Meeting Design Specs in the z-plane

ECE 461/661 Controls Systems

Jake Glower - Lecture #33

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Meeting Design Specs in the z-plane

Add poles and zeros to K(z) to meet the design specs.

- Add zeros to cancel slow poles
- For every zero you add, you need to add a pole
- One of these poles goes to s=0 (z=1) if you need to make the system type-1, and
- The remaining poles go somewhere out of the way (or are adjusted to meet the design specs).

Things to avoid when designing K(z)

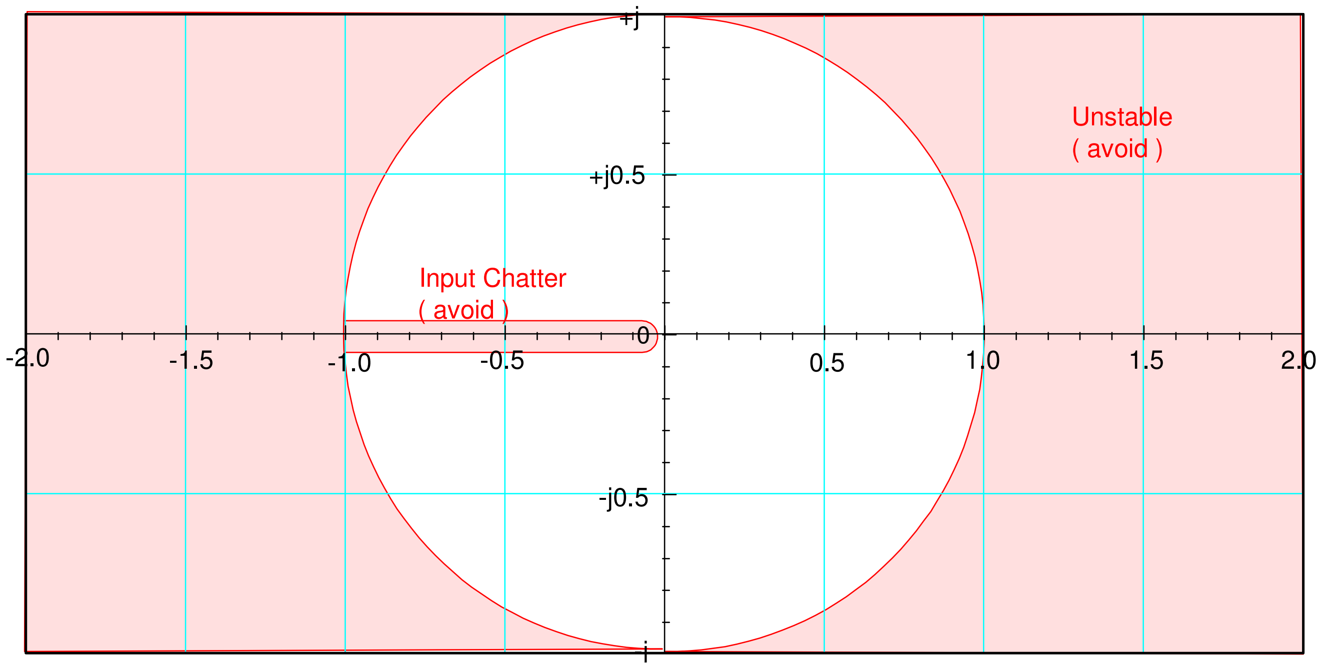
- Avoid placing poles outside the unit circle.

  This results in the open-loop system being unstable

  This makes it difficult (sometimes dangerous) to test and debug.

- Avoid placing poles on the negative real axis (between -1 and 0).

  This results in chatter (+ / - inputs, changing every sample)



Design Example

Design a compensator, K(z), for the following system

$$G(s) = \left(\frac{50}{(s+1)(s+3)(s+10)}\right)$$

that results in

- No error for a step input,
- 20% overshoot for a step input, and
- A 2% settling time of 4 seconds.

Method #1: z-Plane Analysis

Pick T = 200ms:

- Ts = 4 seconds
- T = 200ms gives 20 samples in 4 seconds

Enough for the controller to figure out what the input needs to be

Convert to the z-Plane

$$G(z) \approx \left(\frac{0.1179z}{(z-0.8187)(z-0.5488)(z-0.1353)} \right)$$

The design requirements translate to

- Make the system type-1
- Place the closed-loop dominant pole at s = -1 + j2 (s-plane)
- Place the closed-loop dominant pole at z = 0.7541 + j0.3188

z-plane found from $z = e^{sT}$

$$G(z) \approx \left(\frac{0.1179z}{(z-0.8187)(z-0.5488)(z-0.1353)}\right)$$

$$K(z) = k\left(\frac{(z-0.8187)(z-0.5488)}{(z-1)(z-a)}\right)$$

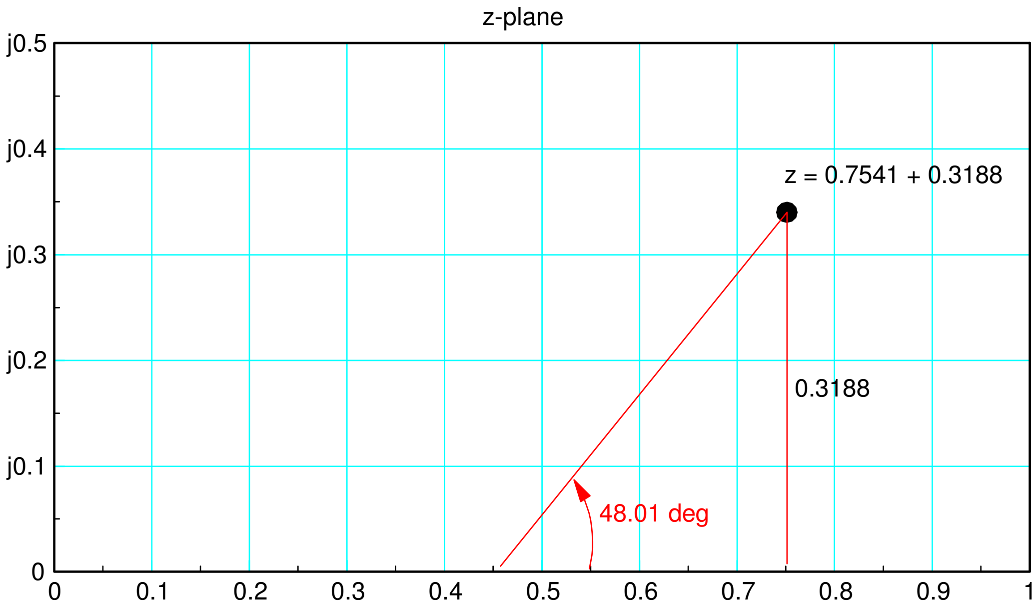
$$GK = \left(\frac{0.1179kz}{(z-1)(z-0.1353)(z-a)}\right)$$

Pick 'a' so that 0.7541 + j0.3188 is on the root locus.

$$\left(\frac{0.1179z}{(z-1)(z-0.1353)}\right)_z = 0.3444\angle -131.98^0$$

$$a = 0.7541 - \left(\frac{0.3188}{\tan\left(48.01^0\right)}\right)$$

$$a = 0.4672$$



To find 'k', set GK = -1 at z = 0.7541 + j0.3188

$$GK = \left(\frac{0.1179z}{(z-1)(z-0.4672)(z-0.1353)} \right)_{z=0.7541+j0.3188} = 0.8029\angle 180^0$$

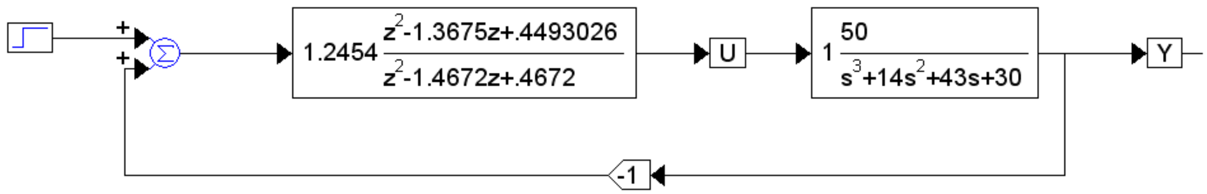
$$k = \frac{1}{0.8029} = 1.2454$$

and

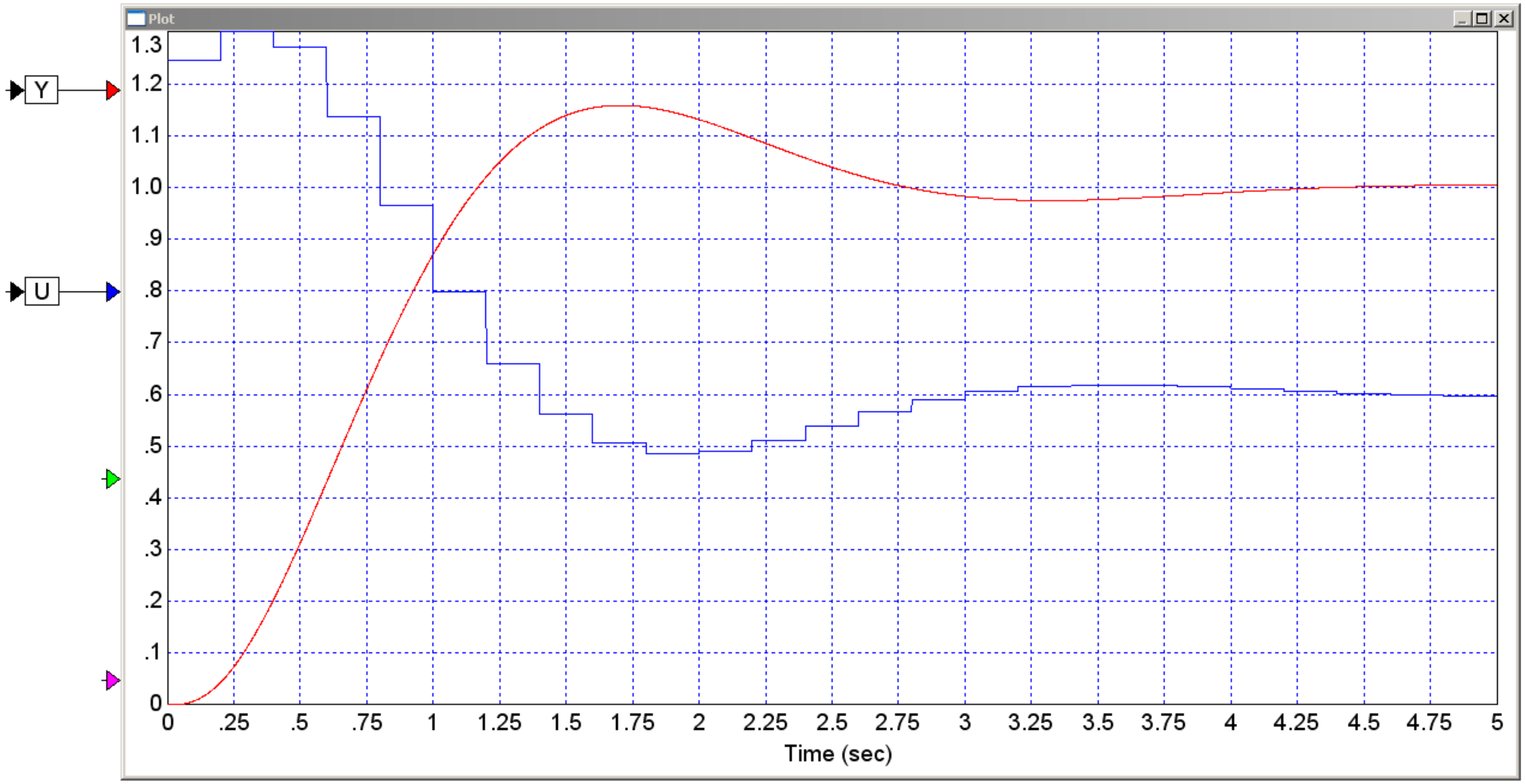
$$K(z) = 1.2454\left(\frac{(z-0.8187)(z-0.5488)}{(z-1)(z-0.4672)} \right)$$

Checking in VisSim:

- Note that the overshoot is a little off.
- This is due to the model for G(z) being slightly off.
- (needs 1.55 zeros at z=0)



+
+
1.2454 z^2-1.3675z+.4493026 / z^2-1.4672z+.4672
U
1 50 / s^3+14s^2+43s+30
Y
-1



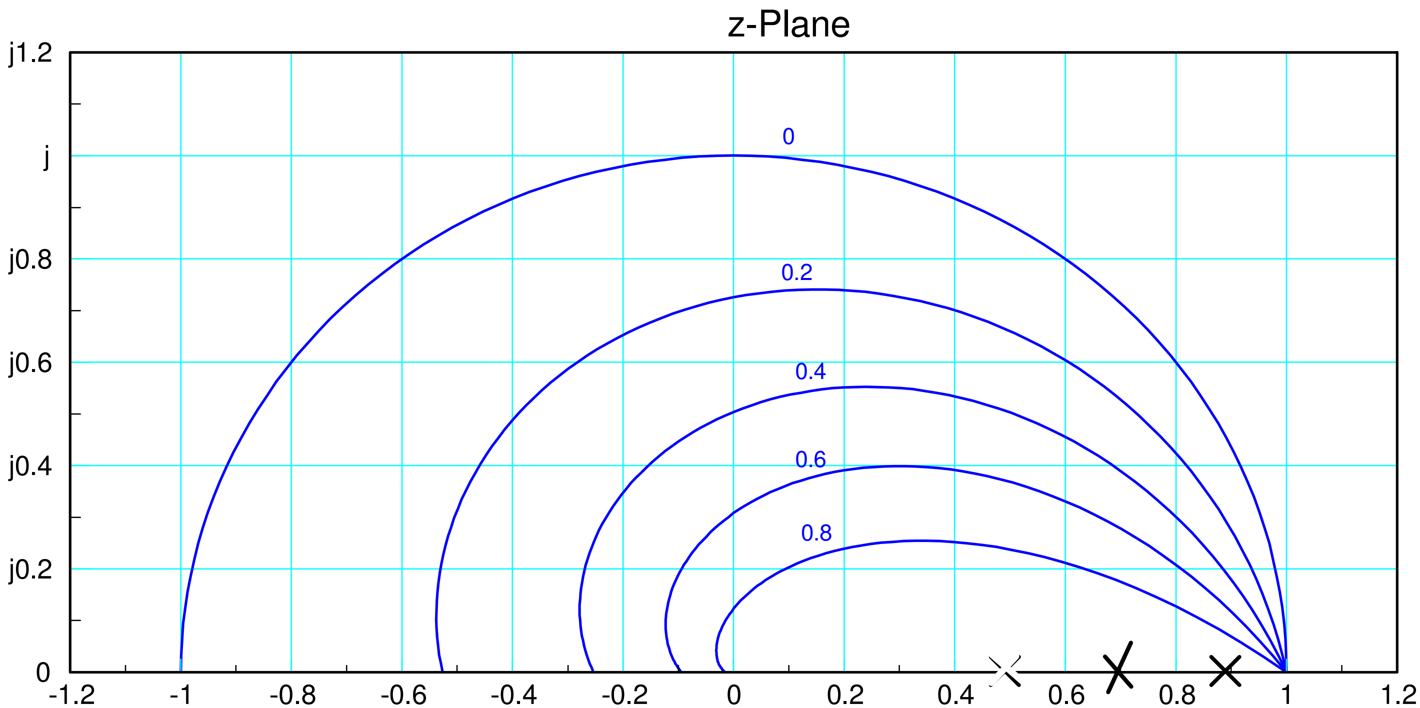
Plot
Y
U
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0 .25 .5 .75 1 1.25 1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75 4 4.25 4.5 4.75 5
Time (sec)

Handout: Given the system

$$G(z) = \left(\frac{0.2z}{(z-0.9)(z-0.7)}\right)$$

design a compensator, K(z), that results in

- A type-1 system, and
- A closed-loop dominant pole at z = 0.6 + j0.4



Method #2: Mixed Plane Analysis

All we care about is one point

- Point on damping line where angles add up to 180 degrees

You don't really need to sketch the root locus

- All we need is that one point

Analyze the hybrid system:

$$G(s) \cdot \exp\left(\frac{-sT}{2}\right) \cdot K(z)$$

Step 1: Decide where you want to place the closed-loop poles. From before

- s = -1 + j2
- z = 0.7541 + j0.3188

Step 2: Model G(s) and the zero-order hold (modeled as a 1/2 sample delay)

$$G(s) \cdot ZOH = \left(\frac{50}{(s+1)(s+3)(s+10)}\right) \cdot e^{-sT/2}$$

Step 3: Pick the form of K(z)

$$K(z) = k\left(\frac{(z-0.8187)(z-0.5488)}{(z-1)(z-a)}\right)$$

$$G \cdot K \cdot ZOH = \left(\frac{50}{(s+1)(s+3)(s+10)}\right) \cdot e^{-sT/2} \cdot k\left(\frac{(z-0.8187)(z-0.5488)}{(z-1)(z-a)}\right)$$

Note that the zeros cancel poles

- s = -1 z = 0.8187
- s = -3 z = 0.5488

To find 'a', evaluate at s (z). Pick 'a' to make the angles add up to 180 degrees

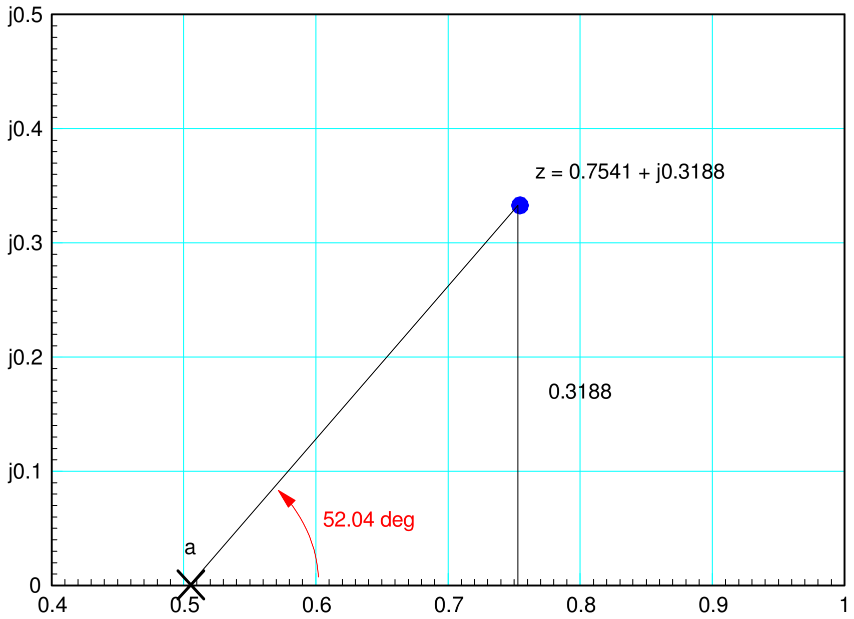
$$\left.\left(\left(\frac{50}{(s+1)(s+3)(s+10)}\right)\cdot e^{-sT/2}\cdot\left(\frac{(z-0.8187)(z-0.5488)}{(z-1)}\right)\right)\right|_{s=-1+j2} =$$

$$= ((0.9587\angle -147^0)\cdot(0.9048\angle -11.46^0)\cdot(0.3063\angle 31.03^0))$$

$$= 0.2657\angle -127.96^0$$

To make the angle 180 degrees, (z-a) contributes 52.04 degrees

$$a = 0.7541 - \left(\frac{0.3188}{\tan\left(52.04^0\right)}\right) = 0.5054$$



This results in

$$K(z) = k\left(\frac{(z-0.8187)(z-0.5488)}{(z-1)(z-0.5054)}\right)$$

To find 'k'

$$\left(\left(\frac{50}{(s+1)(s+3)(s+10)}\right) \cdot e^{-sT/2} \cdot \left(\frac{(z-0.8187)(z-0.5488)}{(z-1)(z-0.5054)}\right)\right)_{s=-1+j2} = 0.8028\angle 180^0$$

so

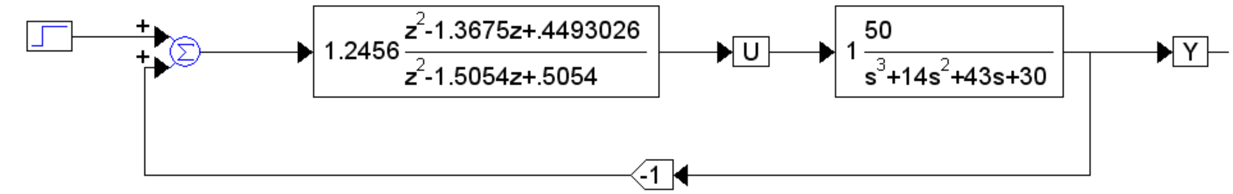
$$k = \frac{1}{0.8028}$$

and

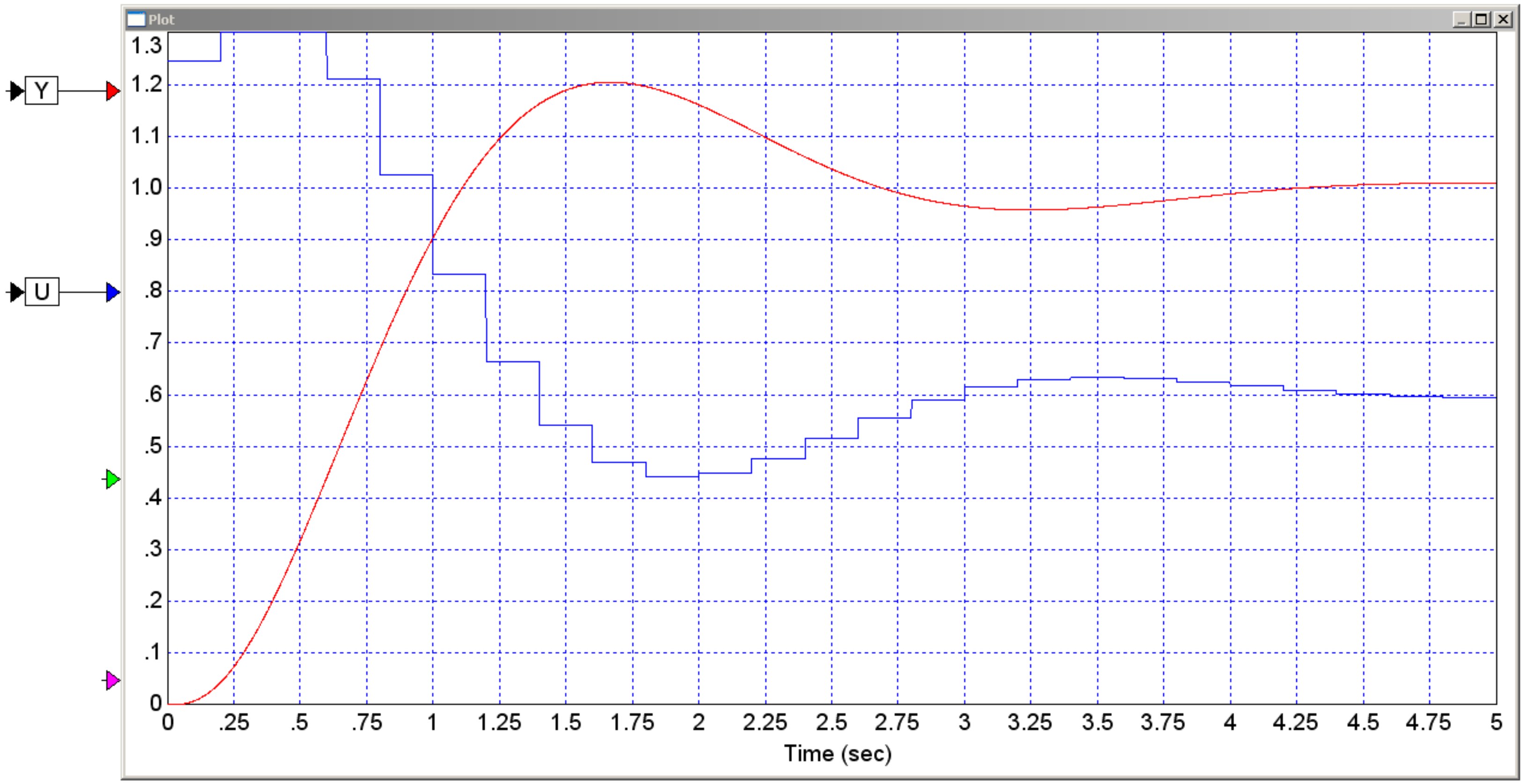
$$K(z) = 1.2456\left(\frac{(z-0.8187)(z-0.5488)}{(z-1)(z-0.5054)}\right)$$

Checking in VisSim:

- The overshoot is closer to 20%
- No s to z conversion (and resulting errors)



+
+
1.2456
z^2-1.3675z+.4493026
z^2-1.5054z+.5054
U
1
50
s^3+14s^2+43s+30
Y
-1



Plot
Y
U
1.3
1.2
1.1
1.0
.9
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.5
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.3
.2
.1
0
0
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1.25
1.5
1.75
2
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2.5
2.75
3
3.25
3.5
3.75
4
4.25
4.5
4.75
5
Time (sec)

Summary:

Designing K(z) to meet the design specs is similar to the design in the s-plane

- Add a pole at s=0 (z=1) to make the system type-1 if needed
- Start cancelling poles with zeros to speed up the system
- Once the system is too fast, start adding poles so that

    The number of poles equals the number of zeros, and

    *G(z)*K(z) = -1 at the design point, or*

    *G(s)*zoh*K(z) = -1 at the design point*