Bode Plots

ECE 461/661 Controls Systems

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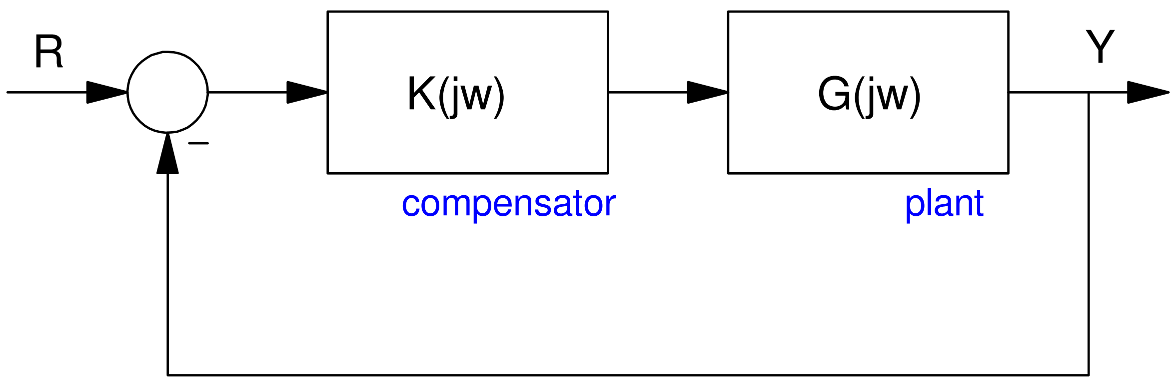
Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Frequency Domain Techniques

Another way to look at a feedback system

- G(jw) is a filter with a gain and a phase shift
- K(jw) is a pre-filter which warps the frequency response
- This turns a poor system into a good system



Frequency domain: Another way of looking at a feedback system

Frequency Domain Origins

- Hedrik Bode: 1938 (Bode Plots)
- Harry Nyquist: 1917 - 1954 Bell Labs
- Nathaniel Nichols: 1940's MIT

Problem: Design feedback amplfiers and anti-aircraft guns for WWII

Sometimes, the feedback made a very good amplifier. Sometimes, the amplifier would just squawk and ring. The idea behind K(jw) was to add a pre-filter to make a bad amplifier behave like a good amplifier.

Vacuum Tube Amplifier (Amazon.com, where else?)

Which technique is Better?

- Students who learn Root Locus first tend to prefer that method
- Students who learn frequency-domain techniques first tend to prefer these methods

They both work.

Some systems are easier to analyze using root locus

Some systems are easier to analyze using frequency domain techniques

These are toos for you to use.

- Use the one that works best for you.

Types of Frequency-Domain Plots

How to plot 3 variables:

- The frequency, w
- The gain, |G(jw|, and
- The phase shift: $\angle G(j\omega)$

Name	X-Axis	Y-Axis
Bode Plot	Frequency on a log scale	Gain in dB
Nichols Chart	Phase in degrees	Gain in dB
Nyquist Diagram	real(G(jw))	imag(G(jw))
Inverse Nyquist Diagram	real(1/G(jw))	imag(1/G(jw))

Bode Plots

When we first started out in this class, we looked at

- Finding the step response given the transfer function, G(s), and
- Finding the transfer function, G(s), given the step response.

In this lecture, we look at

- Finding the frequency response given the transfer function, G(s), and
- Finding G(s) given the frequency response.In this lecture, we're looking at Bode Plots.

Definitions

$$gain = |G(j\omega)|$$

$$dB = 20 \log_{10}(gain)$$

$$gain = 10^{dB/20}$$

Determine the Frequency Response given G(s)

- Numerical Methods (Matlab)
- Graphical Methods

Probably easiest to explain through an example:

Problem: Draw the Bode plot for

$$G(s) = \left(\frac{10s(s+100)}{(s+1)(s+10)}\right)$$

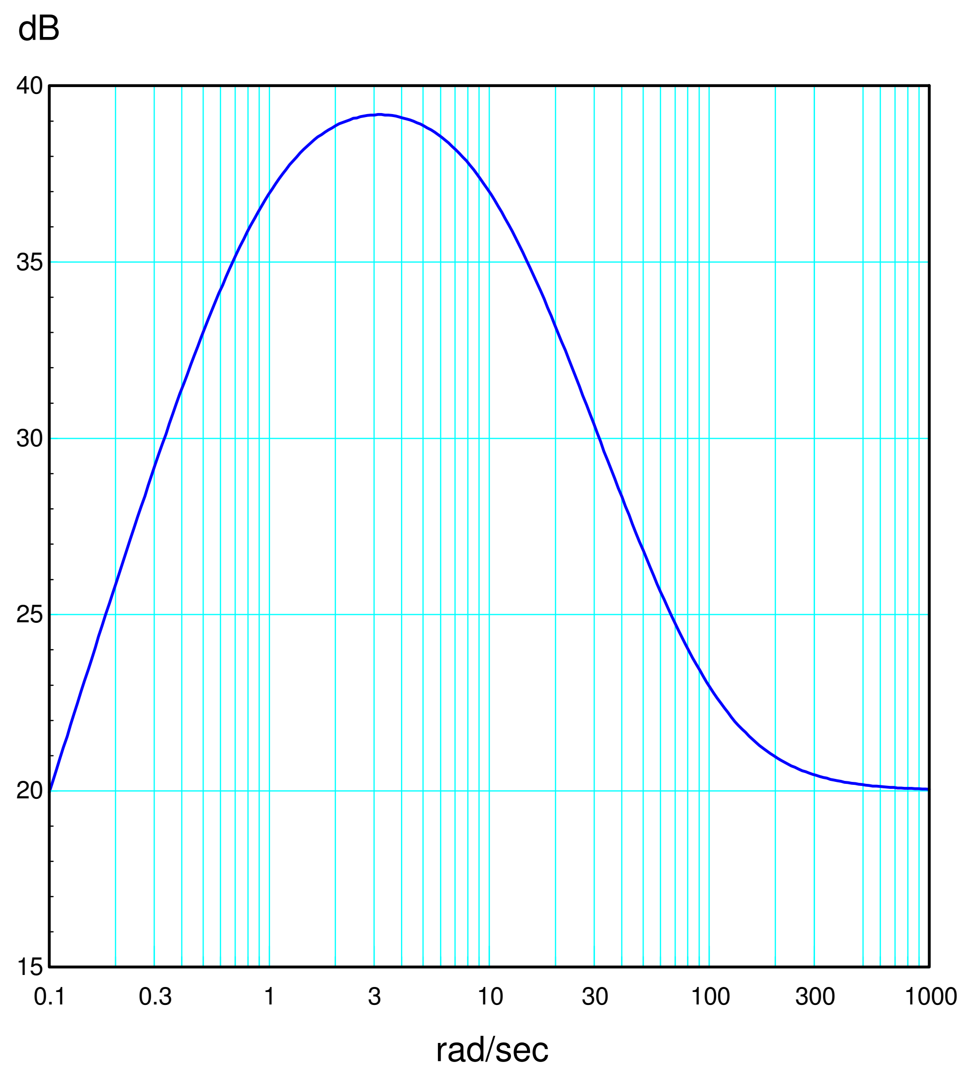
Numerical Solution (Matlab):

Substitute

$$G(s) = \left(\frac{10s(s+100)}{(s+1)(s+10)} \right)_{s \to j\omega}$$

Calculate the gain and plot

- X axis: frequency on a log scale
- Y axis: gain in dB



In Matlab:

```
w = logspace(-1,3,250)';
s = j*w;
Gs = 10*s.*(s+100) ./ ( (s+1) .*
(s+10) );
dB = 20*log10(abs(Gs));
semilogx(w,dB);
```

Matlab Function: Bode2

Bode plots are really useful, so you can also write a Matlab m-file to do this

```
function [Gw ] = Bode2( G, w )
   Gw = 0*w;
   for i=1:length(w)
      Gw(i) = evalfr(G, j*w(i));
      end
   GdB = 20*log10(abs(Gw));
   semilogx(w, GdB);
   end
```

This is called as

```
G = zpk([0,-100],[-1,-10],10);
w = logspace(-1,3,250)';
Gw = Bode2(G,w);
semilogx(w, 20*log10(abs(Gw)));
```

Graphical Solution:

- Works better when using a slide rule
- Offers better insight

First, note that the function

$$G(s) = s^n$$

plots as a straight line. The gain is

$$dB = 20 \log 10(|(j\omega)^n|)$$

$$= 20n \log 10(\omega)$$

which is a slope of 20ndB/decade

example: n = -2

$$G(s) = \frac{1}{s^2}$$

Next, approximate

$$(s+a) \approx \begin{cases} a & |s| < |a| \\ s & |s| > |a| \end{cases}$$

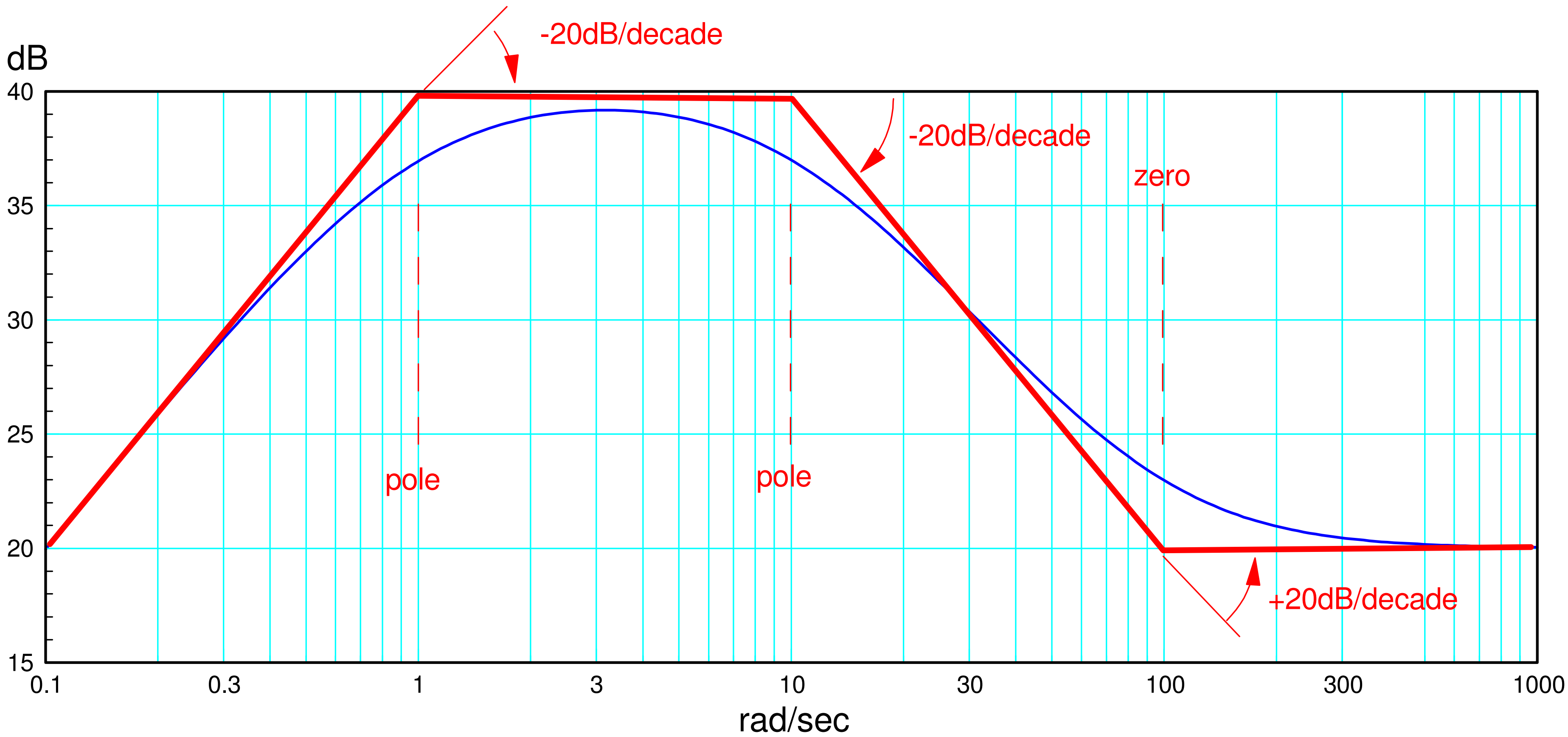
This gives

$$G(s) = \left(\frac{10s(s+100)}{(s+1)(s+10)}\right) \approx \begin{cases} \left(\frac{10s(100)}{(1)(10)}\right) = 100s & |s| < 1 \\ \left(\frac{10s(100)}{s(10)}\right) = 100 & 1 < |s| < 10 \\ \left(\frac{10s(100)}{(s)(s)}\right) = \frac{1000}{s} & 10 < |s| < 100 \\ \left(\frac{10s(s)}{(s)(s)}\right) = 10 \quad 10 & 100 < |s| \end{cases}$$

Note: At the corners, the gain is

- Down 3dB for a pole
- Up 3dB for a zero
- 3dB = $\sqrt{2}$

$$G(s) = \left(\frac{10s(s+100)}{(s+1)(s+10)} \right)$$



Graphical approximation for the gain vs. frequency (red line) vs. actual gain vs. frequency (blue line)

Bode Plots with Complex Poles

Complex poles come in pairs

- There are two poles with the same magnitude,
- The slope changes by -40dB/decade

The gain at the corner tells you ζ

$$G(s) = \left| \frac{1}{s^2+2\zeta s+1} \right|_{s=j} = \frac{1}{2\zeta}$$

For complex poles, the gain at the corner is $\frac{1}{2\zeta}$

Complex Pole Example

$$G(s) = \left(\frac{1000s^2}{\left(s^2+1.4s+1\right)\left(s^2+5s+100\right)} \right)$$

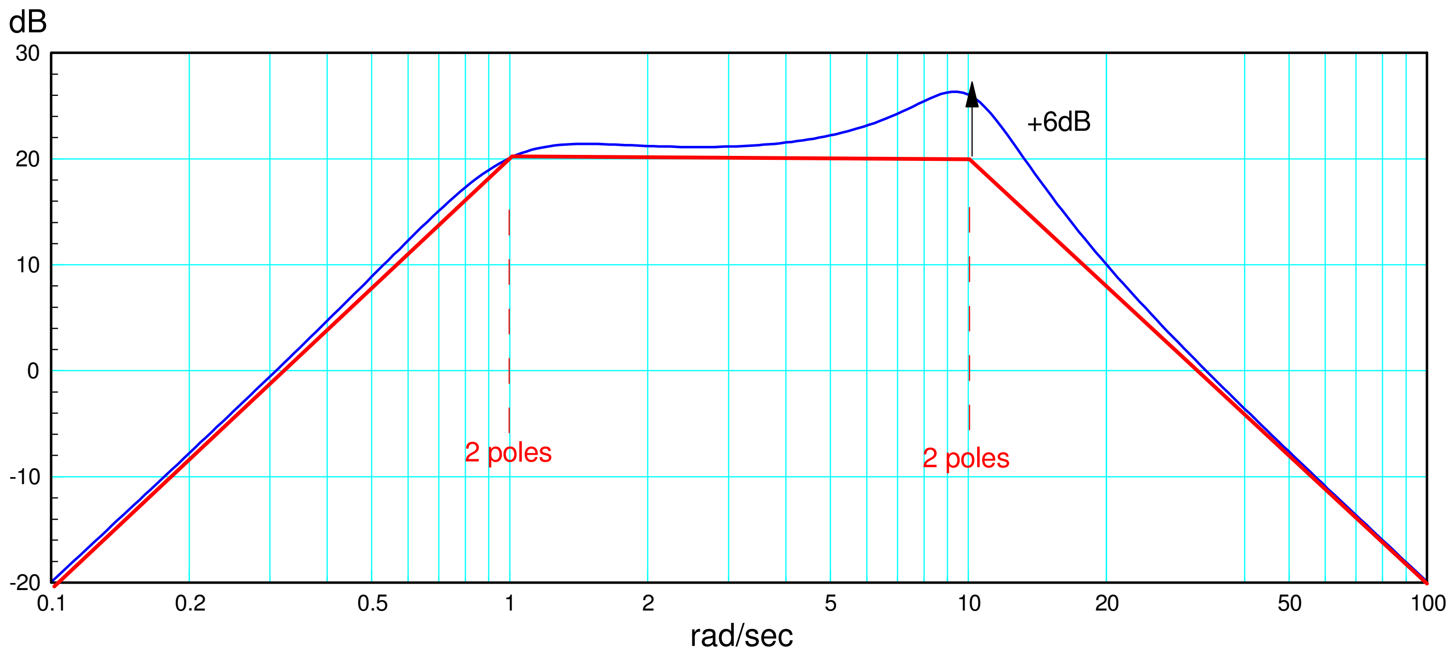
At 1 rad/sec

- ζ=0.5
- Gain at corner = 0dB above the corner

At 10 rad/sec

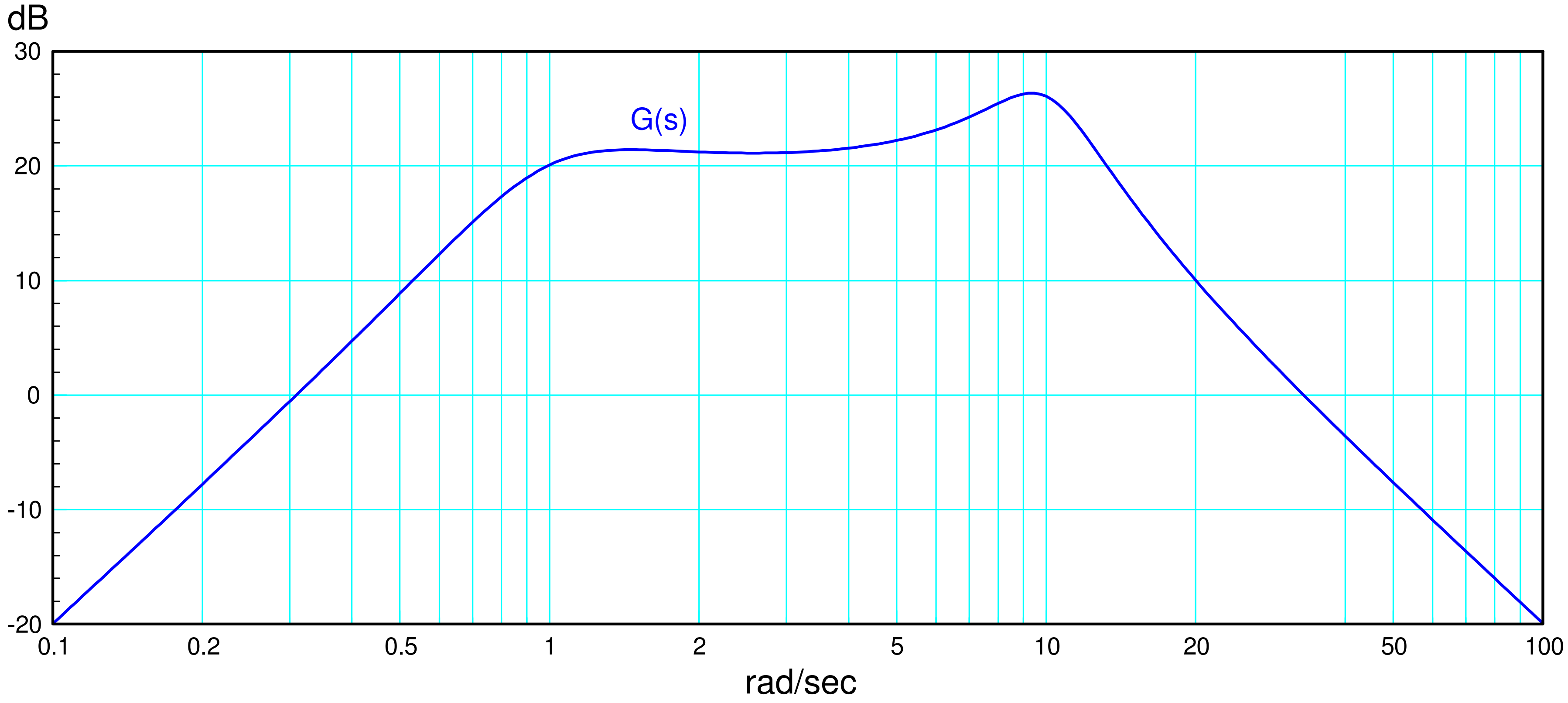
$\zeta = 0.25$

Gain at corner = +6dB



Determining G(s) from its Bode Plot

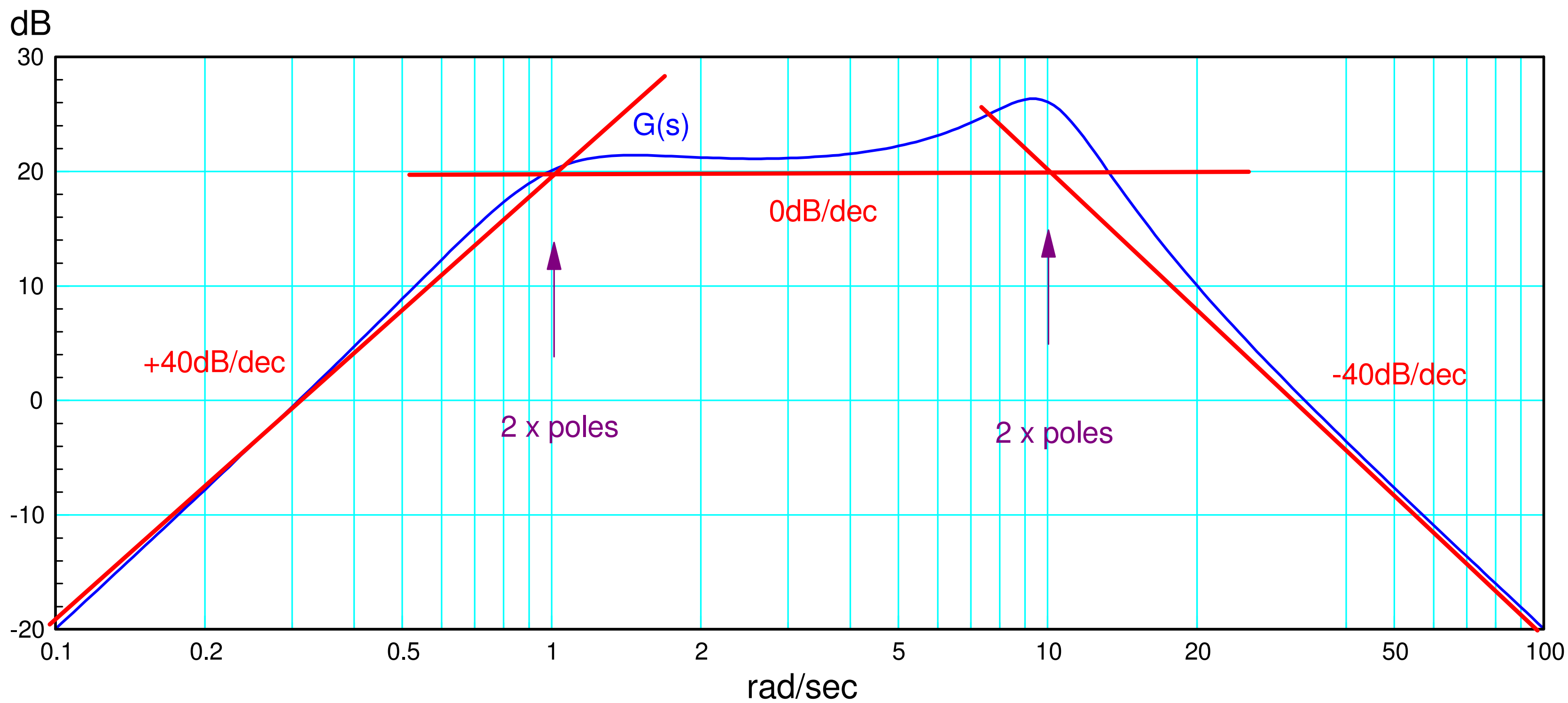
Find G(s) given the Bode plot



Step 1: Draw in the asymptotes.

$$G(s) \approx \left(\frac{ks^2}{(s+1\angle\pm\theta)(s+10\angle\pm\phi)} \right)$$

- Note: Each line *must* have a slope of 20n dB/decade
- 10dB/decade, for example, implies half of pole or zero, meaning a half derivative.



$$G(s) \approx \left(\frac{ks^2}{\left(s+1\angle\pm 60^0\right)\left(s+10\angle\pm 75.5^0\right)} \right)$$

1 rad/sec

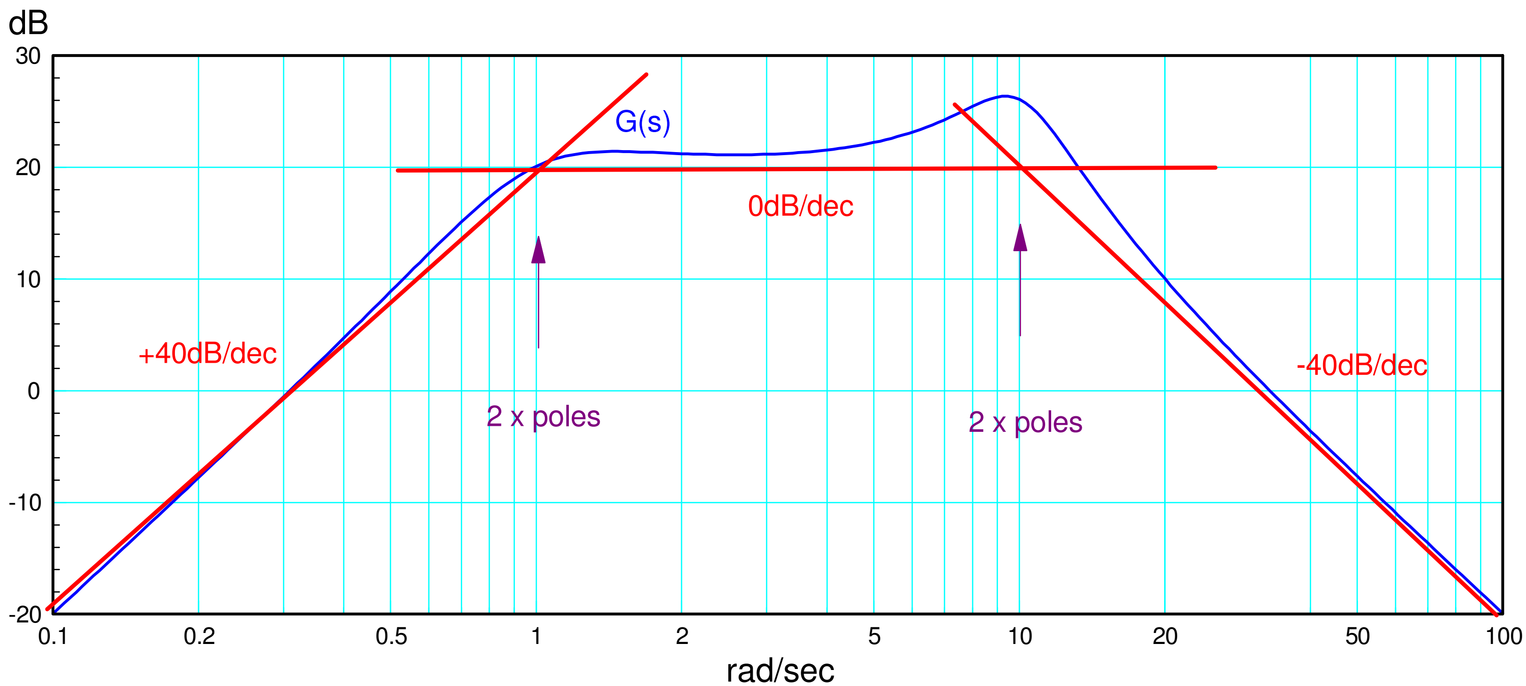
gain at corner = 0dB above corner

$\zeta = 0.5 \qquad \theta = 60^0$

10 rad/sec

gain at corner = +6dB

$\zeta = 0.25 \qquad \phi = 75.5^0$

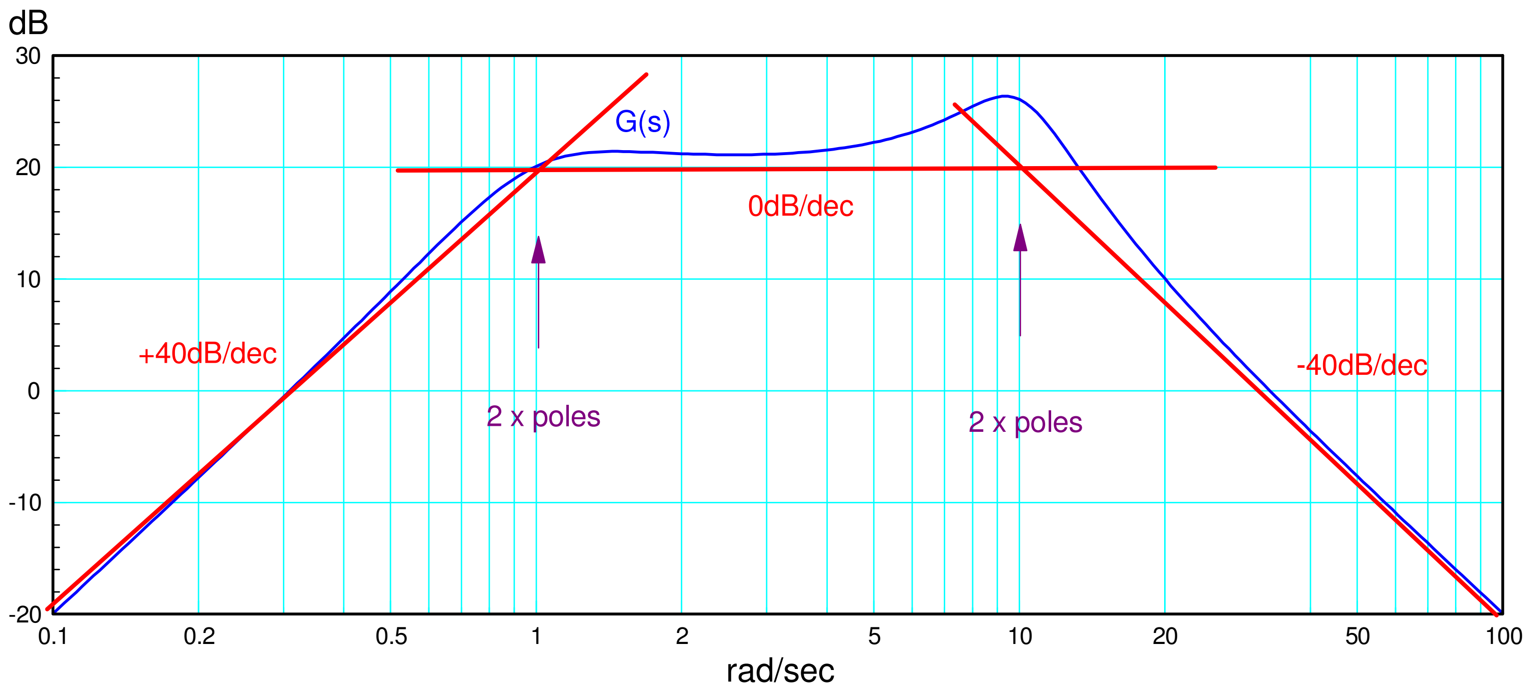


Step 3: Solve for k:

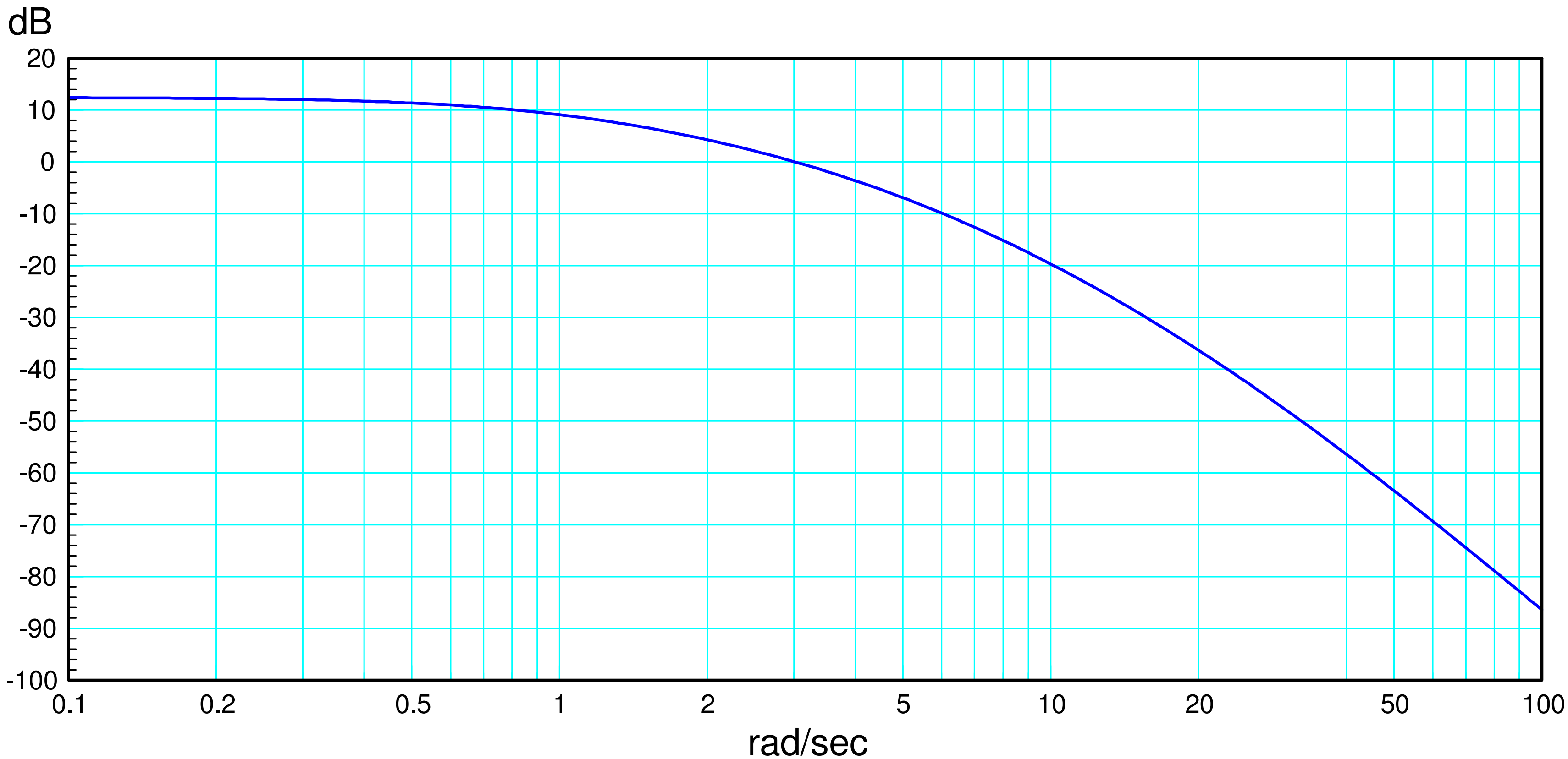
At 0.1 rad/sec, the gain is -20dB (0.1)

$$-20dB = 0.1 = \left| \frac{ks^2}{\left(s+1\angle\pm60^0\right)\left(s+10\angle\pm75.5^0\right)} \right|_{s=j0.1}$$

k = 9949.5



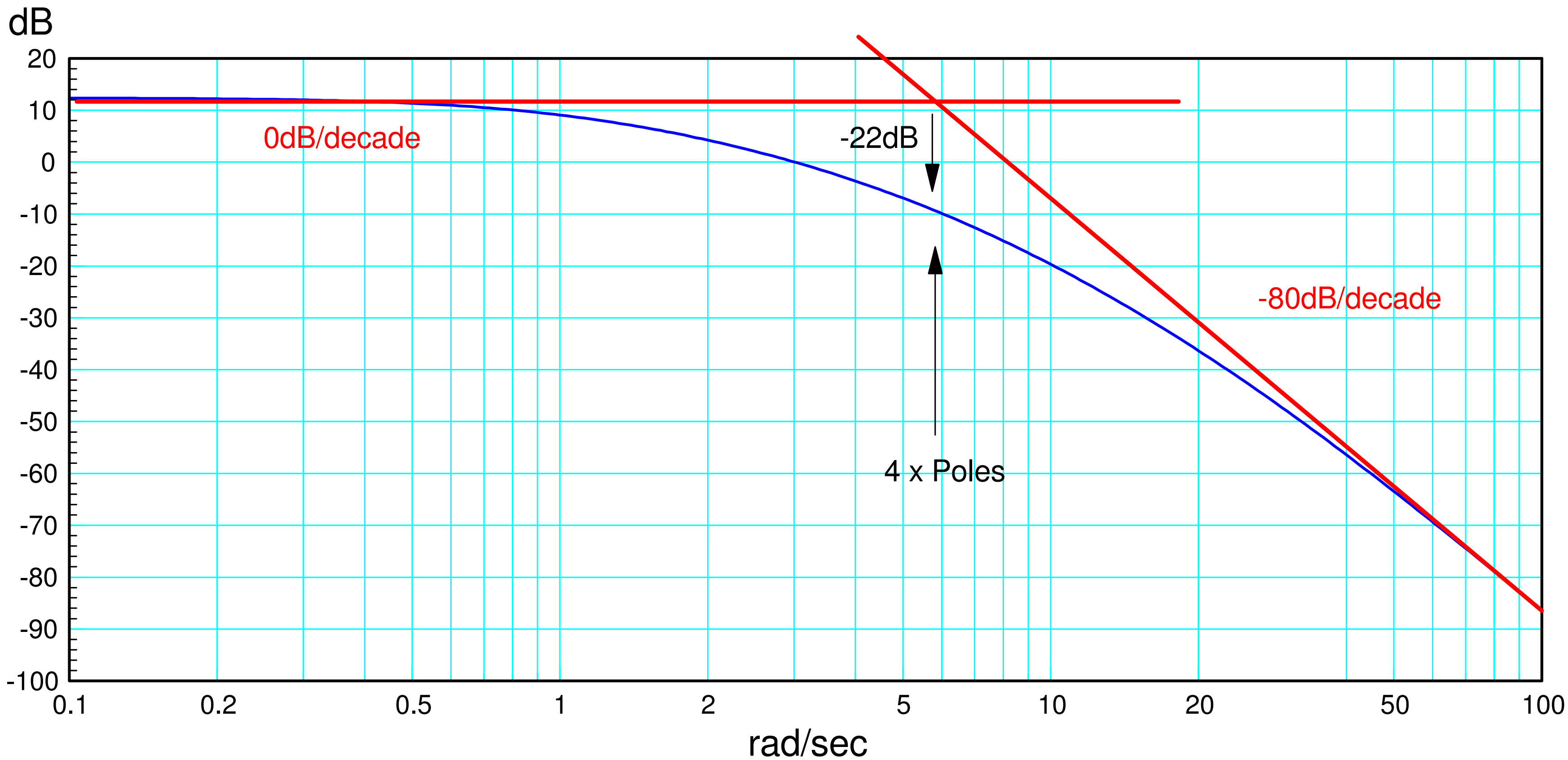
Example 2: Determine G(s) given G(jw)



Example 2: Find G(s)

Step 1: Draw in the asmptotes.

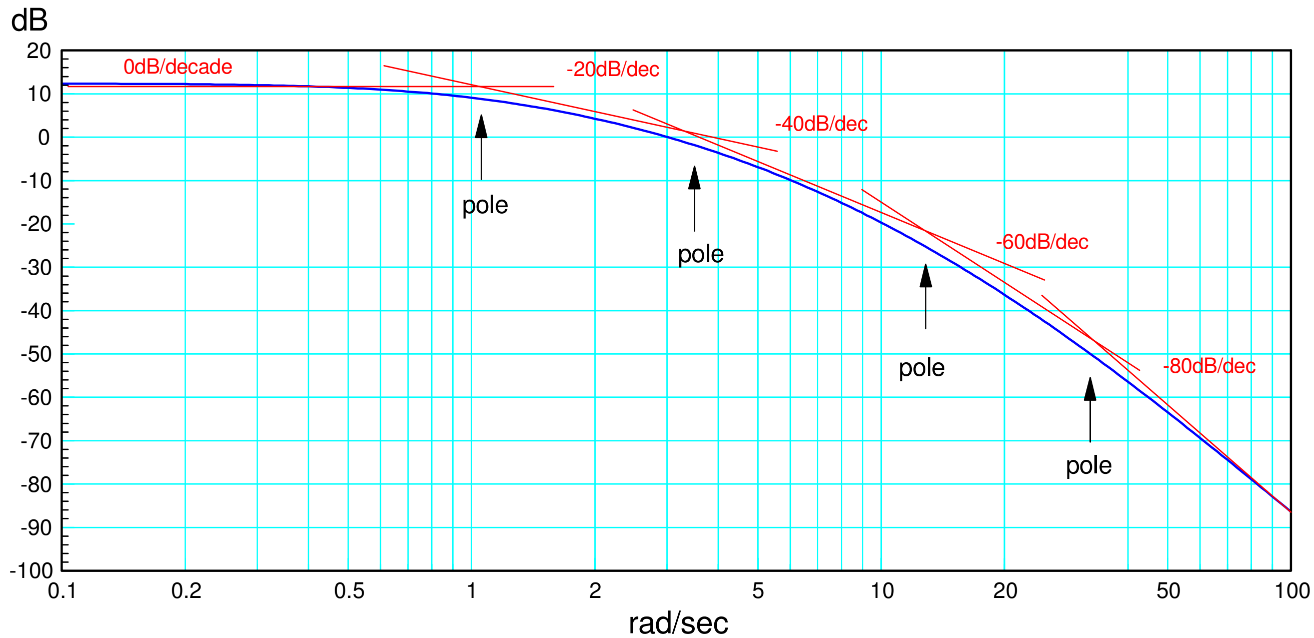
- Start with two. $\zeta = 1.77$



Approximating G(s) with two asymptotes. The gain at the corner is down too much

Add more asymptotes

$$G(s) \approx \left(\frac{k}{(s+1.1)(s+3.3)(s+12)(s+30)} \right)$$



Find k

- Match the gain somewhere

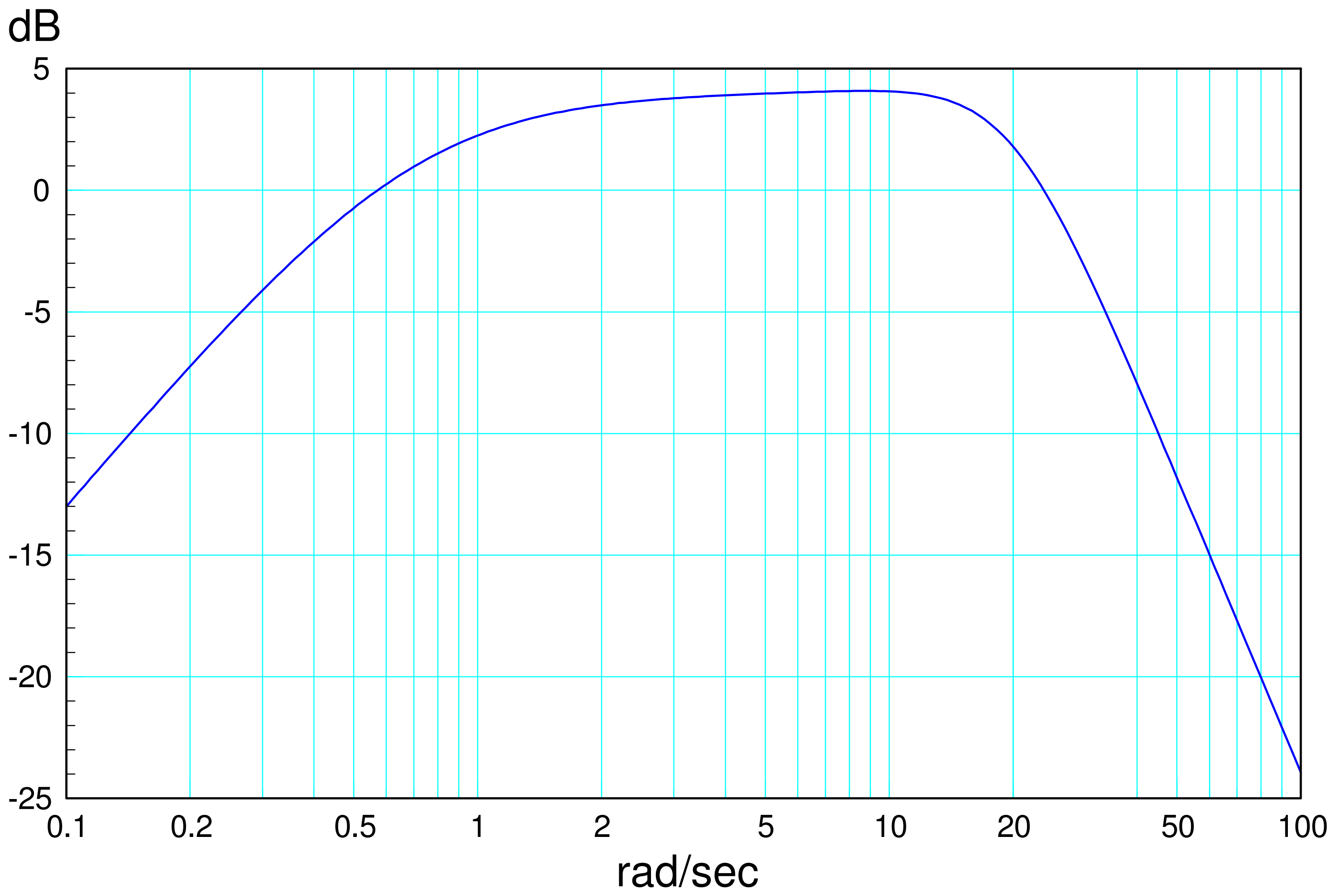
$$G(j0.1) = 12dB = 3.98$$

$$= \left(\frac{k}{(s+1.1)(s+3.3)(s+12)(s+30)} \right)_{s=j0.1}$$

k = 5225

$$G(s) \approx \left(\frac{5225}{(s+1.1)(s+3.3)(s+12)(s+30)} \right)$$

Handout: Determine G(s) given the Bode plot



Summary:

Given G(s), you can find the frequency response

- Substitute s = jw
- Use Matlab to evaluate at a bunch of points

Given the frequency response, you can find G(s)

- Plot as a Bode plot (log-w vs. dB)
- Add asymptotes at multiples of 20dB / decade
- Corners tell you where the poles & zeros are
- Gain at the corners tell you the damping ratio