Unstable Systems and Multi-Loop Feedback

ECE 461/661 Controls Systems

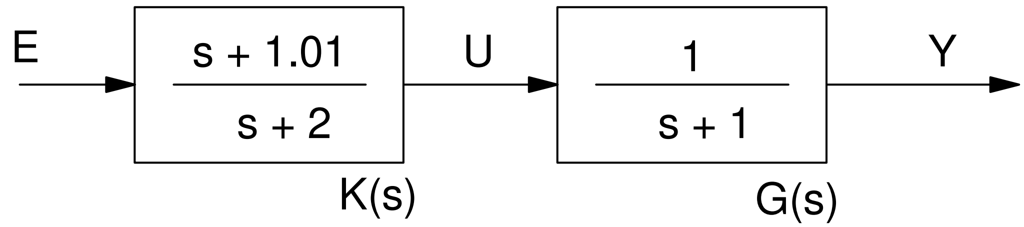
Jake Glower - Lecture #28

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Pole-Zero Cancellation:

Pole-Zero cancellation just makes the initial condition small

Example: Find the step response



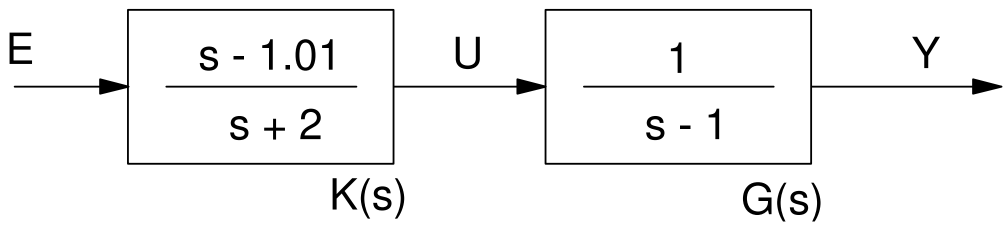
$$Y = \left(\frac{1}{s+1}\right)\left(\frac{s+1.01}{s+2}\right)\left(\frac{1}{s}\right) = \left(\frac{0.505}{s}\right) + \left(\frac{0.01}{s+1}\right) + \left(\frac{0.495}{s+2}\right)$$

$$y(t) = 0.505 + 0.01e^{-t} + 0.495e^{-2t} \qquad t > 0$$

Ignoring the pole at s = -1 doesn't change the results significantly

Unstable Poles

This doesn't work with unstable poles



$$Y = \left(\frac{1}{s-1}\right)\left(\frac{s-1.01}{s+2}\right)\left(\frac{1}{s}\right) = \left(\frac{0.505}{s}\right) + \left(\frac{-0.00333}{s+1}\right) + \left(\frac{0.50167}{s+2}\right)$$

$$y(t) = 0.505 - 0.00333e^{t} + 0.050167e^{-2t} \qquad t > 0$$

The unstable term blows up (you can't ignore it)

Result:

You cannot cancel unstable poles

If you miss by the slightest amount, they'll blow up

Design Problem:

Design a compensator for the following system:

$$G(s) = \left(\frac{10}{(s-1)(s+1)(s+5)}\right)$$

(similar dynamics to a rocket)

that results in

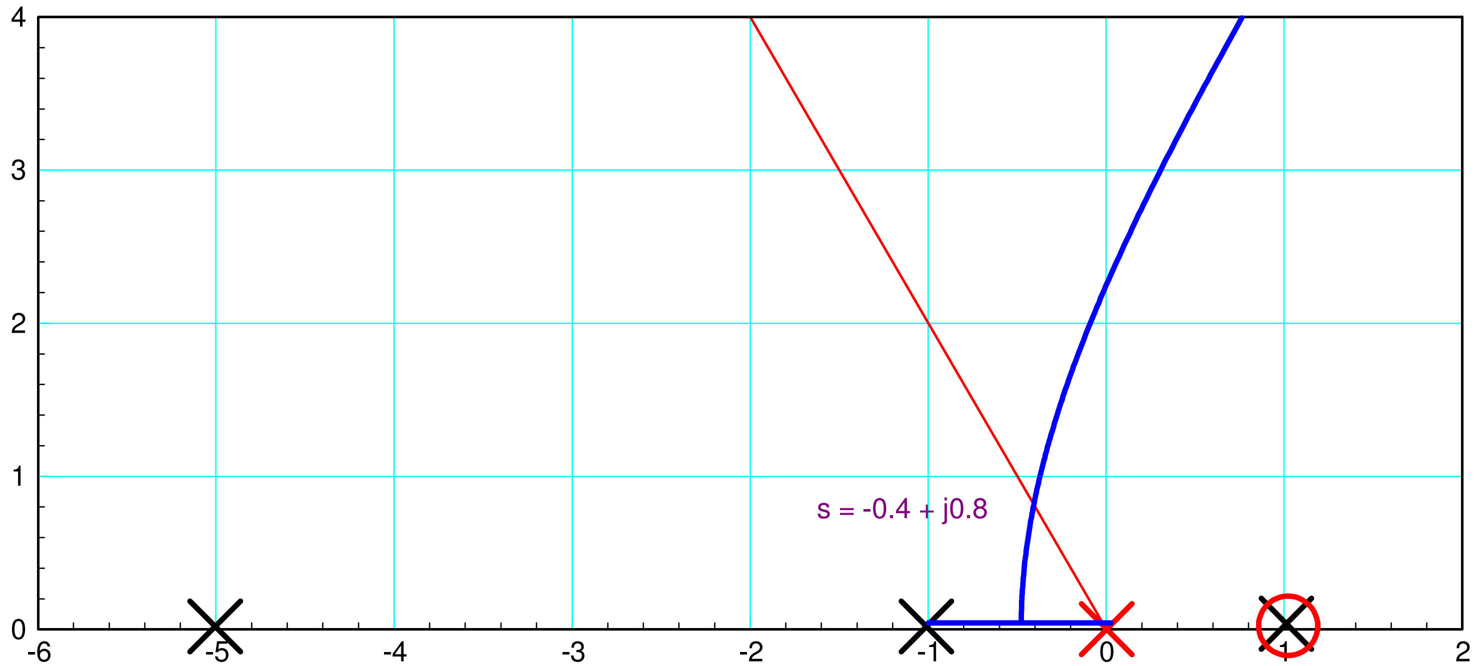
- No error for a step input, and
- 20% overshoot for a step input

Verify your design with VisSim (or Simulink)

Method #1 (which won't work) .

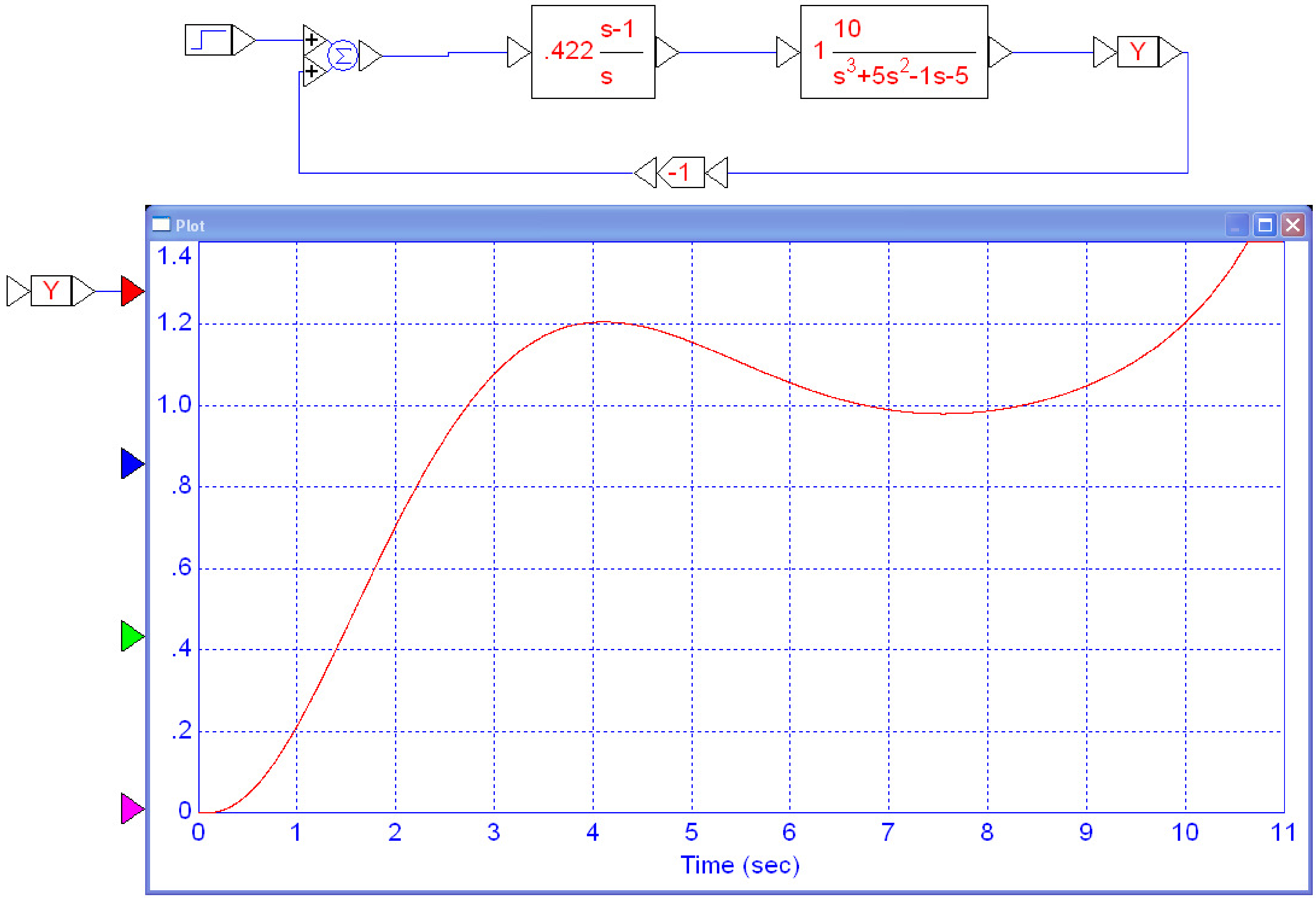
- Cancel the pole at s = +1 since it's causing problems.
- Add a pole at s = 0 to make the system type-1.
- Add a gain of 0.4220 to place the closed-loop poles at s = -0.4031 + j0.8062

$$K(s) = 0.4220\left(\frac{s-1}{s}\right)$$



VisSim Result:

- Unstable



NASA Result:

- Tried many times in the 1950's
- Always ended up with an unstable system

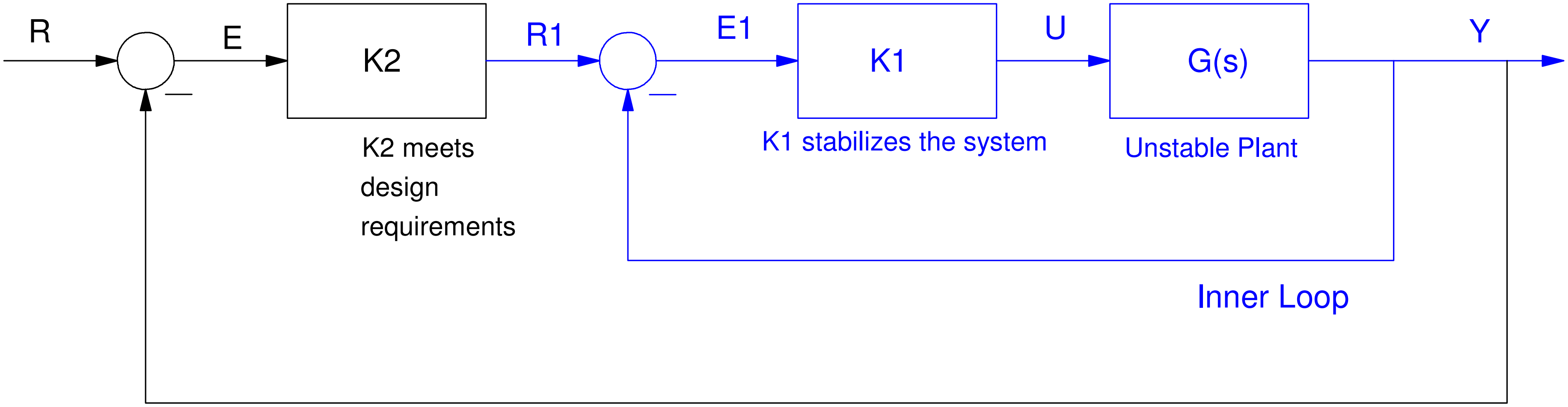
Early US rocket and space launch failures and expl video

Method #2: Multi-Loop Feedback.

Force the problem to fit the solution

- Add a feedback loop (K1) to stabilize the system
- *Then* worry about meeting the design specs

We know how to design controllers for systems which are open-loop stable



Design Problem (repeat)

Design a controller for

$$G(s) = \left(\frac{10}{(s-1)(s+1)(s+5)}\right)$$

that results in

- No error for a step input,
- 20% overshoot for a step input, and
- A 2% settling time of 4 seconds.

Verify your design with VisSim.

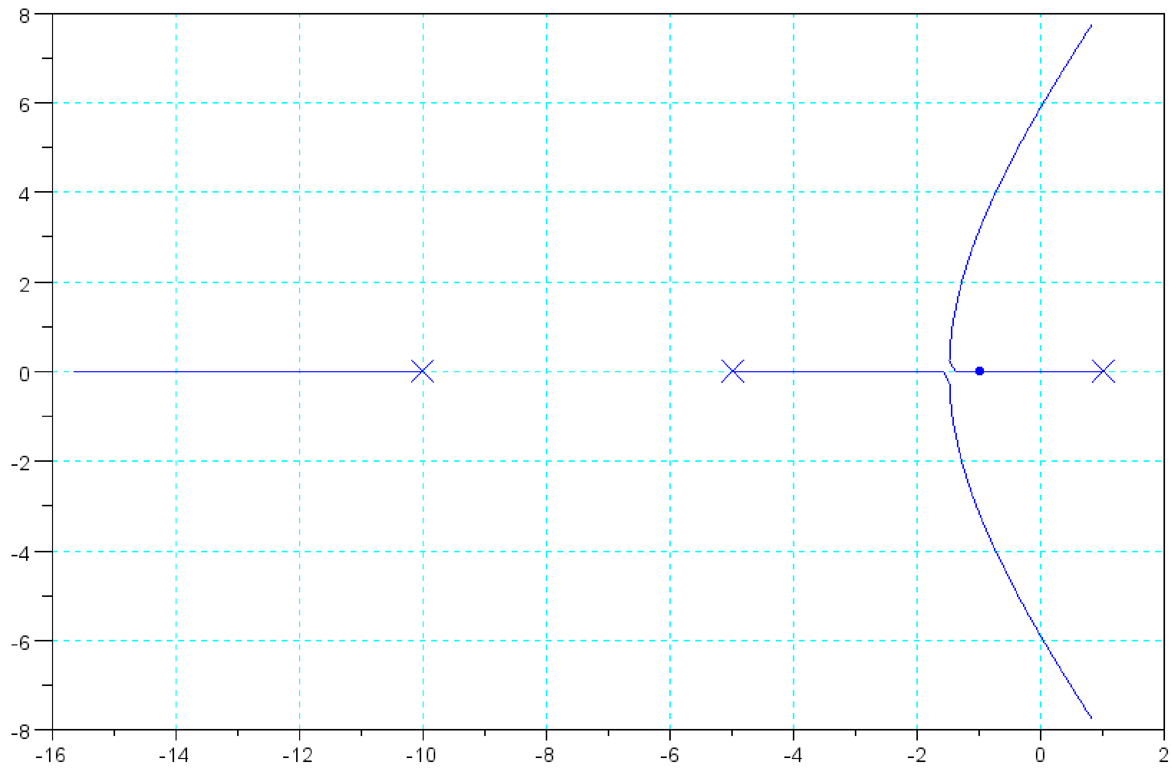
Step 1: Stabilize the system

Add a compensator, K1(s), to stabilize the system.

- Don't cancel the pole at s = +1: it's unstable
- Cancel the pole at s = -1 instead

$$K_1(s) = k\left(\frac{s+1}{s+10}\right)$$

$$GK_1 = \left(\frac{10k}{(s-1)(s+5)(s+10)}\right)$$

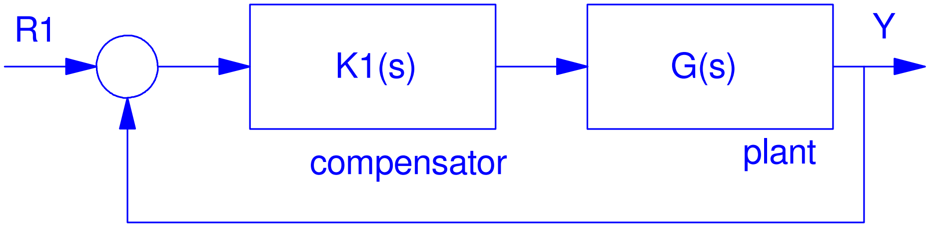


Pick a spot that's stable

- s = -1

Find k to place the closed-loop poles there

Finding K1(s)



$$\left(\frac{10k}{(s-1)(s+5)(s+10)}\right)_{s=-1} = 1\angle 180^0$$

$$\left(\frac{10}{(s-1)(s+5)(s+10)}\right)_{s=-1} = 0.1389\angle 180^0$$

so

$$k = 7.200$$

and

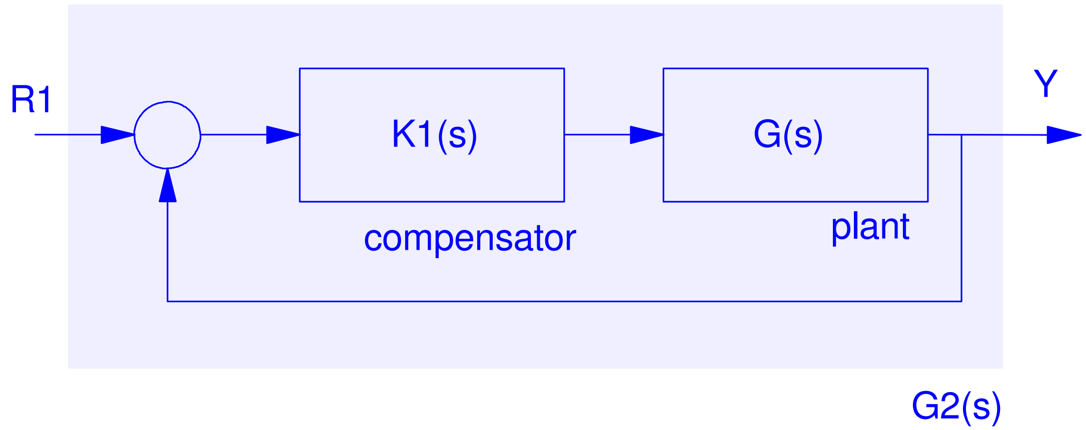
$$K_1(s) = \left(\frac{7.2(s+1)}{s+10}\right)$$

This results in the closed-loop system being

$$G_2 = \left(\frac{GK_1}{1+GK_1}\right) = \left(\frac{72}{(s+1)(s+2)(s+11)}\right)$$

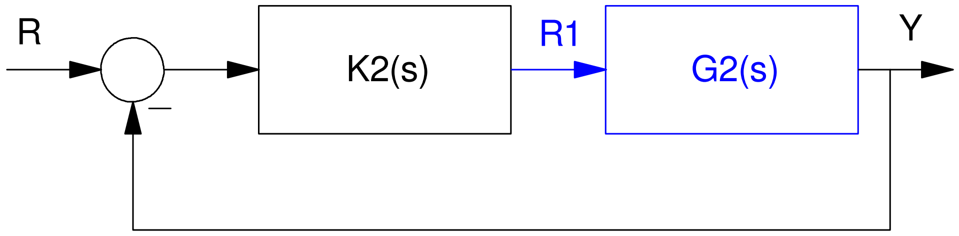
Note that this doesn't meet the requirements in any way. At least it's stable though.

```
G = zpk([],[1,-1,-5],10)

          10
-----------------
(s-1) (s+1) (s+5)

K1 = zpk(-1,-10,7.2);
G2 = minreal(G*K1 / (1+G*K1));
zpk(G2)

           72
------------------
(s+1) (s+2) (s+11)
```



Step 2: Add K2(s) to meet the design specs.

$$G_2 = \left(\frac{72}{(s+1)(s+2)(s+11)}\right)$$



To meet the design requirements,

- The system should be type-1
- The closed-loop dominant pole should be at s = -1 + j2

To meet this requirement,

- Add a pole at s = 0 to make the system type-1
- Cancel the poles at -1 and -2
- Add a pole at -a so that -1 + j2 is on the root locus

$$K_2(s) = k\left(\frac{(s+1)(s+2)}{s(s+a)}\right)$$

To solve for 'a'

$$G_2K_2 = \left(\frac{72k}{s(s+11)(s+a)}\right)$$

Evaluating what we know

$$\left(\frac{72}{s(s+11)}\right)_{s=-1+j2} = 3.1574\angle -127.87^0$$

To make this 180 degrees

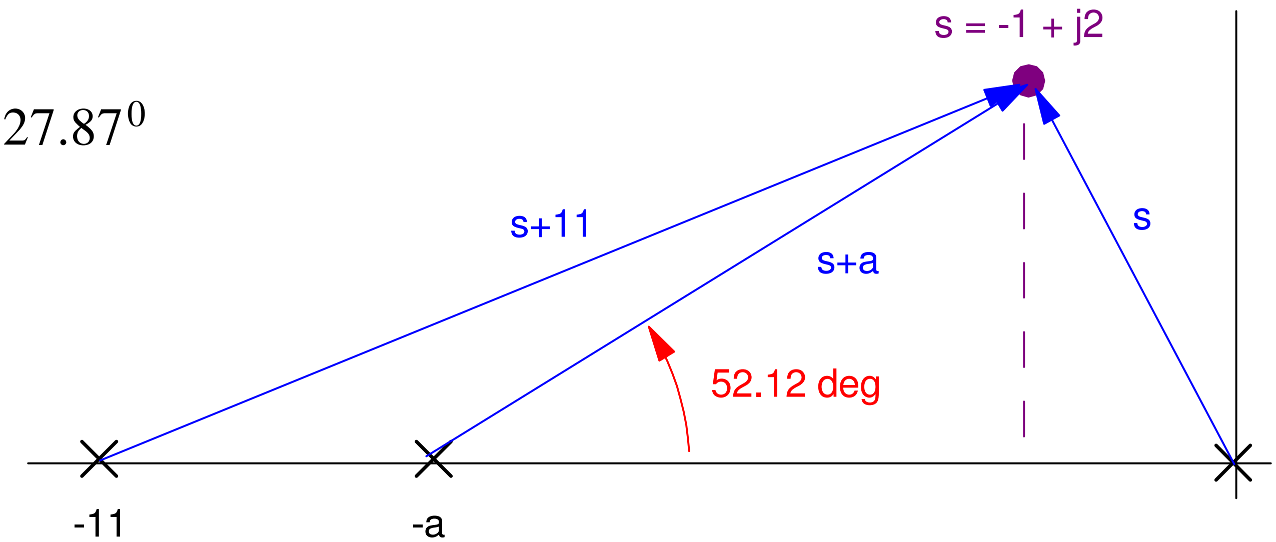
$$\angle(s+a) = 52.125^0$$

$$a = \frac{2}{\tan(52.125^0)} + 1$$

$$a = 2.5556$$

$$K_2(s) = k\left(\frac{(s+1)(s+2)}{s(s+2.5556)}\right)$$

$$G_2K_2 = \left(\frac{72k}{s(s+11)(s+2.5556)}\right)$$

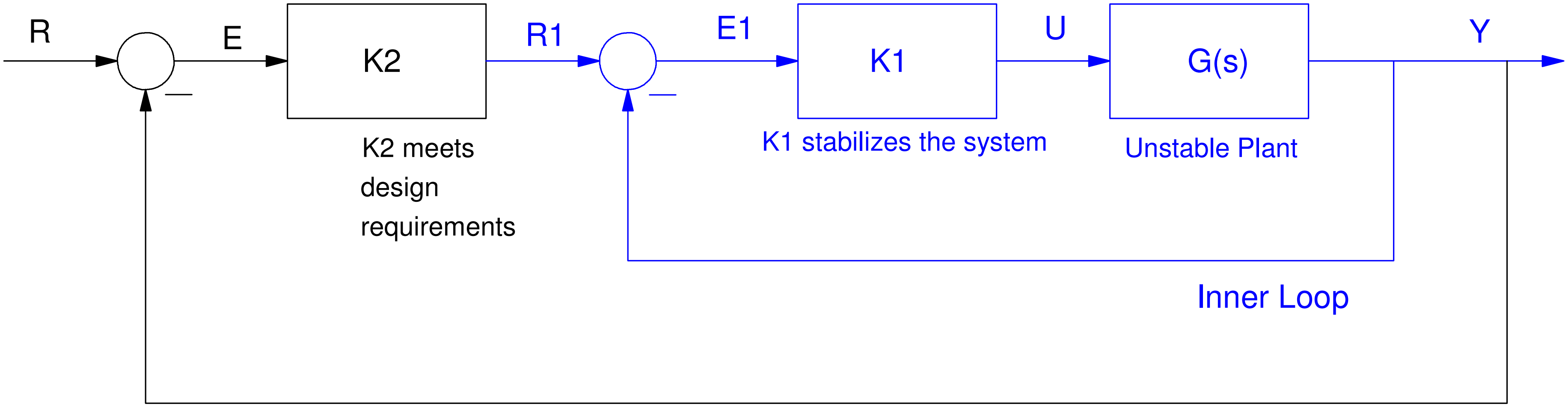


To find 'k'

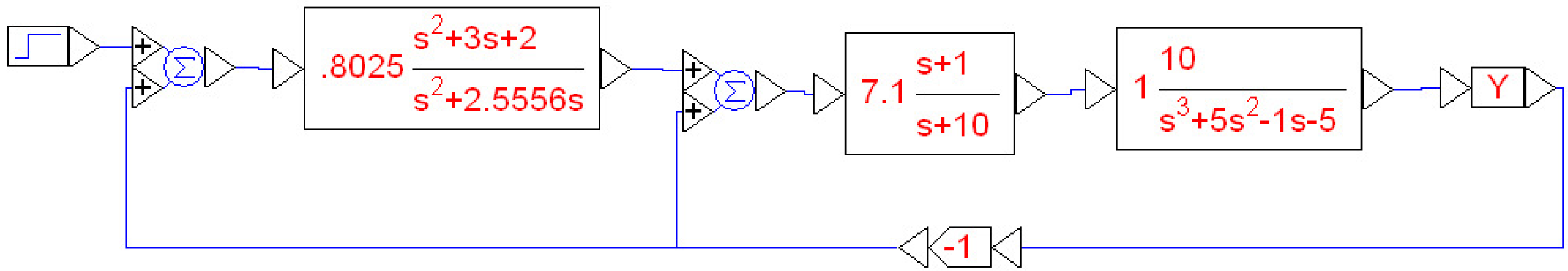
$$\left(\frac{72}{s(s+11)(s+2.5556)}\right)_{s=-1+j2} = 1.2461\angle 180^0$$

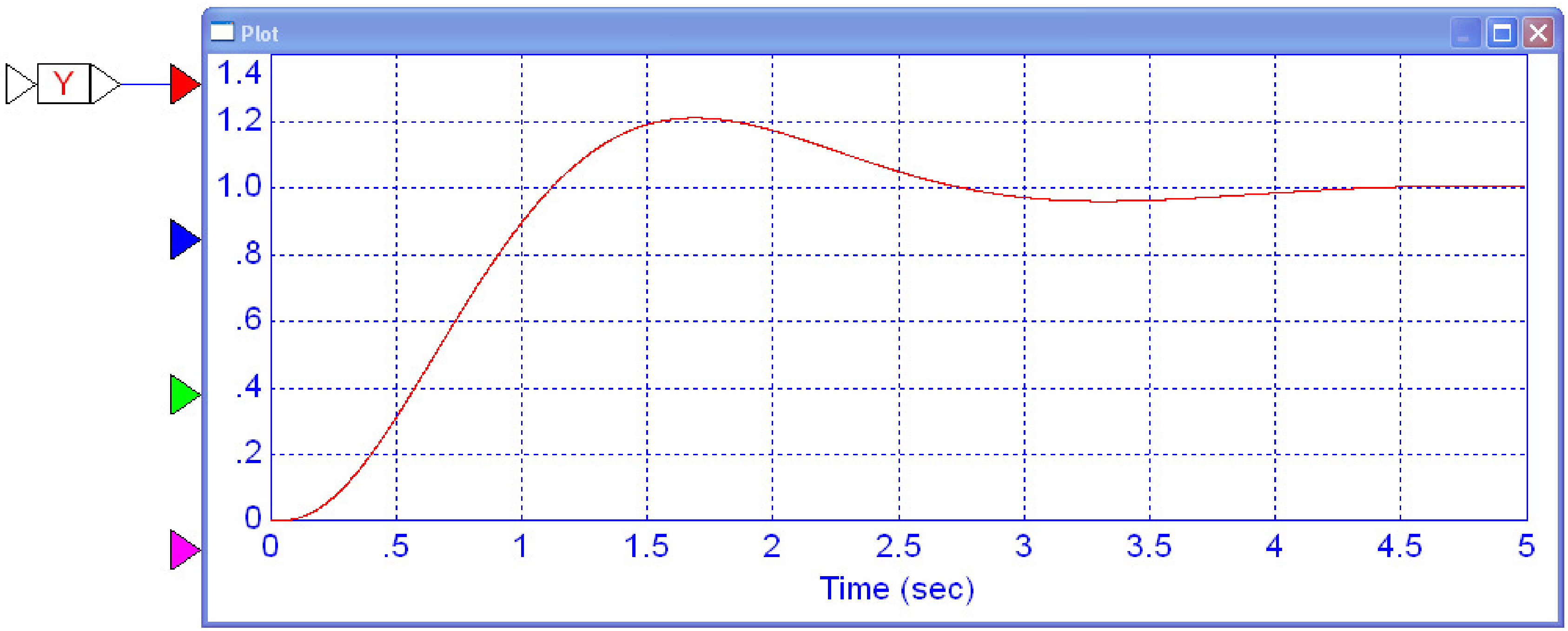
$$k = \frac{1}{1.2461} = 0.8025$$

$$K_2(s) = 0.8025\left(\frac{(s+1)(s+2)}{s(s+2.5556)}\right)$$



Step 3: Validation (VisSim works)

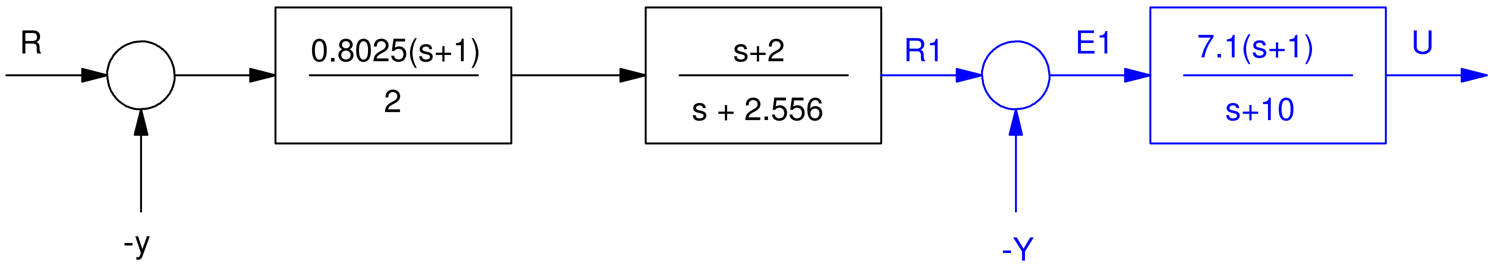


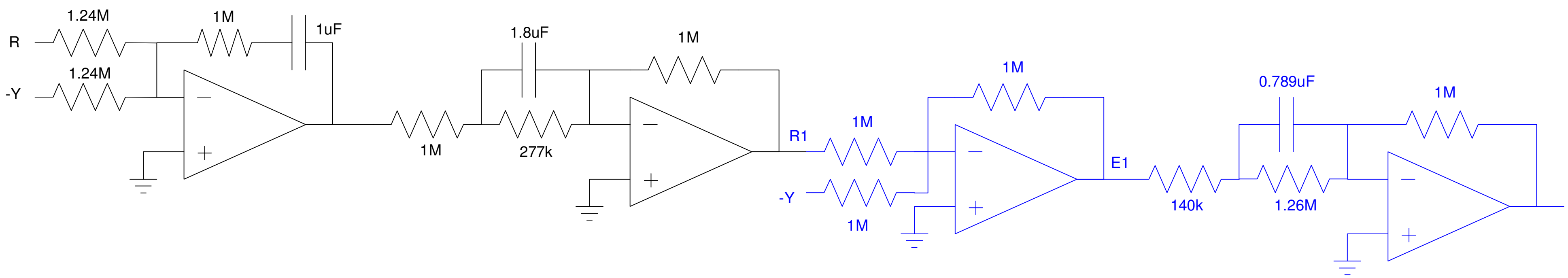


Note

- It doesn't really matter where you place the poles in Step 1: you're just going to cancel them in Step 2. Likewise, these poles were placed on the real axis at { -1, -2, -11 }. Real poles are easier to cancel than complex ones when using an op-amp circuit.
- The closed-loop system is stable in spite of the open-loop system being unstable. Unstable open-loop systems are OK to use - as long as your feedback control law is working.
- No unstable poles were canceled. The system remains stable if you run it out to 100 seconds.

Implementation:

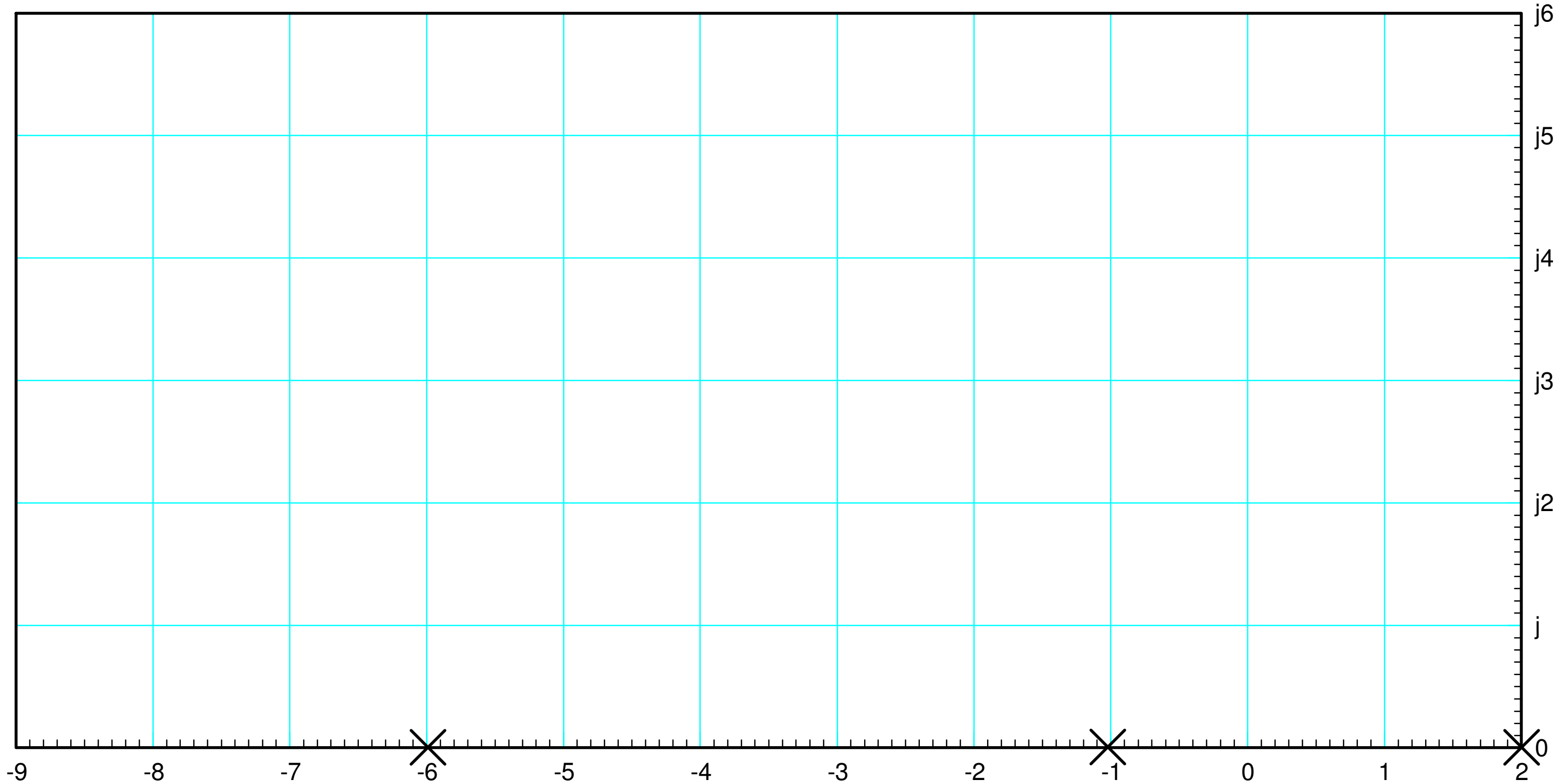




Handout: Design a gain compensator, K(s), so that the following system is

- Closed-loop stable,
- With 2% settling time of 5 seconds

$$G(s) = \left(\frac{20}{(s-2)(s+1)(s+6)}\right)$$



Summary:

You *cannot* cancel unstable poles

- Cancelling the unstable pole doesn't make it go away
- It just makes that pole uncontrollable from your input

You *can* stabilize unstable systems using root locus techniques

- Close the feedback loop twice
- First time: stabilize the system
- Second time: Meet the design specs