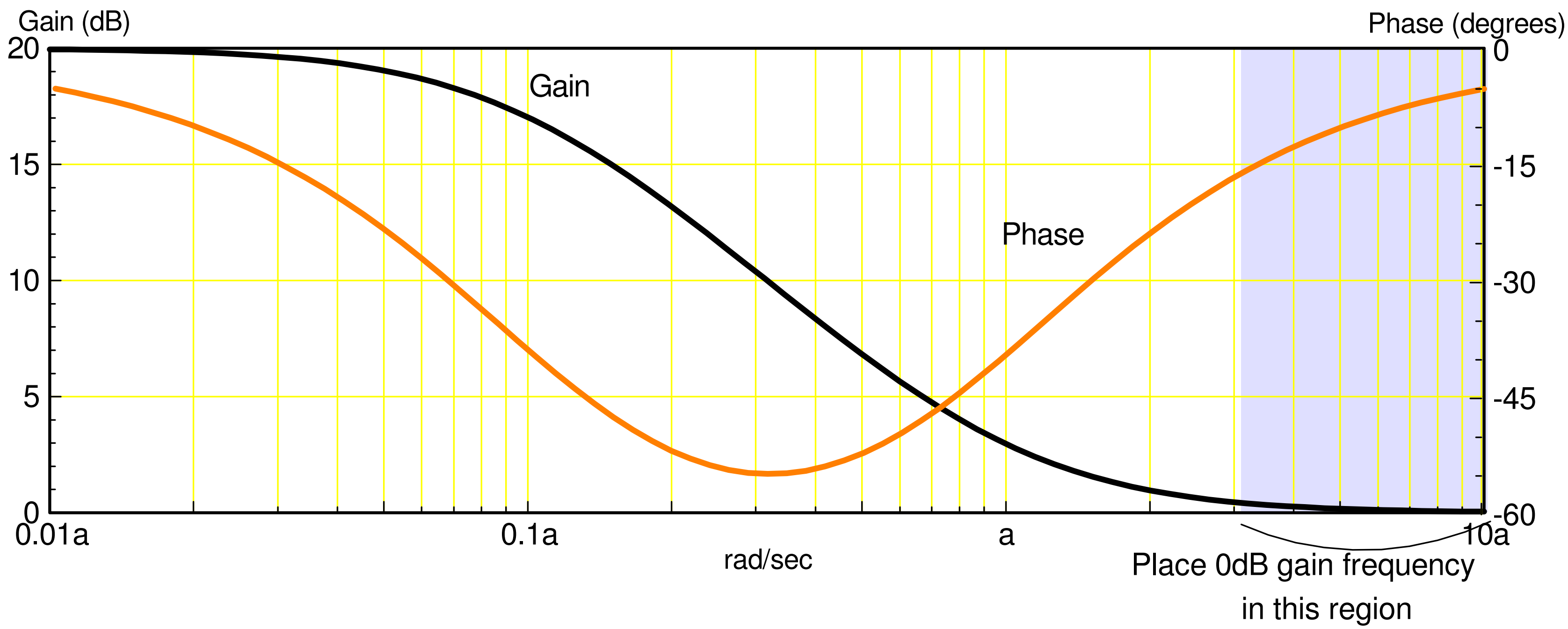
- Gain & Lead compensation meets all design requirements *except* the error constant
- Add a filter (Lag compensator) to

  Increase the DC gain (meet the error constant requirement),

  And leave the rest of your design alone

Lag Compensator: $K(s) = \left(\frac{s+a}{s+0.1a}\right)$

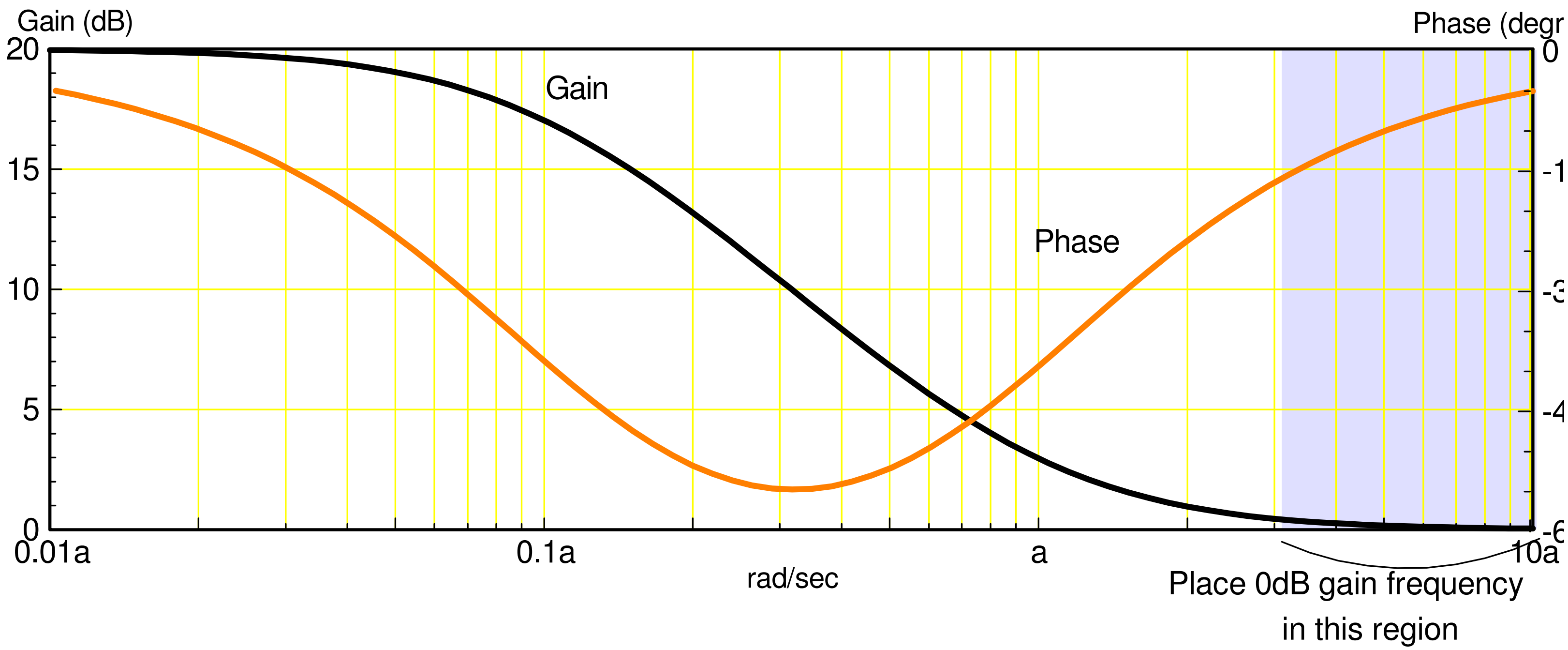
- DC gain = 10 (20dB: good)
- Phase is negative (lag: bad)



Lag Compensator Design

Place the zero 1/3rd to 1/10th of the 0dB gain frequency

- 1/3rd: Faster system (good) but hurts the phase margin by 16.3 degrees (bad)
- 1/10th: Slower system (bad) but hurts the phase margin by 5.1 degrees (not so bad)



Design Procedure

Step 1: Pick a design rule

- Pick the zero to be 1/3rd or 1/10th of the 0dB gain frequency.)

Step 2: Design a gain compensator

- Design for an extra 5.1 degrees phase margin (if using the 1/10 rule)
- Design for an extra 16.3 degrees phase margin (if using the 1/3 rule)

Determine the 0dB gain frequency.

Step 3: Add the lag compensator.

- This should bring the phase margin back to where you wanted it.

Lag Compensator Design Example

- $G(s) = \left(\frac{2000}{s(s+5)(s+20)}\right)$
- 40 degree phase margin

Solution:

1) Assume the lag zero will be 1/10th of the 0dB gain frequency.

2) Find w for (40 degree phase margin plus an extra 5.1 degrees).

$$G(j3.4983) = 4.6144\angle -134.9^0$$

3) Add the lag compensator

$$K_{lag} = \left(\frac{s+0.34983}{s+0.034983}\right)$$

4) Crank up the gain until $GK = 1\angle -140^0$

$$GK = \left(\frac{2000}{s(s+5)(s+20)}\right)(k)\left(\frac{s+0.34983}{s+0.034983}\right)$$

$$GK(j3.4983) = 4.6372k\angle -140^0$$

$$k = 0.2156$$

and the compensator is

$$K(s) = \left(\frac{0.2156(s+0.34983)}{s+0.034983}\right)$$

Note

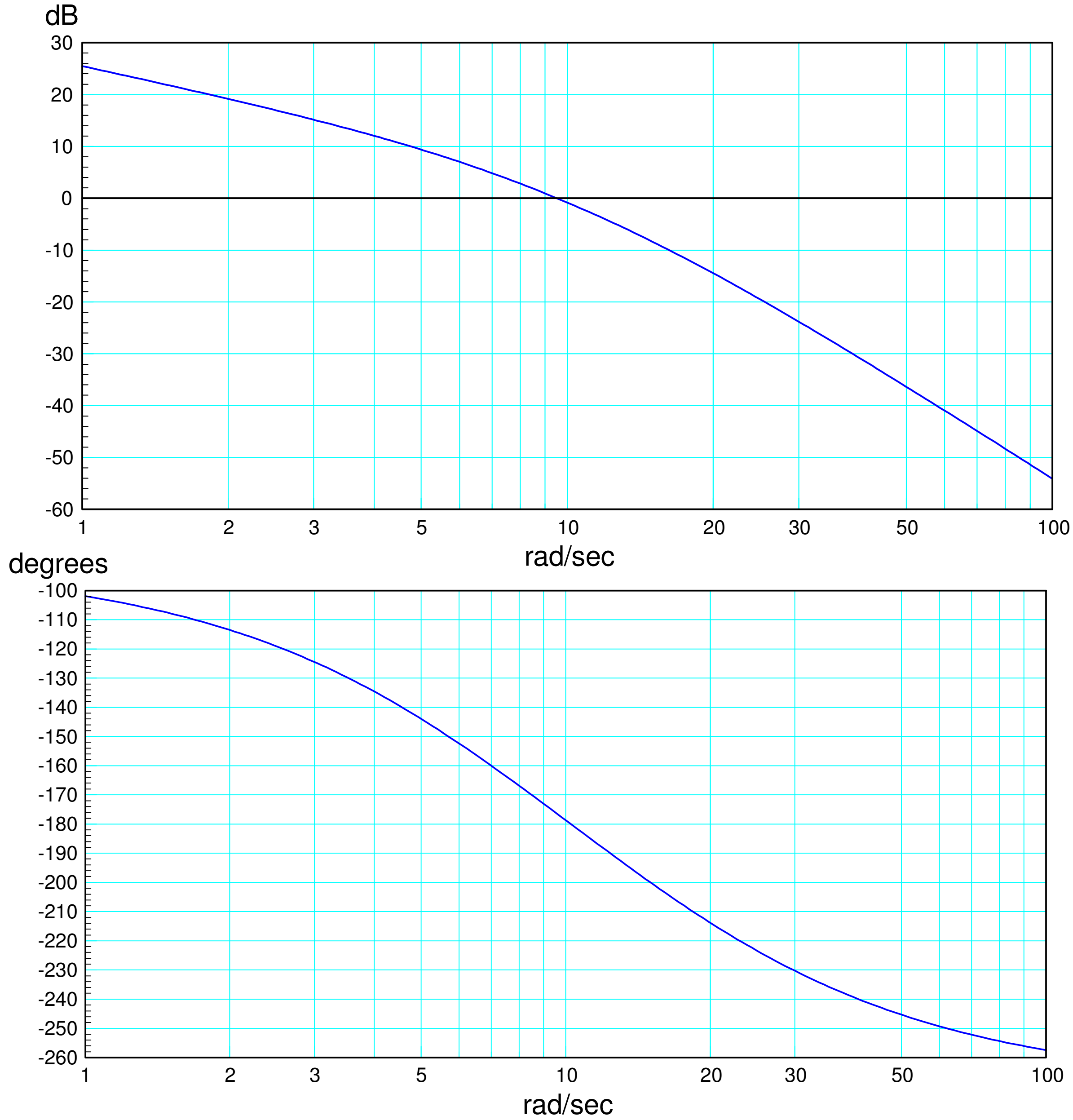
- The DC gain is 8x larger
  - *10x from the lag*
  - *A little less since had to design for an extra 5.1 degree phase margin*

Summary

	Gain	Lead	Lead & Gain	Lag & Gain
K(s)	0.2615	$\left(\frac{2.615(s+4)}{s+40}\right)$	$\left(\frac{7.2171(s+4)}{s+40}\right)$	$\left(\frac{0.2156(s+0.34983)}{s+0.034983}\right)$
Phase Margin	40 degrees	72 degrees	40 degrees	40 degrees
0dB Gain Frequency	4.0031 rad/sec	5.7165 rad/sec	13.8325 rad/sec	3.4983 rad/sec
Kv	5.23	5.23	14.43	43.12

Handout:

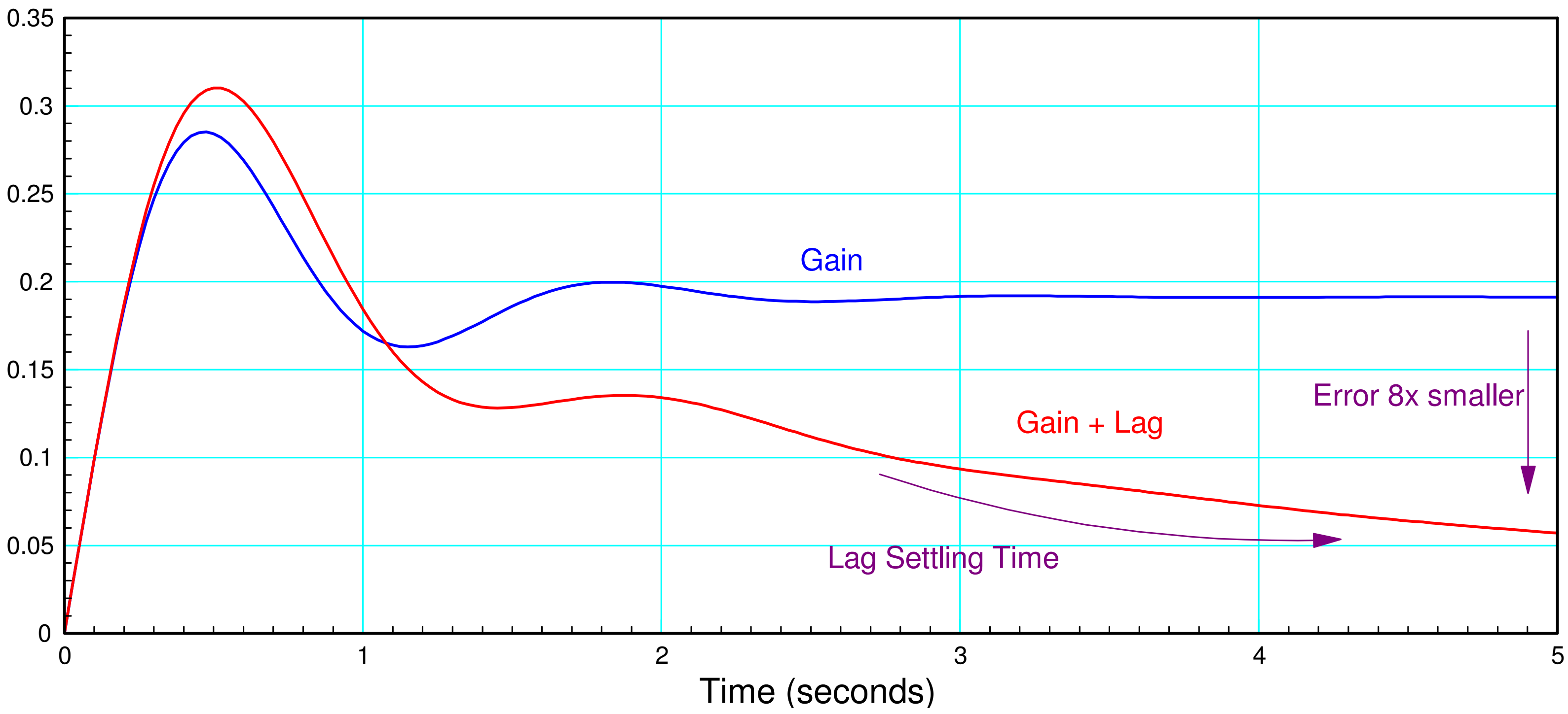
- Design a lag compensator for a 50 degree phase margin
- Determine the resulting error constant, Kv



Comments on Lag Compensators

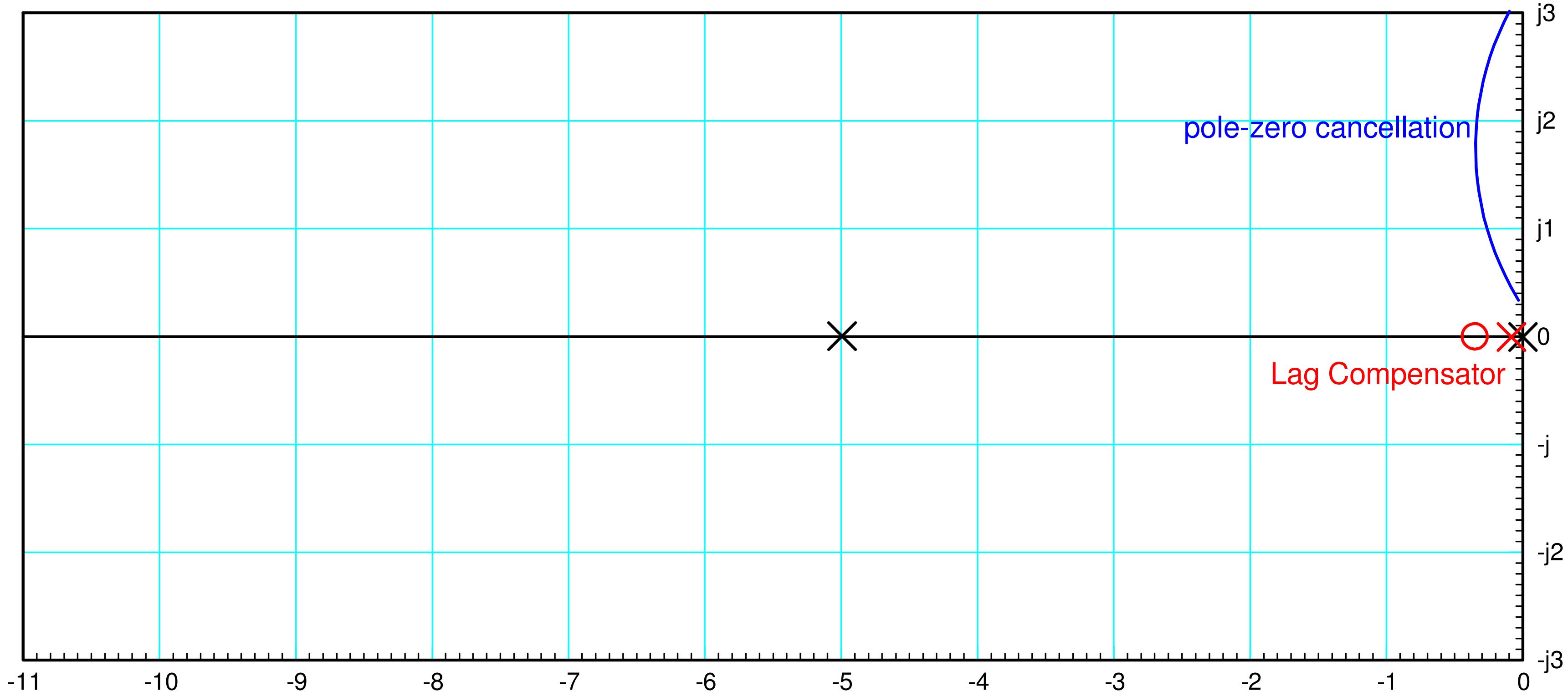
My personal opinion is that lag compensators are not that useful.

- The lag pole and zero are really slow, almost by definition
- Assumes the set point (R) does't vary:



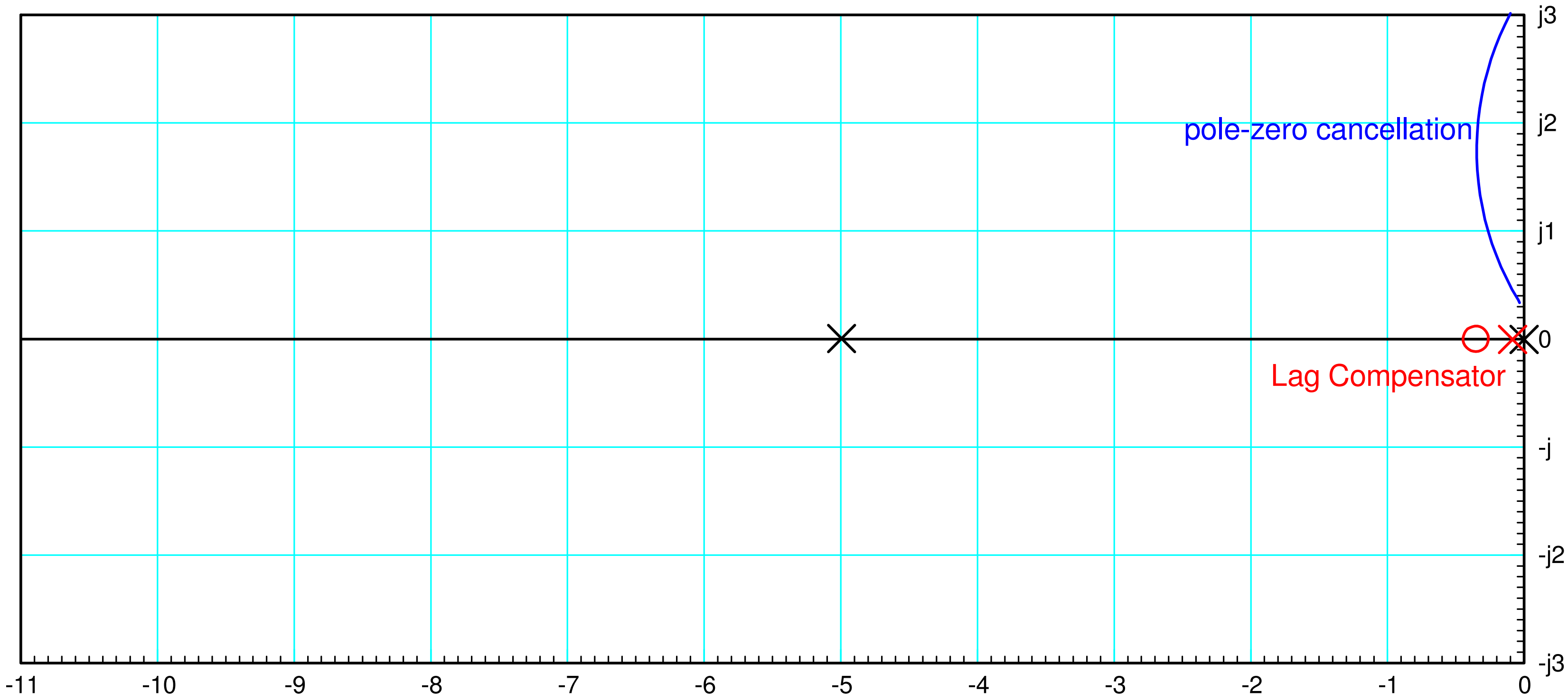
Lag only works if R(jw) is frequency limited to 0.03 rad/sec

- Lag looks like pole-zero cancellation above this



Just place the pole at s = 0

- $\left(\frac{s+0.35}{s+0.035}\right) \approx \left(\frac{s+0.35}{s}\right)$
- Not much affect on the root-locus plot
- Now a type-2 system (error = 0)



Summary:

Lag compensators are a band-aid

- If your compensator, K(s), meets all design requirements *except* the error constant
- Add a lag compensator

The resulting system technically works

- The error contant is increased
- With only a small degradation in the response

But...

- The increased gain only applies to very low frequency signals