Gain and Lead with Bode Plots

ECE 461/661 Controls Systems

Jake Glower - Lecture #37

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Gain Compensation

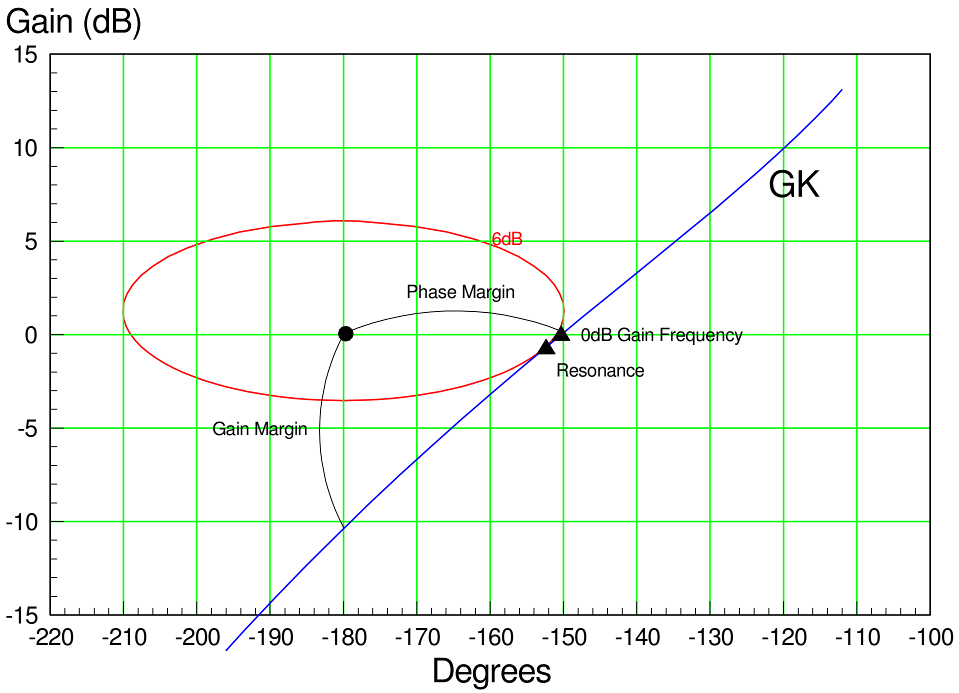
Nichols charts are useful

- They show directly what you are trying to do: keep away from -1

Nichols charts are a pain to use:

- They are difficult to draw
- The resonance is difficult to find

  the frequency where G(jw) is tangent to the M-circle changes as you change the gain.



Phase Margin

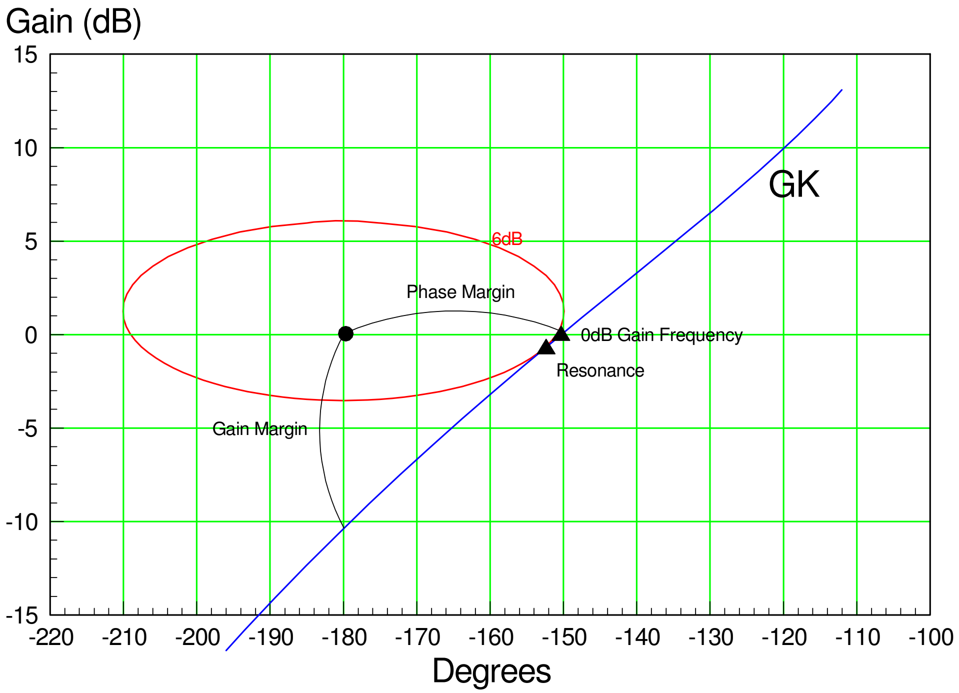
- Distance to -1 when open loop gain is 0dB

Assume the resonance is the same as the 0dB gain frequency

- Close but a little off

If so,

- Designing for a phase margin is equivalent to designing for a resonance (Mm)



Relationship between Mm and Phase Margin:

The phase margin which approximately corresponds to a certain resonance is from

$$G(j\omega) = 0dB\angle\phi = 1\angle\phi$$

$$\left|\frac{G}{1+G}\right| = \left|\frac{1\angle\phi}{1+1\angle\phi}\right| = M_m$$

$$\left|1 + (\cos\phi + j\sin\phi)\right| = \frac{1}{M_m}$$

$$(1+\cos\phi)^2 + (\sin\phi)^2 = \frac{1}{M_m^2}$$

$$1 + 2\cos\phi + \cos^2\phi + \sin^2\phi = \frac{1}{M_m^2}$$

$$\boxed{2 + 2\cos\phi = \frac{1}{M_m^2}}$$

$$\text{Phase Margin} = 180^0 - |\phi|$$

Example: Find K so that $G(s) = \left(\frac{2000}{s(s+5)(s+20)}\right)$ has

- Mm < 6dB, and
- Minimal error for a step and ramp input.

Solution:

i) Find the phase margin corresponding to Mm = 6dB:

$$\cos\phi = \left(\frac{\frac{1}{M_m^2}-2}{2}\right)$$

$$\phi = -151.04^0$$

Phase Margin = 28.96^0

ii) Find the frequency where G(jw) has a phase shift of -151.04^0

$$G(j5.2362) = 2.5474\angle - 151.04^0$$

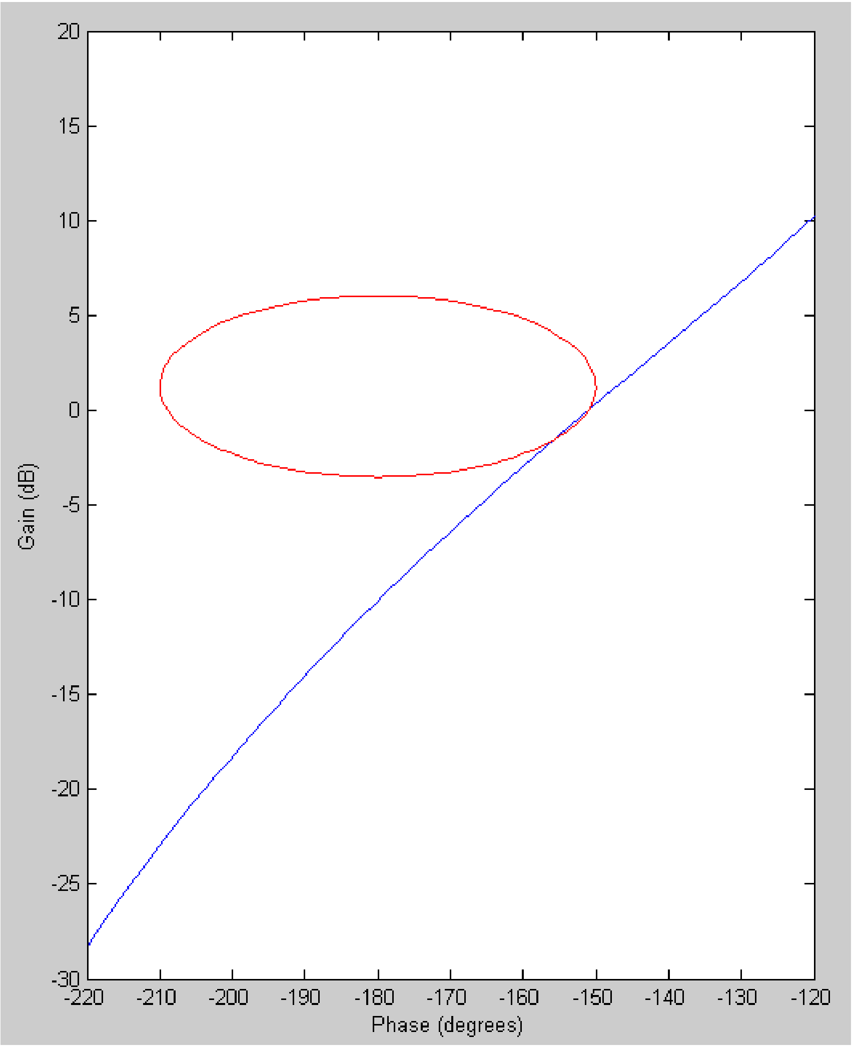
iii) Adjust k so that the gain at this frequency is one.

$$k = \frac{1}{2.5474}$$

$$\boxed{k = 0.3926 = -8.1dB}$$

Note:

- GK intersects the M-circle at 0dB
- The actual resonance is slightly more than +6dB



Resulting Closed-Loop System:

- Phase margin tells you Mm
- Mm tells you ζ
- 0dB gain frequency tells you imag(s)

K(s)	Kv	0dB Gain Freq	Phase Margin	Mm	CL Dom Poles (approx)
0.3926	7.85	5.24 rad/sec	28.96 deg	2.00 (6dB)	-1.41 + j5.24

Example 2: Design a gain compensator for Mm = 6dB

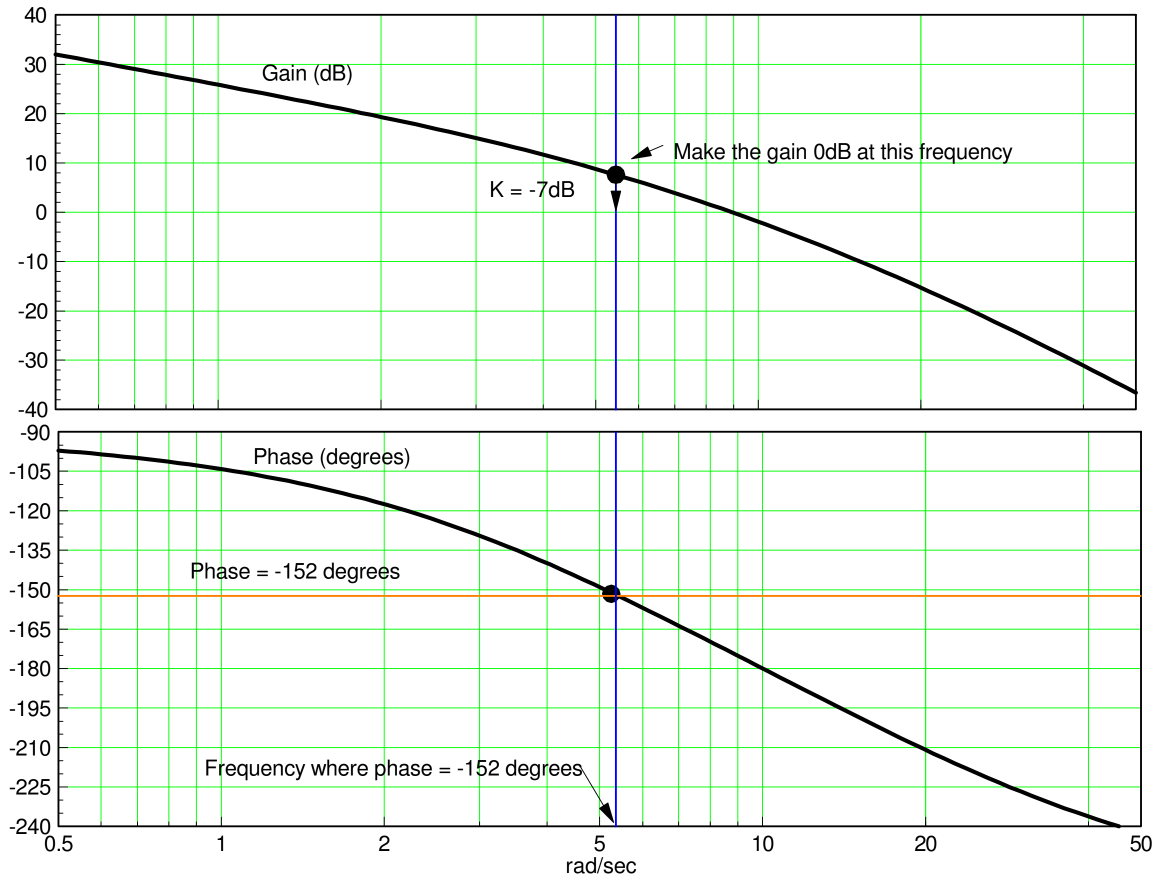
- G(s) isn't given

Step 1:

- Convert Mm to phase
- $\phi = -151.04^0$
- Phase Margin = 28.96^0

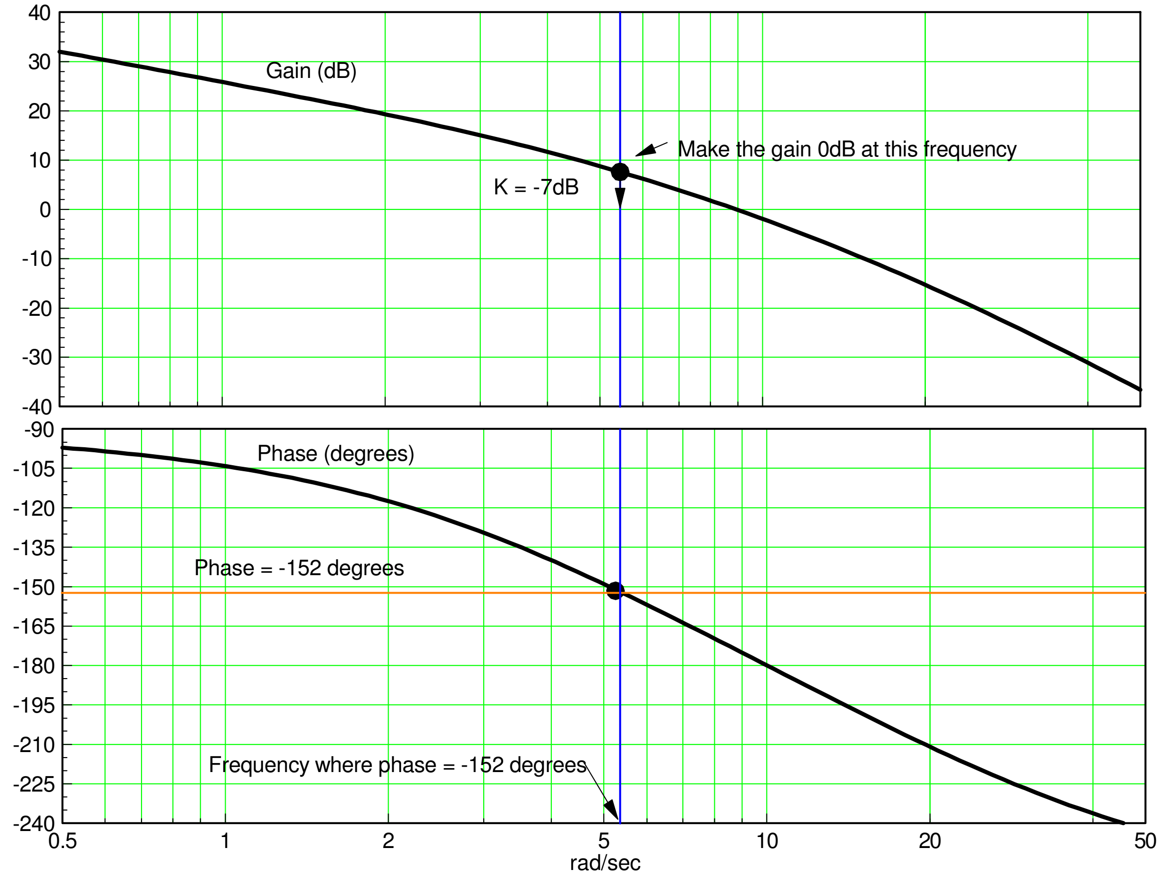
Step 2:

- Find the frequency where G(jw) has a phase shift of -152^0.
- w = 5.2 rad/sec



Step 3:

- Adjust k
- Gain = 1 when phase = -152 degrees
- G(j5.2) = 7dB
- k = - 7dB



Lead Compensator Design using Bode Plots

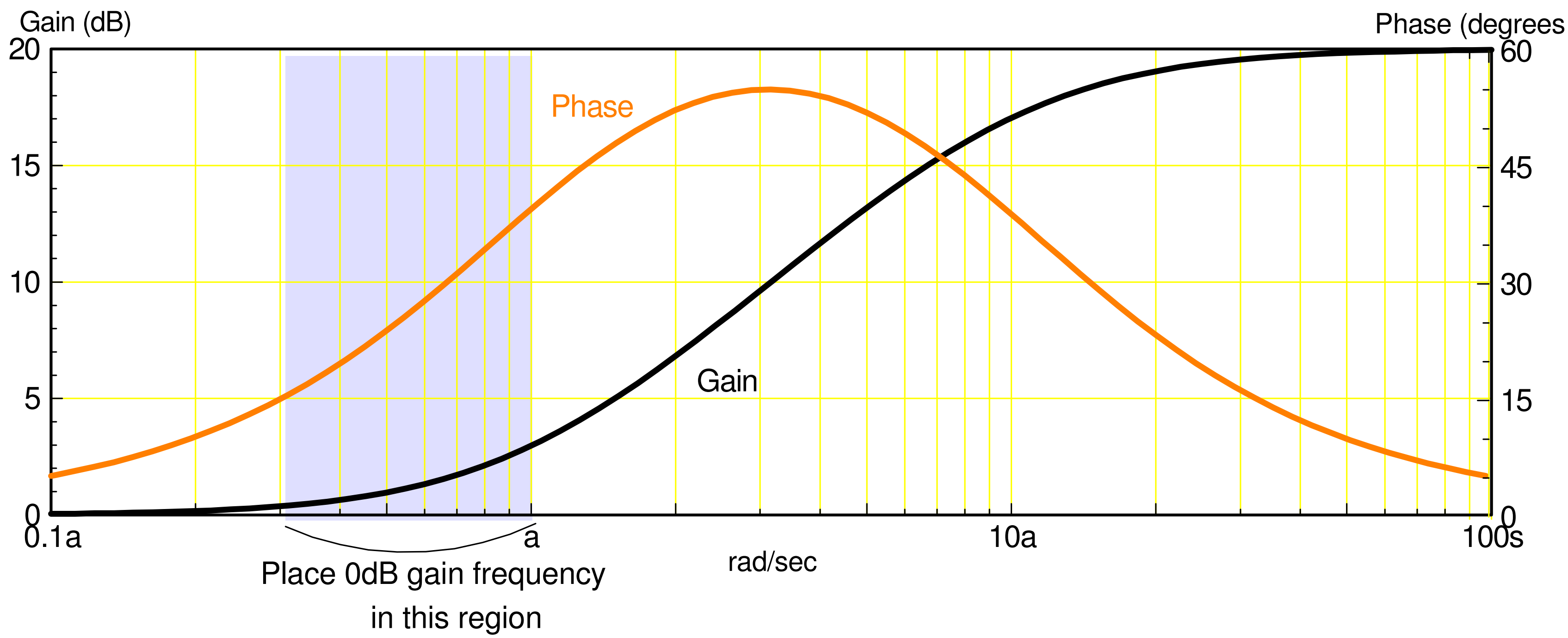
Purpose:

- Push G(jw) away from -1
- Add phase lead (good)
- Without too much gain (bad)
- At the resonance

    The frequency that's too close to -1

Generic Lead Compensator: $K(s) = 10\left(\frac{s+a}{s+10a}\right)$

- Adds phase lead (good: pushes you away from -1)
- Adds gain (bad: pushes you closer to -1)
- Rule: Place the zero 1..3 times the resonance frequency



Example: Design a lead compensator for a phase margin of 28.96 degrees (Mm = 2)

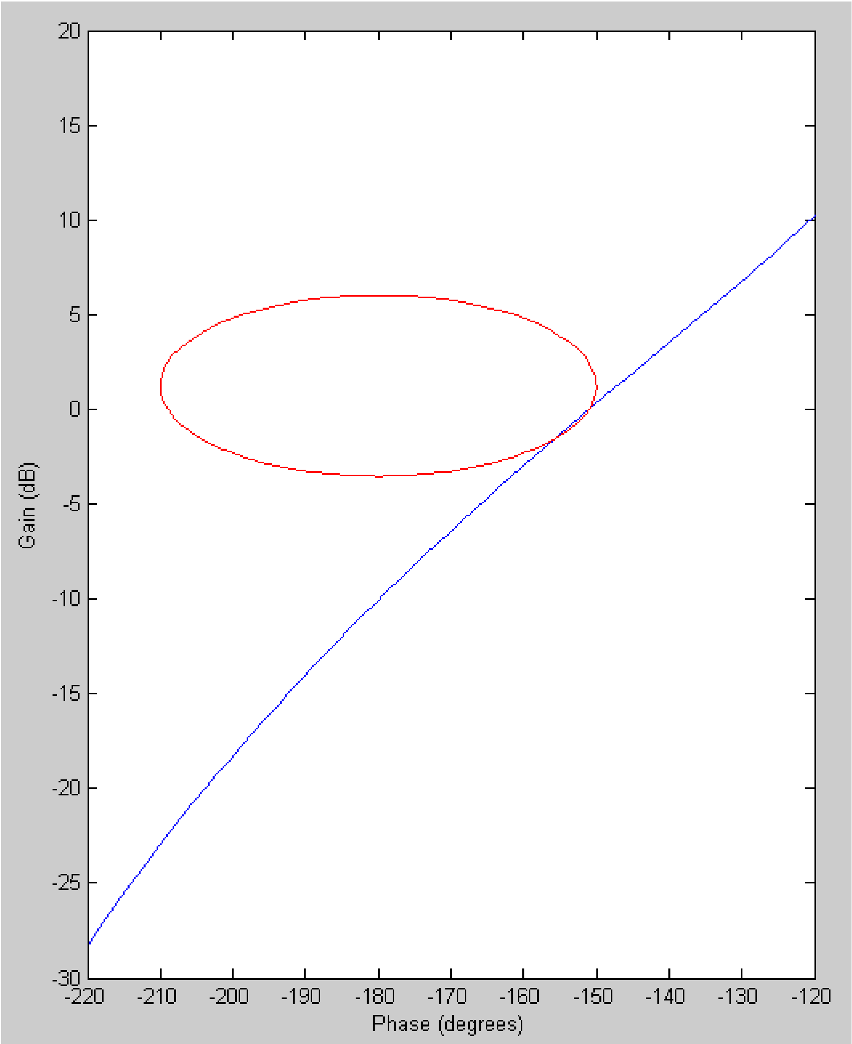
$$G(s) = \left(\frac{2000}{s(s+5)(s+20)}\right)$$

Step 1: Add gain until you're too close to -1 (phase margin = 28.96 degrees)

- K(s) = k = 0.3926
- Resonance = 5.24 rad/sec

```
w = logspace(-2,2,250)';
G = zpk([],[0,-5,-20],2000);
Gw = Bode2(G,w);

Nichols2(Gw*0.3926, 2);
```



Step 2: Add a lead compensator of the form

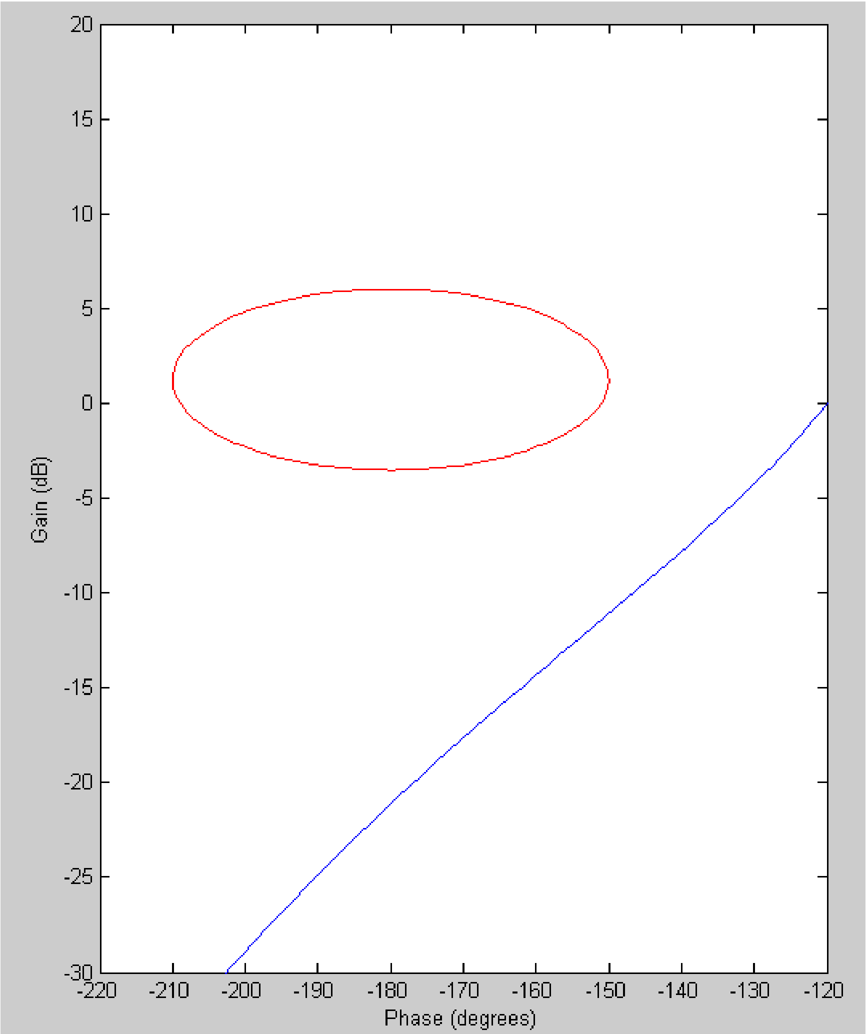
$$K(s) = 10\left(\frac{s+a}{s+10a}\right) \qquad a = (1..3)\cdot 5.24\ \frac{rad}{\sec}$$

Let a = 6

$$K(s) = k\left(\frac{s+6}{s+60}\right)$$

$$GK = \left(\frac{2000k(s+6)}{s(s+5)(s+20)(s+60)}\right)$$

```
K = zpk(-6,-60,10);
Kw = Bode2(K,w);
Nichols2(Gw.*Kw*0.3926, 2);
```

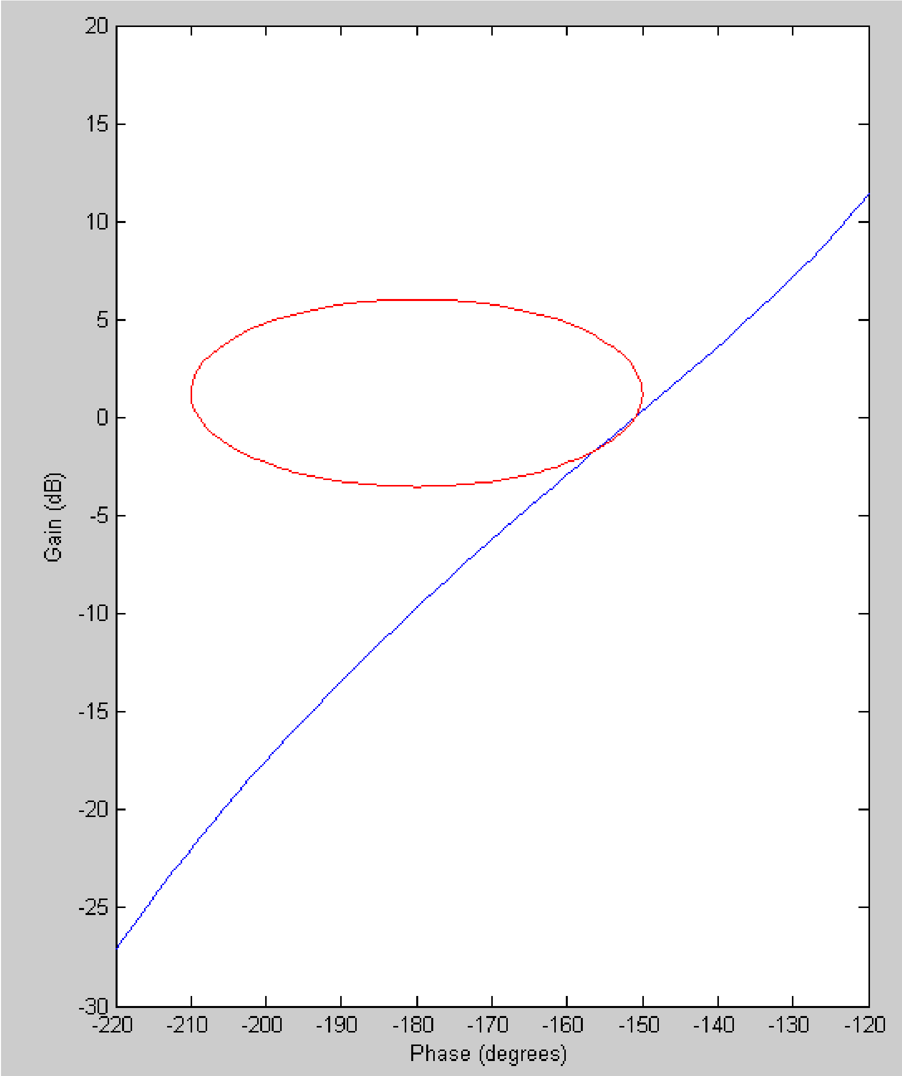


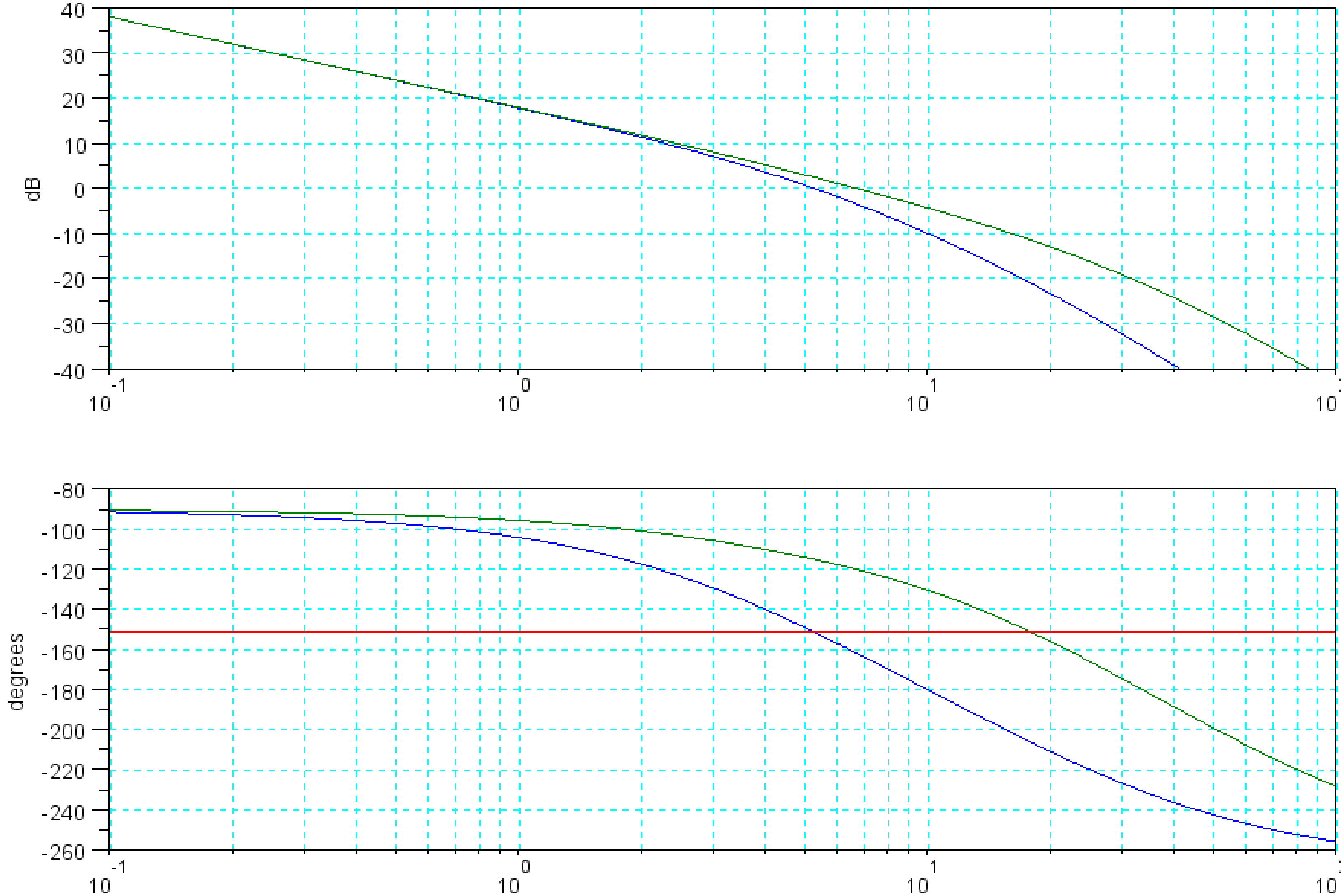
Note:

- K(s) pushes GK(jw) away from -1

Step 3: Add gain

- Phase margin = 29 degrees
- $K(s) = k\left(\frac{s+6}{s+60}\right)$
- $GK = \left(\frac{2000(s+6)}{s(s+5)(s+20)(s+60)}\right)$
- $GK(j17.76) = 0.0684\angle -151.04^{0}$
- $k = \frac{1}{0.0684} = 14.62$
- $K(s) = 14.62\left(\frac{s+6}{s+60}\right)$




40
30
20
10
0
-10
-20
-30
-40
dB
-80
-100
-120
-140
-160
-180
-200
-220
-240
-260
degrees
10^-1
10^0
10^1
10^2

Results:

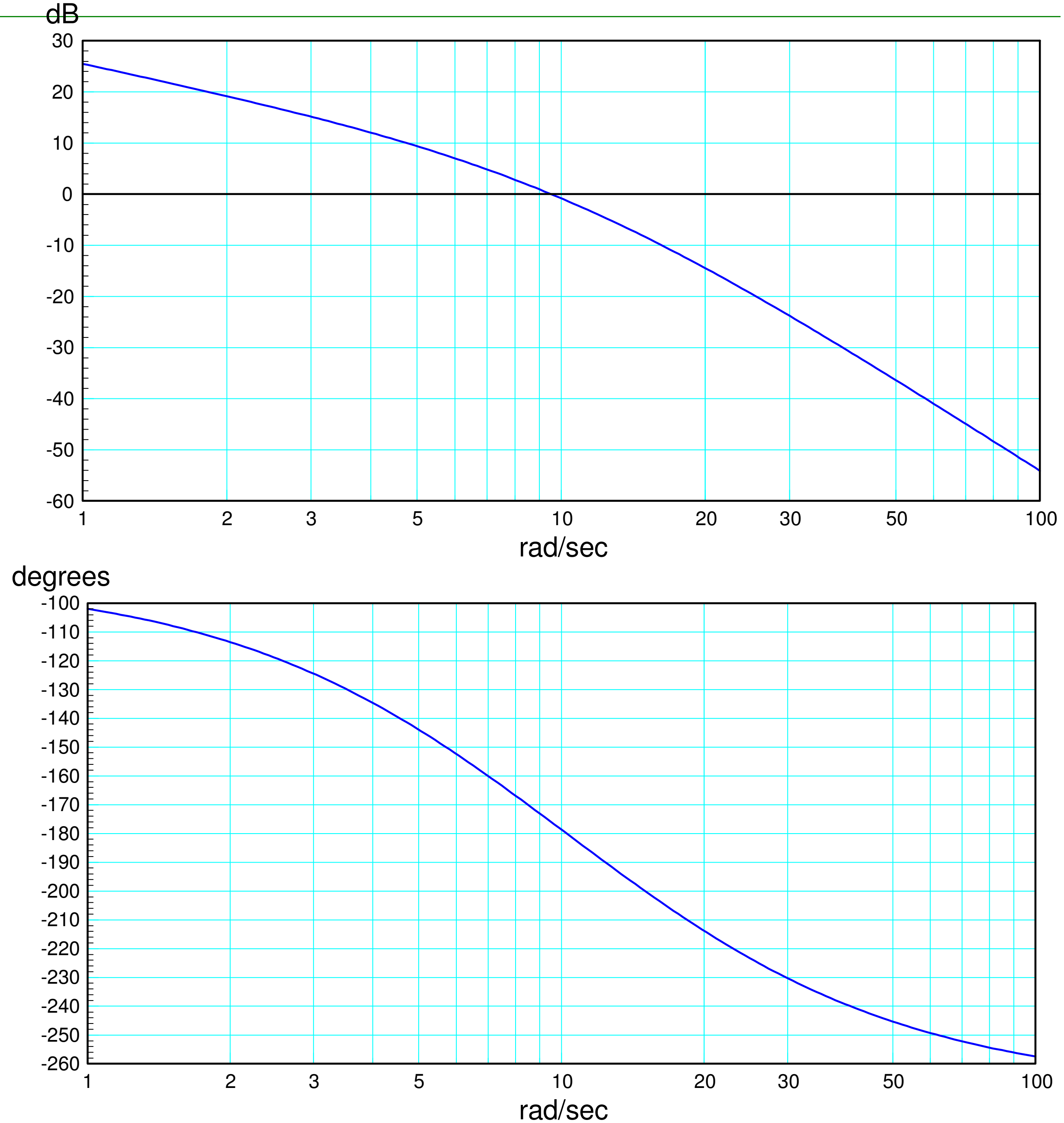
- Lead increases the phase margin
- Lead & Gain increases Kv and 0dB gain frequency

    Better tracking

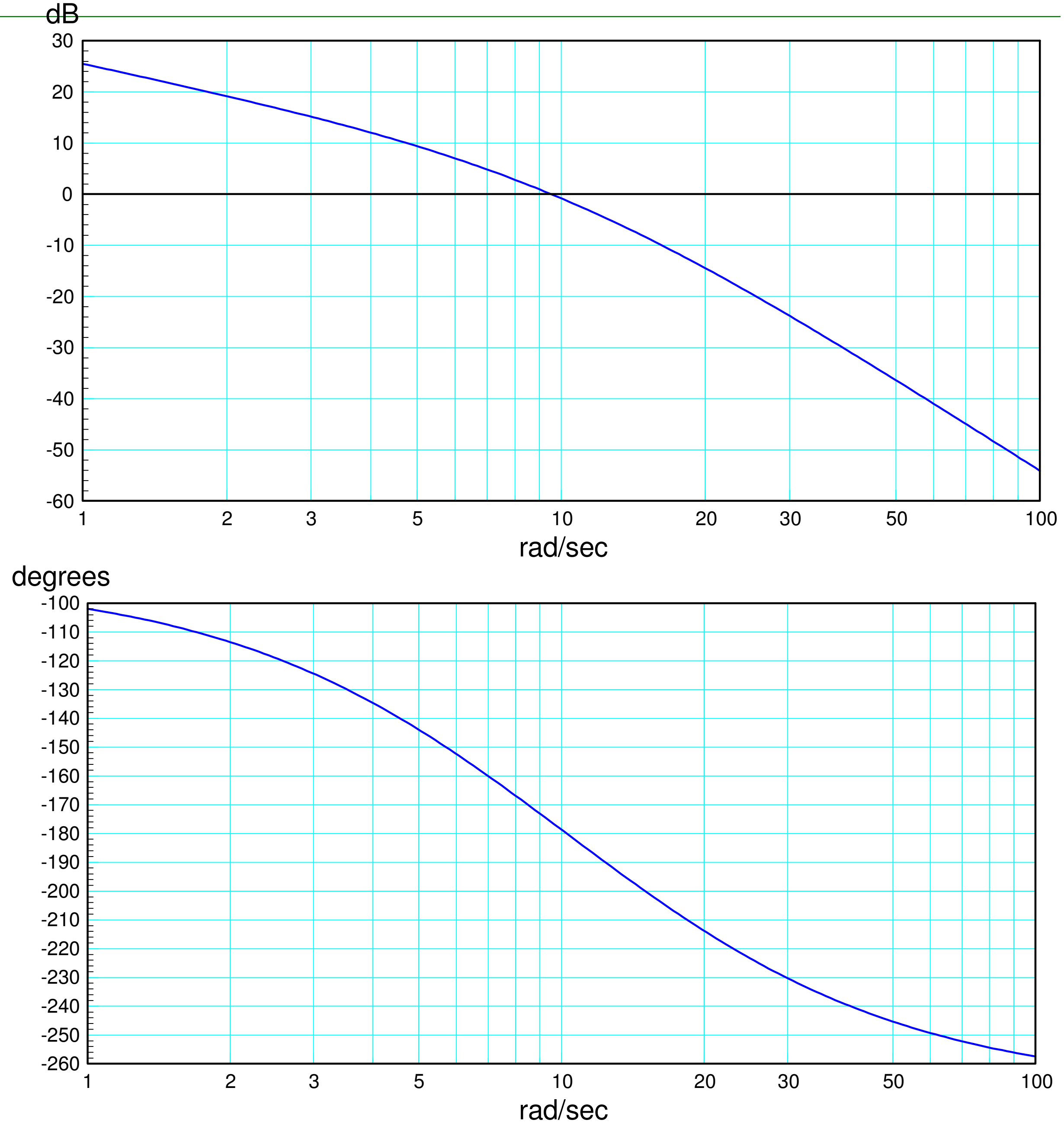
    Faster system

K(s)	Kv	0dB Gain Freq	Phase Margin	Mm	CL Dom Poles (approx)
0.3926	7.85	5.24 rad/sec	28.96 deg	2.00 (6dB)	-1.41 + j5.24
$;3.93\left(\frac{s+6}{s+60}\right)$	7.85	6.64 rad/sec	60.18 deg	1.00 (0dB)	-6.64 + j0
$;14.93\left(\frac{s+6}{s+60}\right)$	29.86	17.76 rad/sec	28.96 deg	2.00 (6dB)	-4.78 + j17.76

Handout: Design a gain compensator for a 50 degree phase margin



Handout: Design a lead compensator for a 50 degree phase margin



Lead Compensator Shortcut:

Given G(s)

$$G(s) = \left(\frac{2000}{s(s+5)(s+20)}\right)$$

Place the zero to cancel the 2nd slowest pole

- Same rule we used with root locus
- Same result we got using Nichols charts and Bode plots

5.2 < a < 15.6

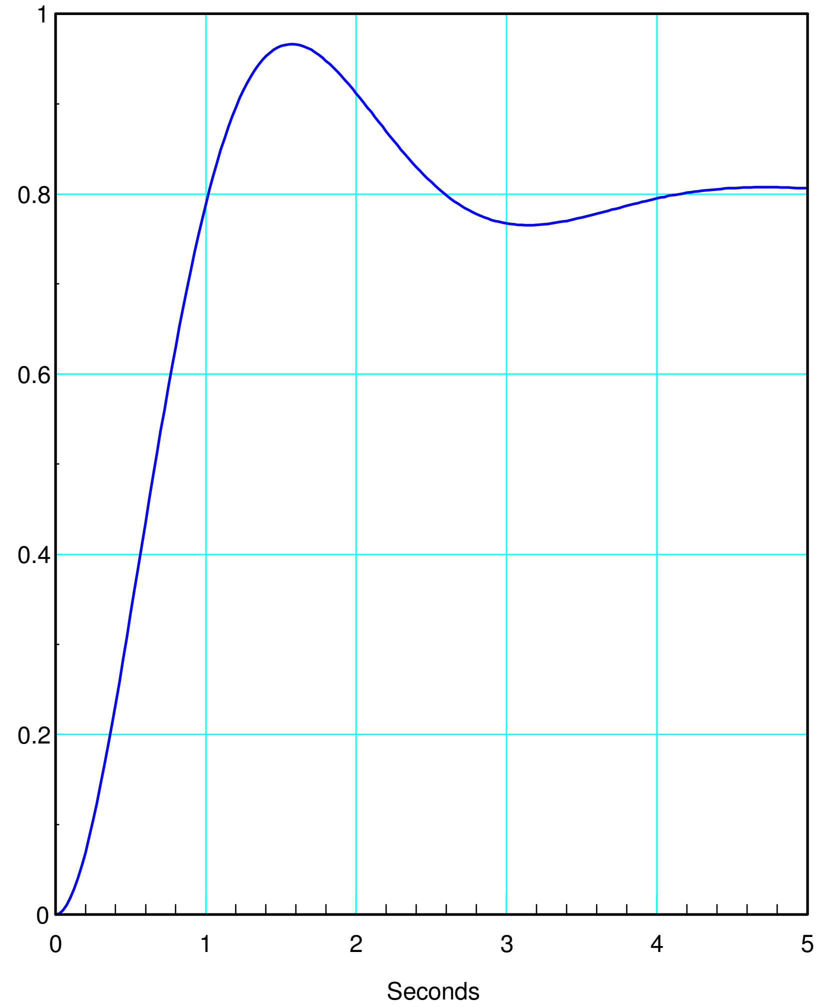
If you're going to miss the pole, miss on the high-side

$$K(s) = k\left(\frac{s+5}{s+50}\right)$$

Add gain to get the desired phase margin

Lead Compensator Design by Inspection

The following response is obtained with gain compensation: K(s) = k.



Design a lead compensator.

Root Locus:

- No idea where to start

Nichols Chart

- Y(jw) has a strong signal at 2 rad/sec
- The resonance is close to 2 rad/sec
- The phase margin is too small at 2 rad/sec
- Pick 'a' to be 1..3 x 2

$$K(s) = k\left(\frac{s+2}{s+20}\right)$$

Summary:

The purpose of a lead compensator is to

- Pull the root locus left (root locus techniques), or
- Add phase lead, moving G(jw) away from -1

Place the zero 1..3 times the frequency which is too close to -1

Add a pole 3-10 times the zero to add phase lead