Gain Compensation and Nichols Charts

ECE 461/661 Controls Systems

Jake Glower - Lecture #36

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

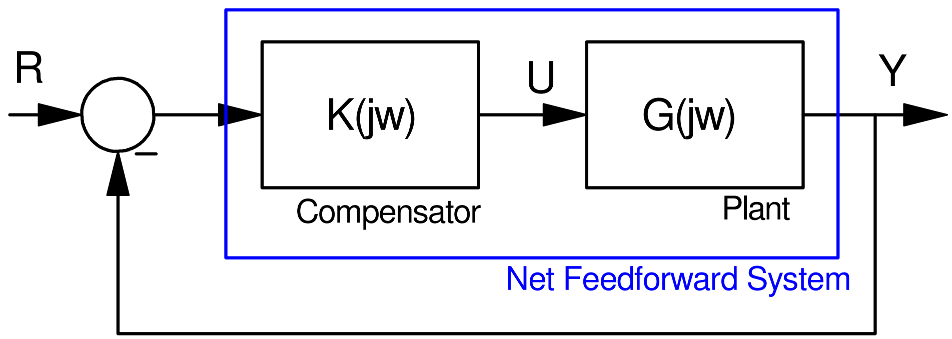
Stability & Feedback

Consider the following feedback system.

- If the net phase shift is 0 degrees, you have positive feedback
- If the loop gain at that frequency is more than one, the gain is infinity

$$Gain = a + a^2 + a^3 + ...$$

The loop gain must be less than one when the open-loop phase is 180 degrees



Gain Margin: How much you can increase the gain before the system just goes unstable.

Resonance: The maximum closed-loop gain

- A measure of the damping ratio

$$M_m = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad 0 < \zeta < 0.7$$

- A measure of the distance to -1

M-Circle:

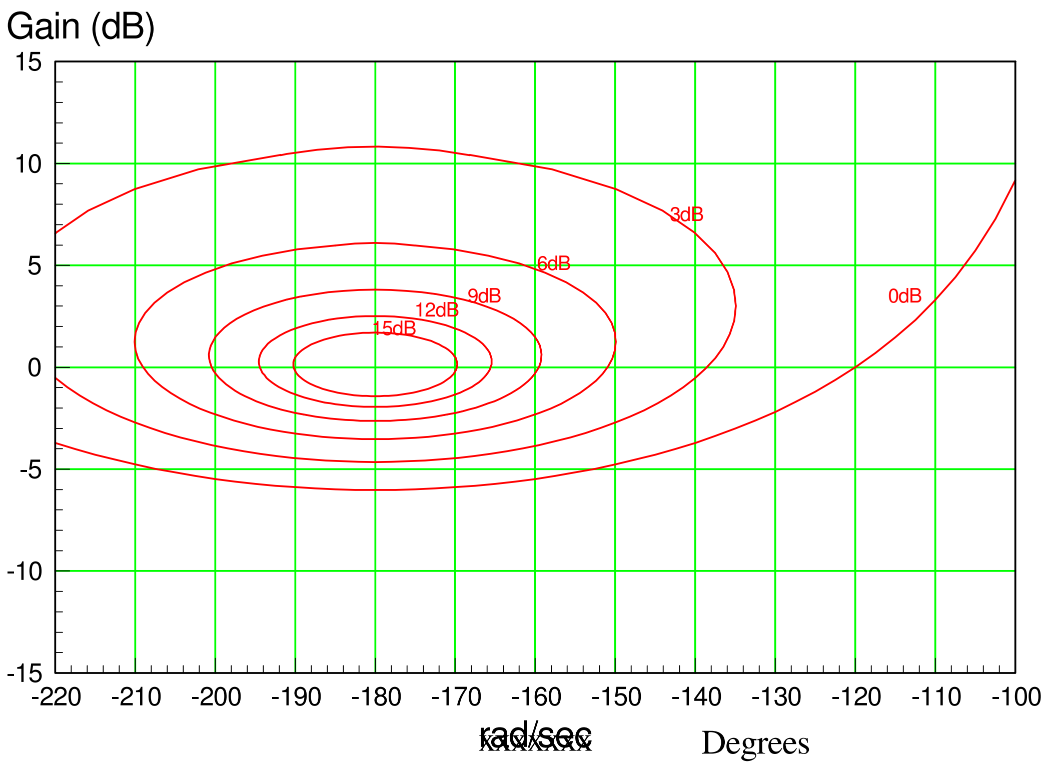
- All points where the closed-loop gain is conatant: Mm
- "Keep Away" region if the resonance must be < Mm

Nichols Charts:

Plots G(jw) as

- X-axis: Phase in degrees
- Y-axis: Gain in dB

M-circles show the corresponding closed-loop gain



Nichols Charts and Gain Compensation

Adjust the gain of G(jw) until

- You are stable

  Gain < 0dB when the phase is 180 degrees

- You are tangent to the M-circle

  Maximum closed-loop gain = resonance = Mm

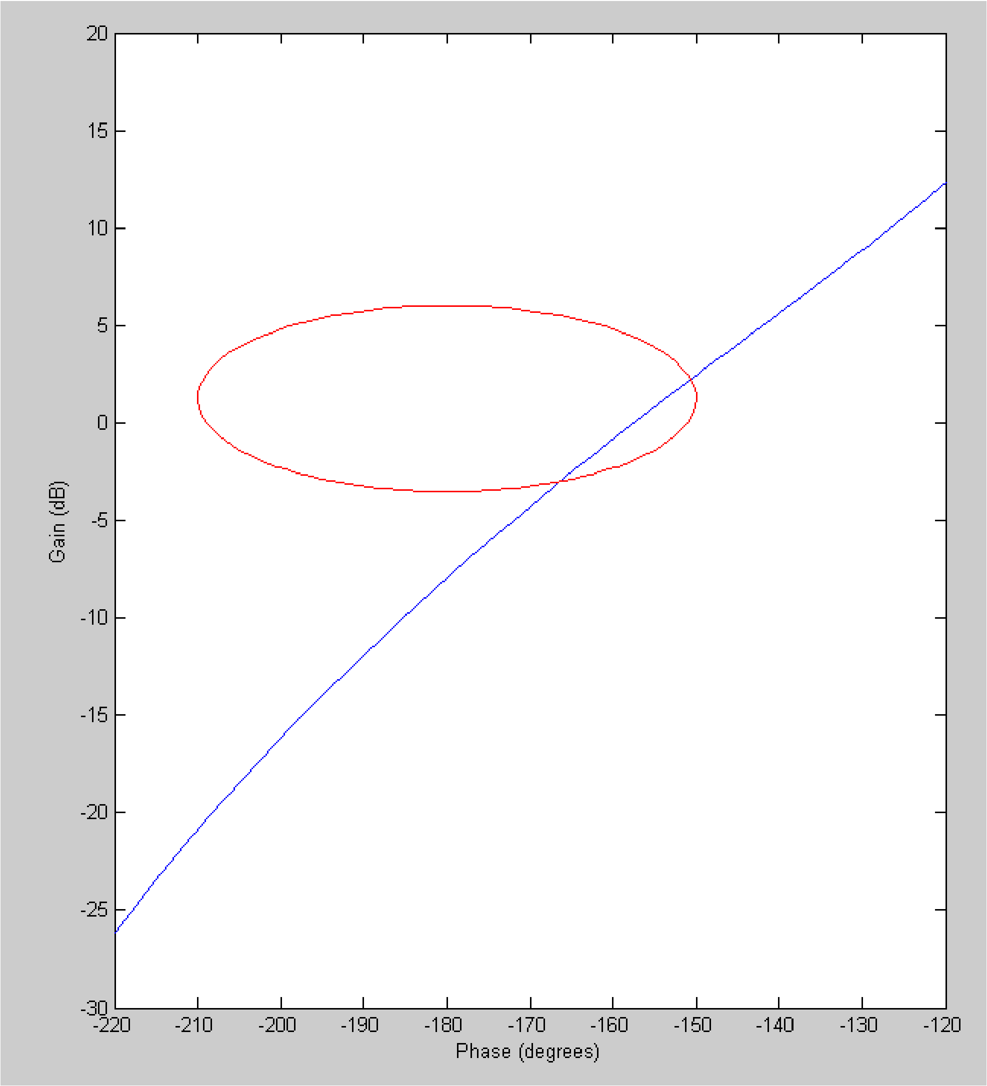
```
function [] = Nichols2(Gw, Mm)
   Gwp = unwrap(angle(Gw))*180/pi;
   Gwm = 20*log10(abs(Gw));
% M-Circle
   phase = [0:0.01:1]' * 2*pi;
   Mcl = Mm * exp(j*phase);
   Mol = Mcl ./ (1 - Mcl);
   Mp = unwrap(angle(Mol))*180/pi - 360;
   Mm = 20*log10(abs(Mol));
   plot(Gwp,Gwm,'b',Mp,Mm,'r');
   xlabel('Phase (degrees)');
   ylabel('Gain (dB)');
   xlim([-220,-120]);
   ylim([-30,20]);
   end
```

Example: Find a gain, k, so that

$$G(s) = \left(\frac{2000}{s(s+5)(s+20)}\right)$$

- Has a resonance of 6dB or less, and
- K is as large as possible.,

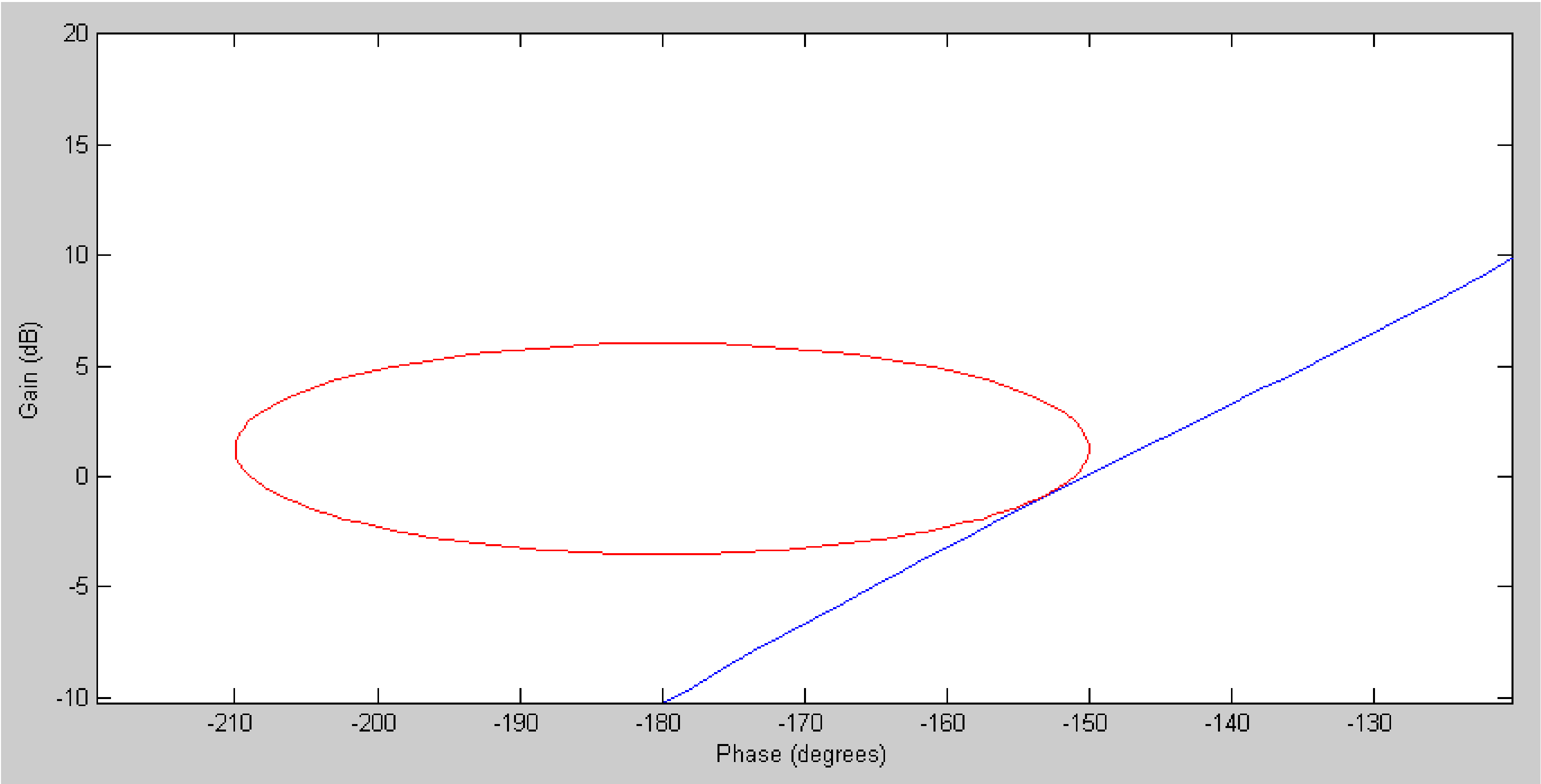
```
G = zpk([],[0,-5,-20],1000);
w = logspace(-2,2,200)';
Gw = Bode2(G,w);

Nichols2(Gw, 2);
```



Adjust the gain until you are tangent

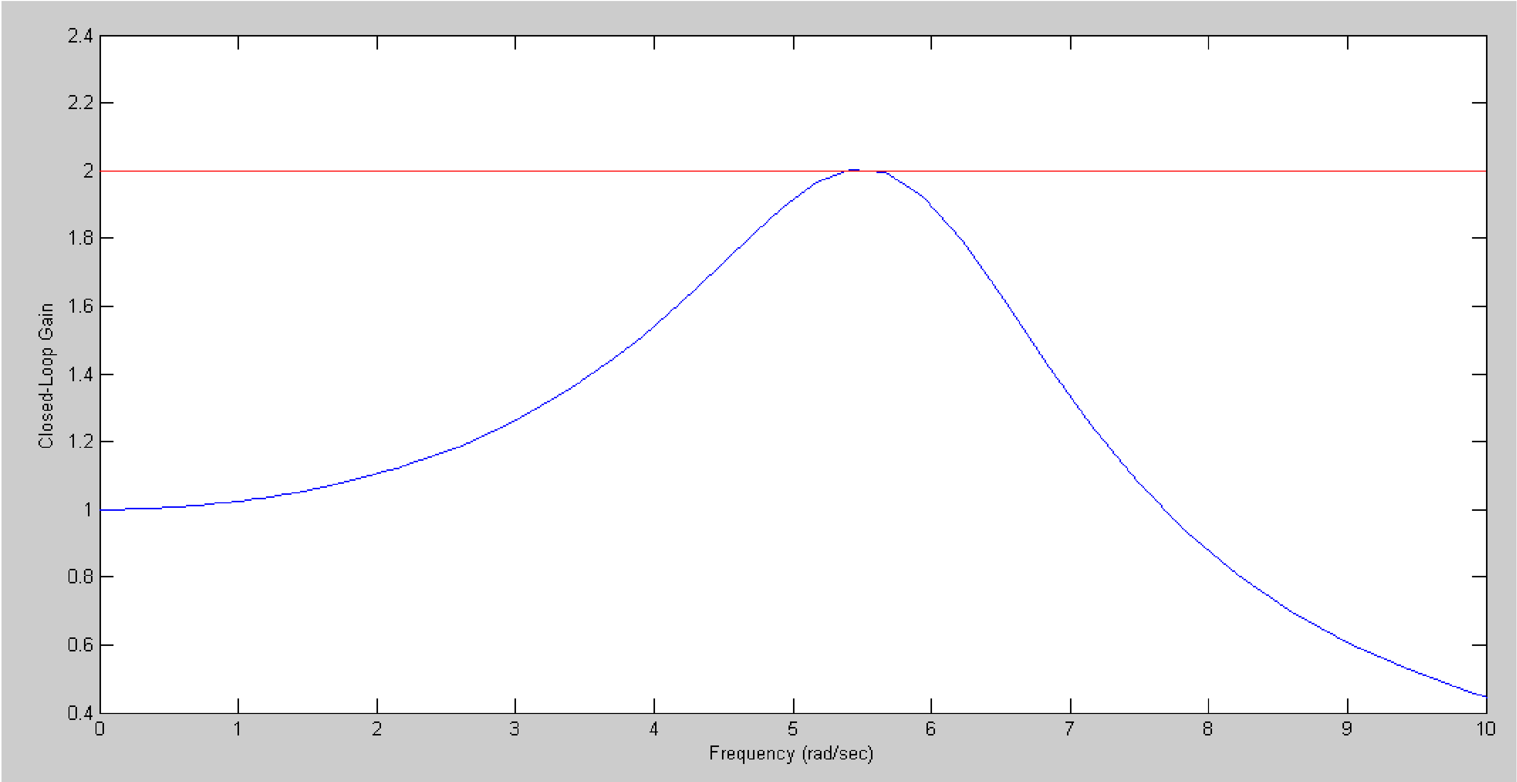
- and stable

```
Nichols2(Gw*0.9, 2);
Nichols2(Gw*0.8, 2);
Nichols2(Gw*0.78, 2);
Nichols2(Gw*0.77, 2);
```



You can check your answer by plotting the closed-loop gain with this value of K:

```
k = 0.77;
Gw_cl = Gw*k ./ (1 + Gw*k) ;
plot(w,abs(Gw_cl));
xlabel('Frequency (rad/sec)');
ylabel('Closed-Loop Gain');
```



Closed-Loop Gain of G*K. K was choses so that the maximum gain was 2.000

Gain Compensation When G(s) Isn't Specified

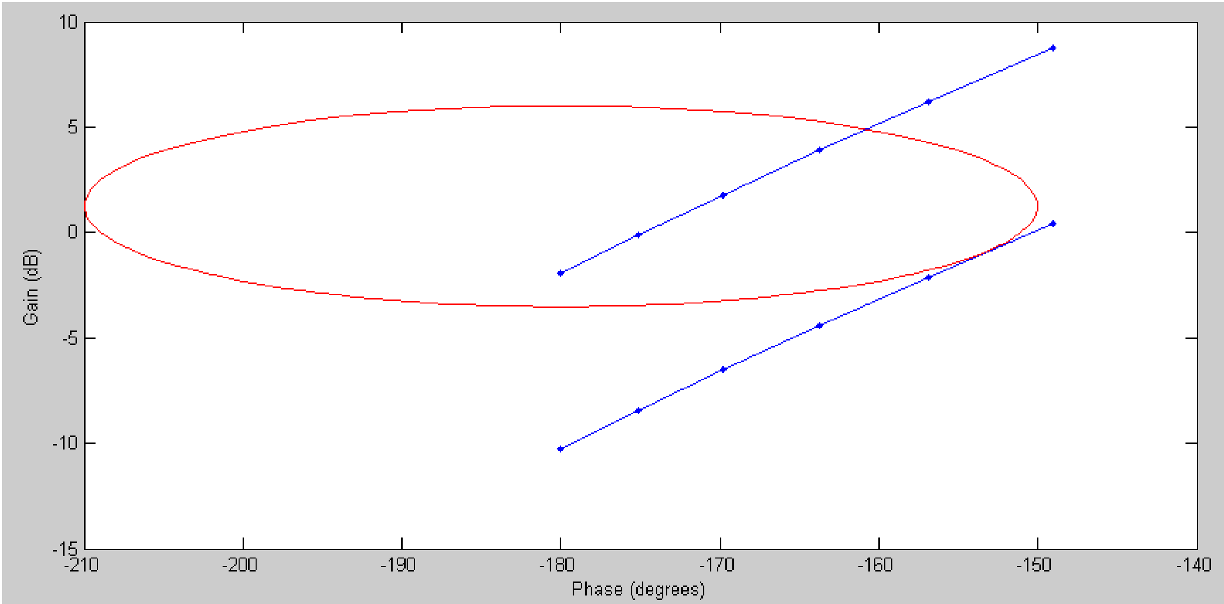
Given

Freq	5	6	7	8	9	10
Gain (dB)	8.77	6.21	3.9	1.8	-0.14	-1.94
Phase (deg)	-149.04	-156.89	-163.75	-169.8	-175.17	-180

Find k for Mm = 2

```
dB = [ ... ]';
P  = [ ... ]';
Gw = 10.^(dB/20) .*
exp(j*pi*P/180);
Nichols2(Gw, 2);
Nichols2(Gw*0.3, 2);
Nichols2(Gw*0.35, 2);
Nichols2(Gw*0.38, 2);
```

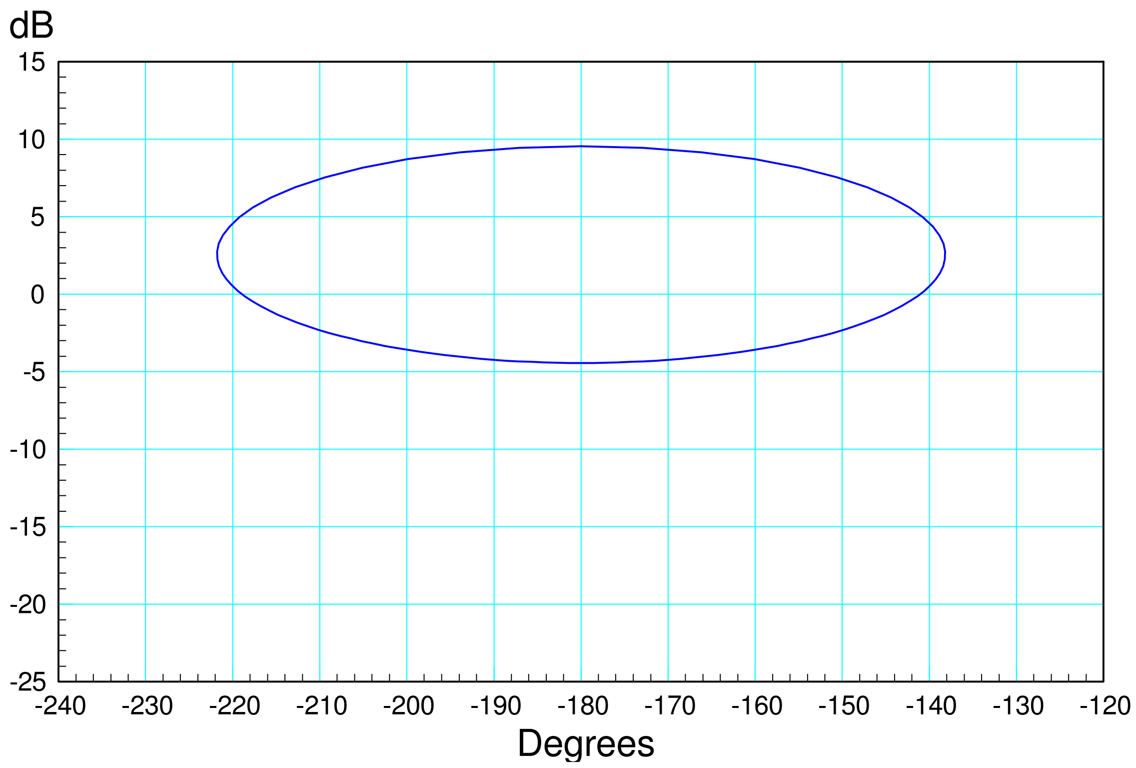
This results in k = 0.38



Handout: The gain vs. frequency for G(jw) is measured:

rad/sec	0	5	6	7	8
Gain	15dB	10dB	0dB	-10dB	-20dB
degrees	0	-120	-150	-170	-190

Determine Gain of M-Circle. Maximum Gain for Stability, k for Mm



Nyquist Diagram

A Nyquist diagram is similar to a Nichols chart. In this case, you plot

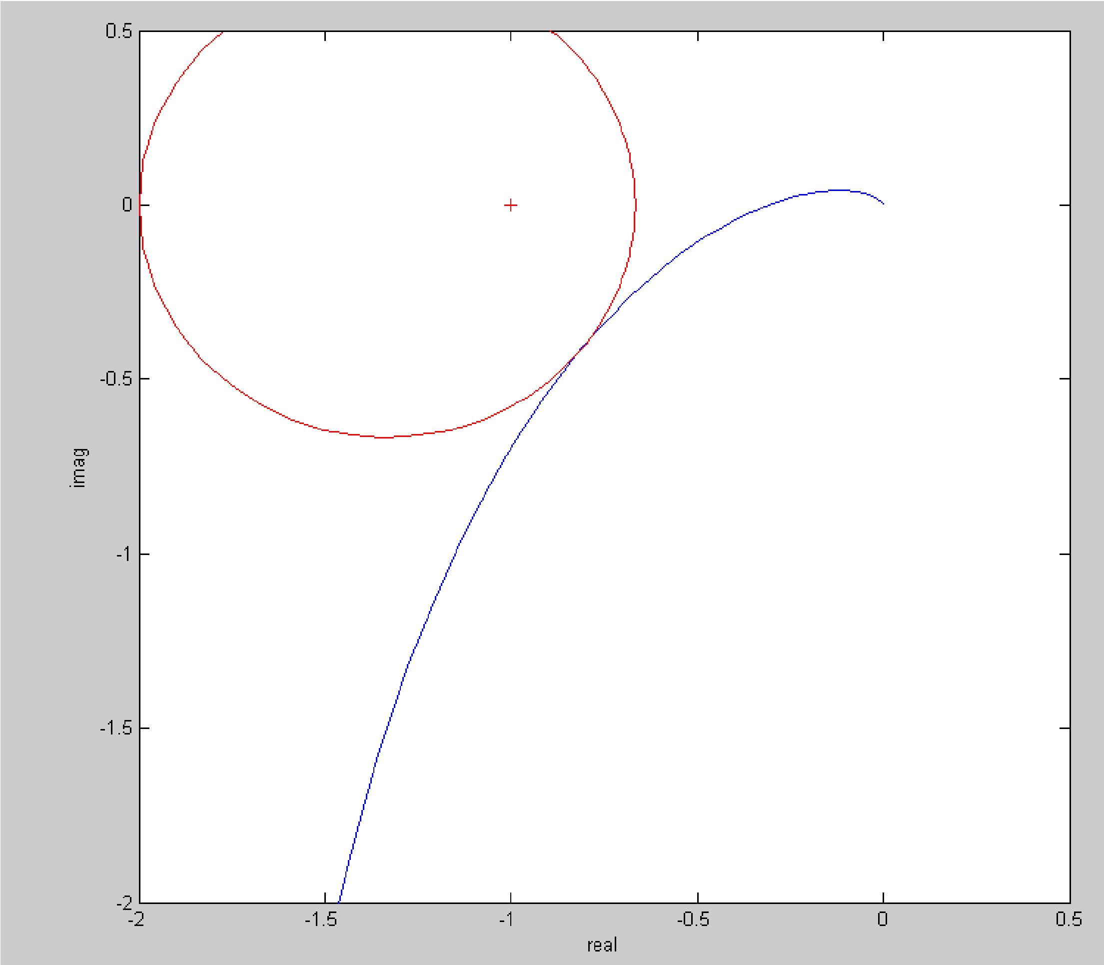
- x-axis: real(G(jw))
- y-axis: imag(G(jw))
- m-circles: constant closed-loop gain

```
function [] = Nyquist2(Gw, Mm)
   Gwp = unwrap(angle(Gw))*180/pi;
   Gwm = 20*log10(abs(Gw));
% M-Circle
   phase = [0:0.01:1]' * 2*pi;
   Mcl = Mm * exp(j*phase);
   Mol = Mcl ./ (1 - Mcl);

   plot(real(Gw), imag(Gw), 'b', real(Mol), imag(Mol),'r',-1,0,'r+');
   xlabel('real');
   ylabel('imag');
   xlim([-2,0.5]);
   ylim([-2,0.5]);
   end
```

Nyquist

Calling sequence

```
w = logspace(-2,2,200)';
G = zpk([],[0,-5,-20],1000);
Gw = Bode2(G,w);
Nyquist2(Gw, 2);
Nyquist2(Gw*0.9, 2);
Nyquist2(Gw*0.77, 2);
```



Note

- The M-circle is a circle

  hence the name
- The circle isn't centered on -1

Inverse Nyquist Diagram

Plot

- x-axis: real(1/G(jw))
- y-axis: imag(1/G(jw))
- m-circles: constant closed-loop gain

```
function [] = InverseNyquist2(Gw, Mm)
   Gwp = unwrap(angle(Gw))*180/pi;
   Gwm = 20*log10(abs(Gw));
% M-Circle
   phase = [0:0.01:1]' * 2*pi;
   Mcl = Mm * exp(j*phase);
   Mol = Mcl ./ (1 - Mcl);

   plot(real(1./Gw), imag(1./Gw), 'b', real(1./Mol),
imag(1./Mol),'r',-1,0,'r+');
   xlabel('real');
   ylabel('imag');
   xlim([-2,0.5]);
   ylim([-1,1.5]);
   end
```

Inverse Nyquist

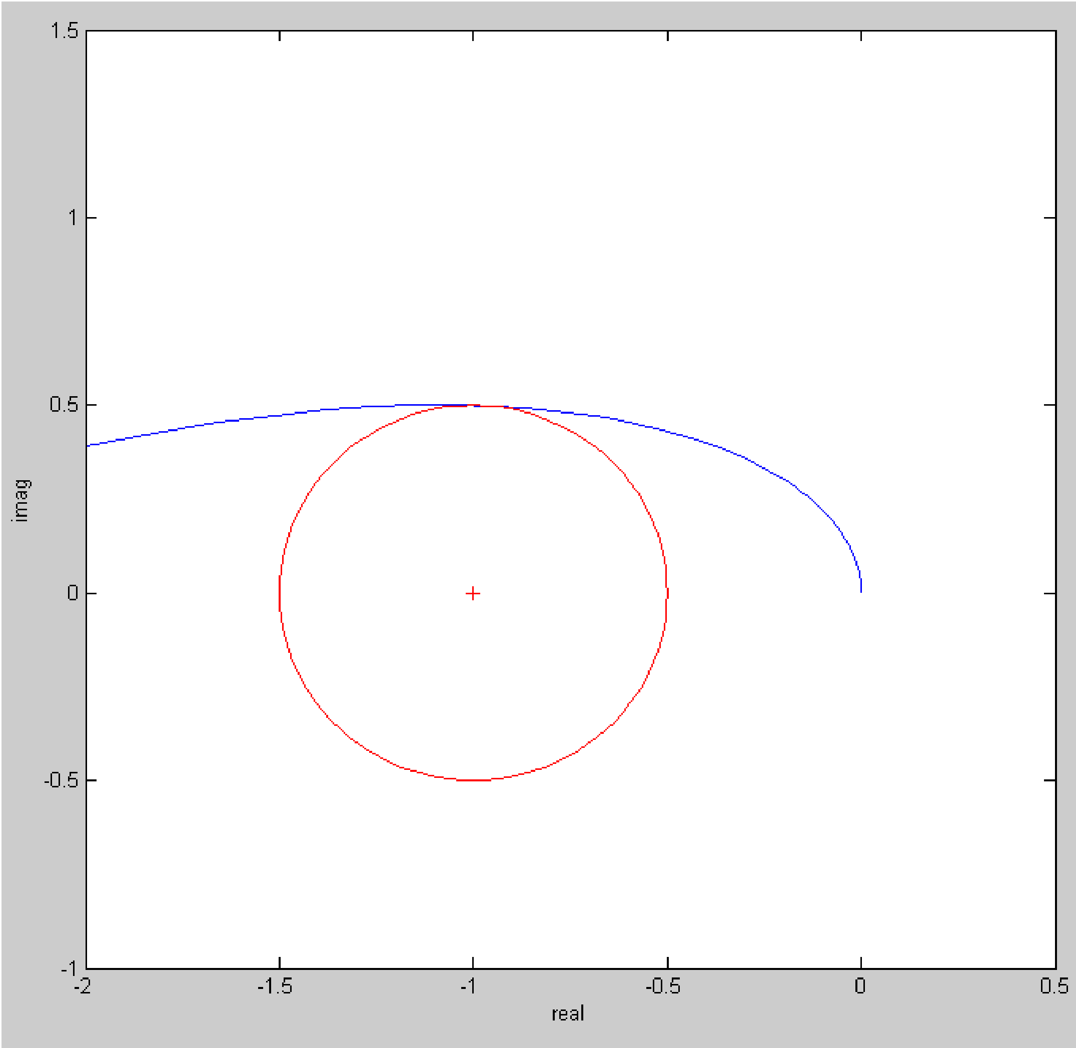
The calling sequence is

```
w = logspace(-2,2,200)';
G = zpk([],[0,-5,-20],1000);
Gw = Bode2(G,w);
InverseNyquist2(Gw, 2);
InverseNyquist2(Gw*0.9, 2);
InverseNyquist2(Gw*0.77, 2);
```

Note:

- M-circles are circles
- Centered on -1

Mm = 1/distance to -1



Summary:

The goal in gain compensation is to keep away from -1

Different charts map this distance using m-circles

- Nichols: phase vs. gain in dB
- Nyquist: real vs. imag for G(jw)
- Inverse Nyquist: real vs. imag for 1/G(jw)

Gain Compensation:

- Crank up the gain until you are tangent to the m-circle,
- And stable