
Control of a DC Servo Motor

ECE 461/661 Controls Systems

Jake Glower - Lecture #25

Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

Problem:

- Control the speed of a DC servo motor
- Control the position of a DC servo motor

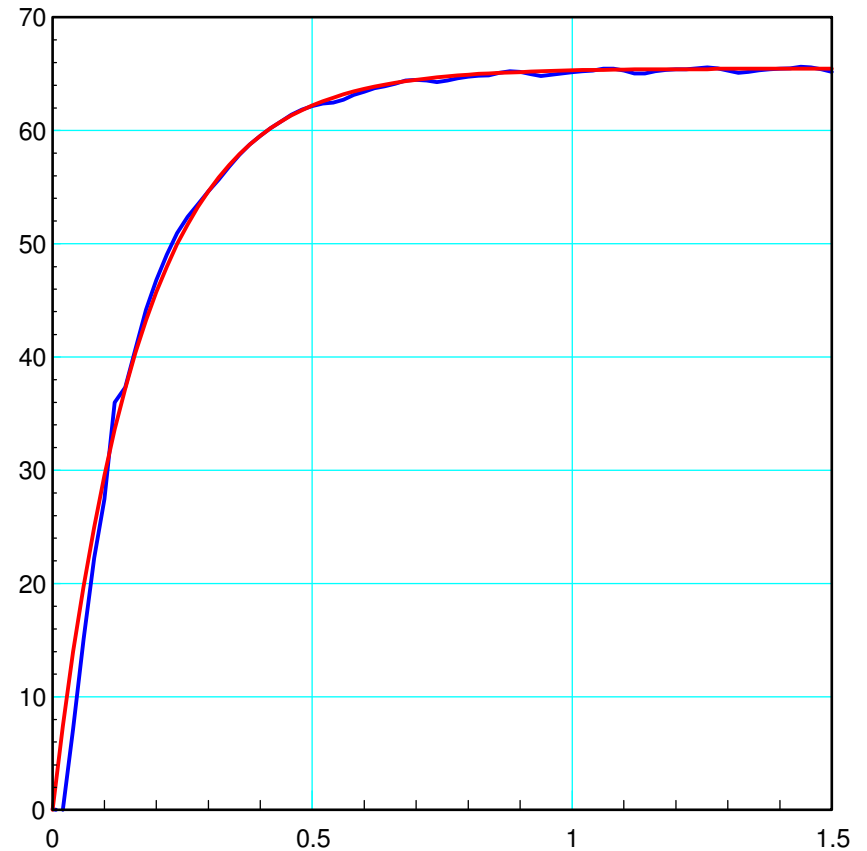
The mathematical model from before:

- Clifton 000-053479-002

$$\omega \approx \left(\frac{39.28}{s+6} \right) V_a$$

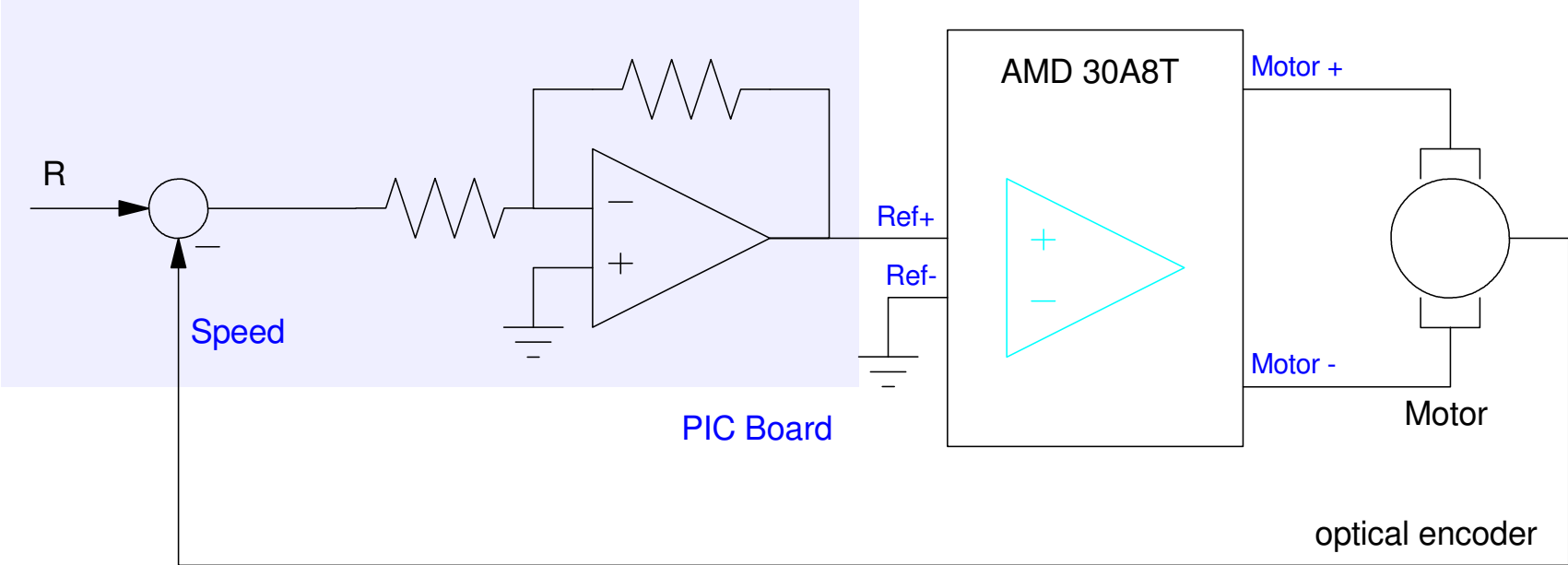
$$\theta \approx \left(\frac{39.28}{s(s+6)} \right) V_a$$

Speed (rad/sec)



Time (seconds)

Hardware Setup

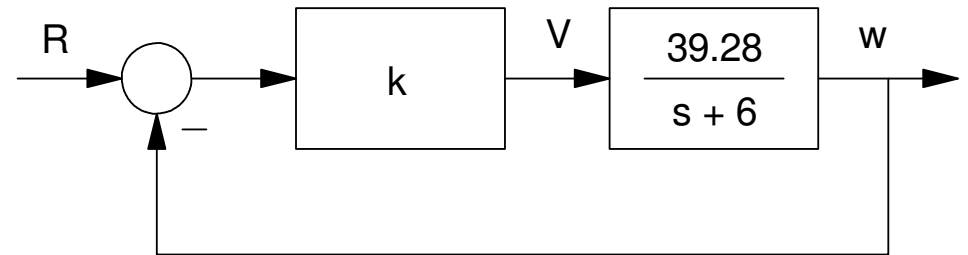


Speed Control: Gain Compensation

$$G(s) = \left(\frac{39.28}{s+6} \right)$$

$$K(s) = k$$

$$GK = \left(\frac{39.28k}{s+6} \right)$$



Type-0 Systems

- $K_p = 6.547 k$
- Steady-State Error for a Step Input

Predicted Response

$$(GK)_s = -1$$

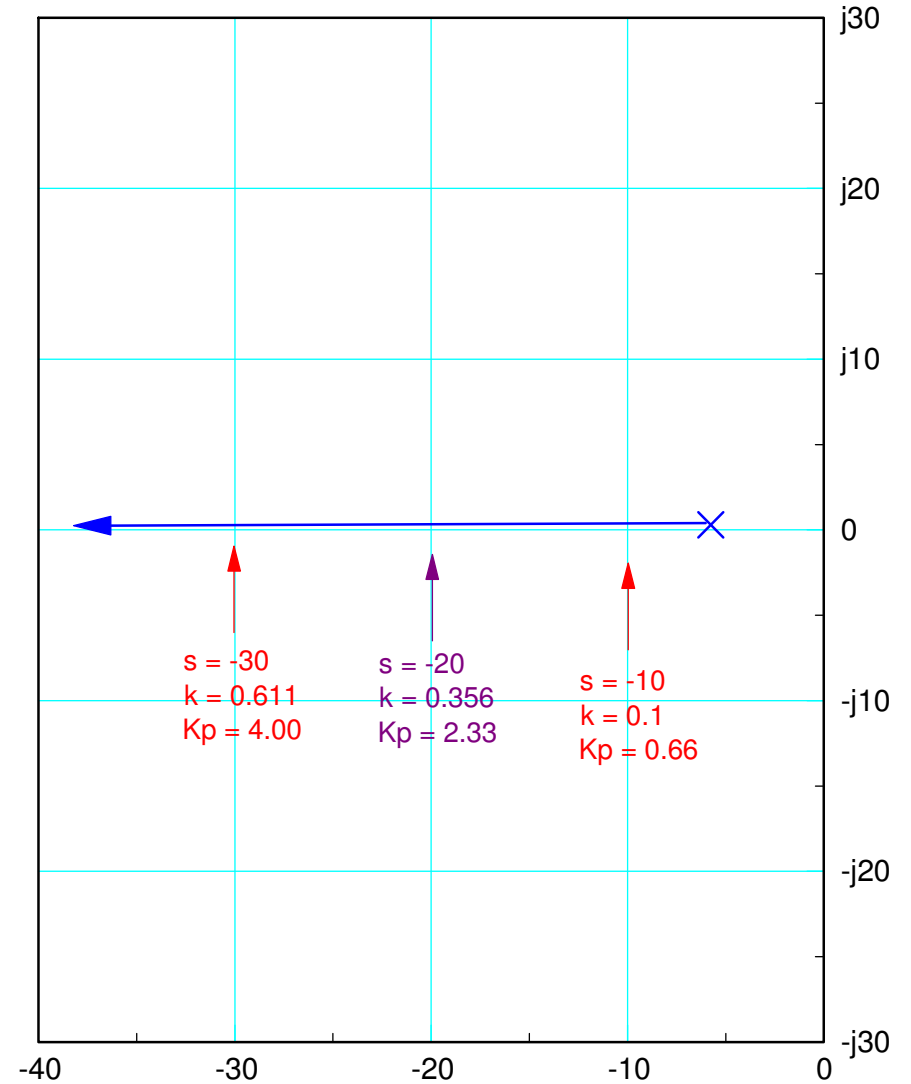
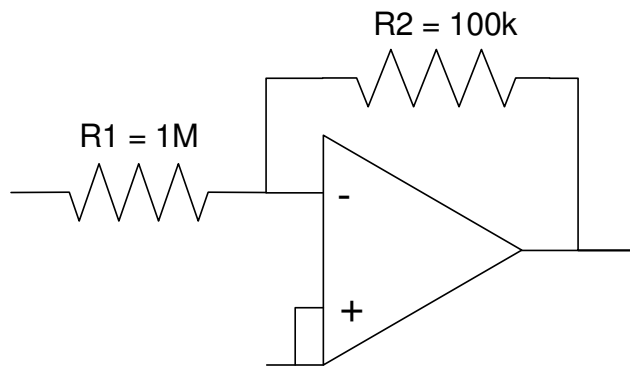
$$s = -10$$

$$\left(\frac{39.28k}{s+6} \right)_{s=-10} = -1$$

$$k = 0.1$$

$$K_p = (GK)_{s=0} = 0.66$$

$$E_{step} = \frac{1}{K_p+1} = 0.602$$



Experimental Results:

$s = -10$

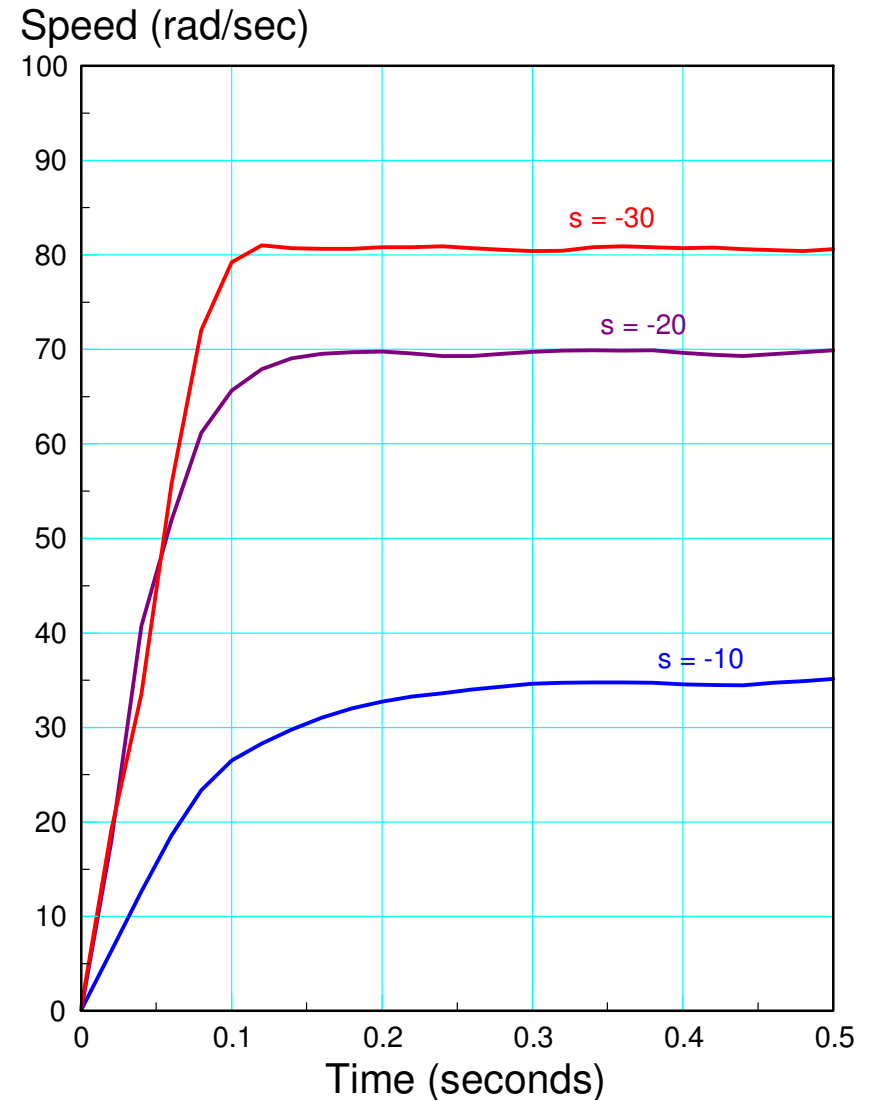
- $k = 0.1$
- $R2 = 100k$

$s = -20$

- $k = 0.356$
- $R2 = 356k$

$s = -30$

- $k = 0.611$
- $R2 = 611k$



Speed Control: I Compensation

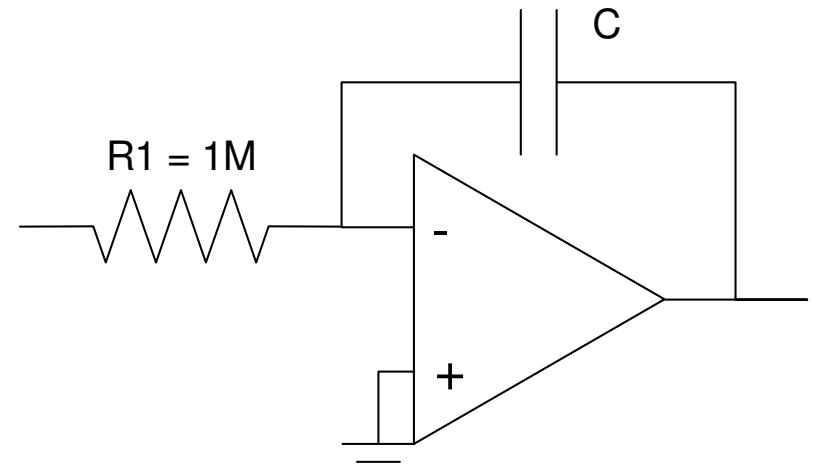
$$K(s) = \left(\frac{k}{s} \right)$$

$$k = \left(\frac{1}{R_1 C} \right)$$

$$GK = \left(\frac{39.68k}{s(s+6)} \right)$$

Type-1 System

- No Error for a Step Input



I Compensation

$$s = -0.5$$

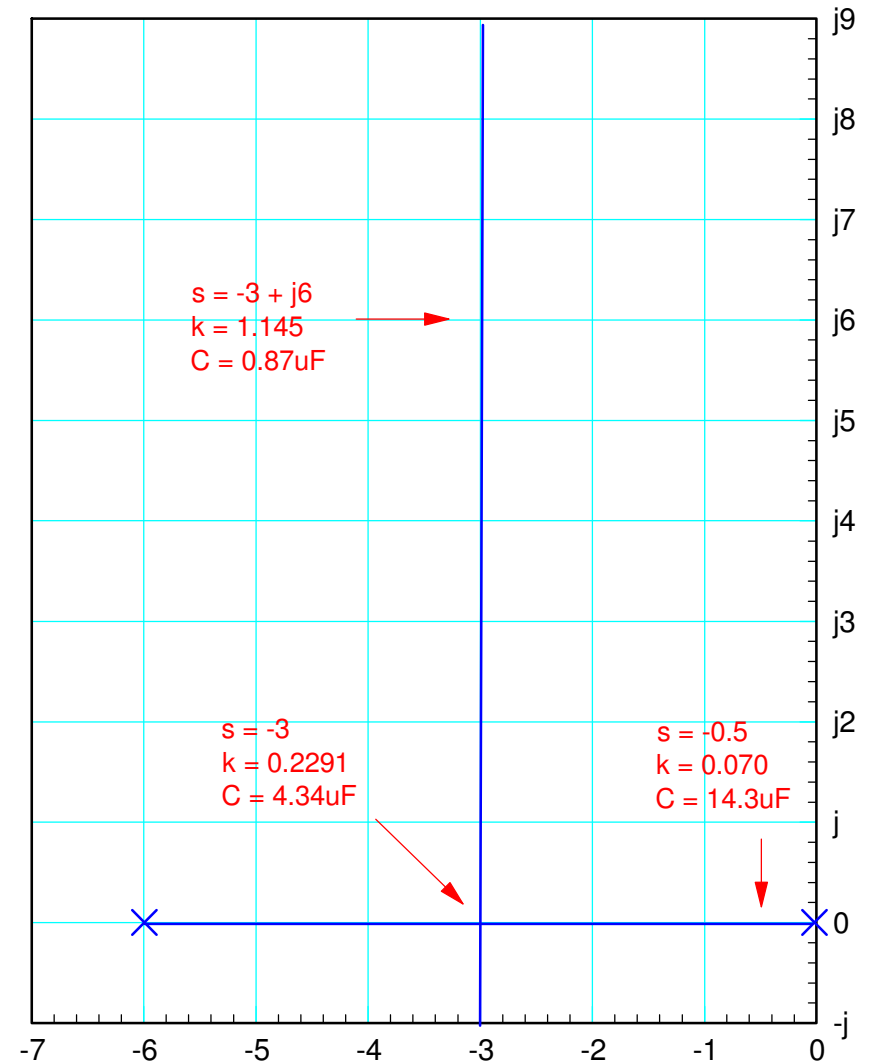
- $\left(\frac{39.68k}{s(s+6)}\right)_{s=-0.5} = -1$
- $k = 0.070$, $C = 14.3\mu\text{F}$

$$s = -3$$

- $\left(\frac{39.68k}{s(s+6)}\right)_{s=-3} = -1$
- $k = 0.2291$, $C = 4.34\mu\text{F}$

$$s = -3 + j6$$

- $\left(\frac{39.68k}{s(s+6)}\right)_{s=-3+j6} = -1$
- $k = 1.145$, $C = 0.87\mu\text{F}$



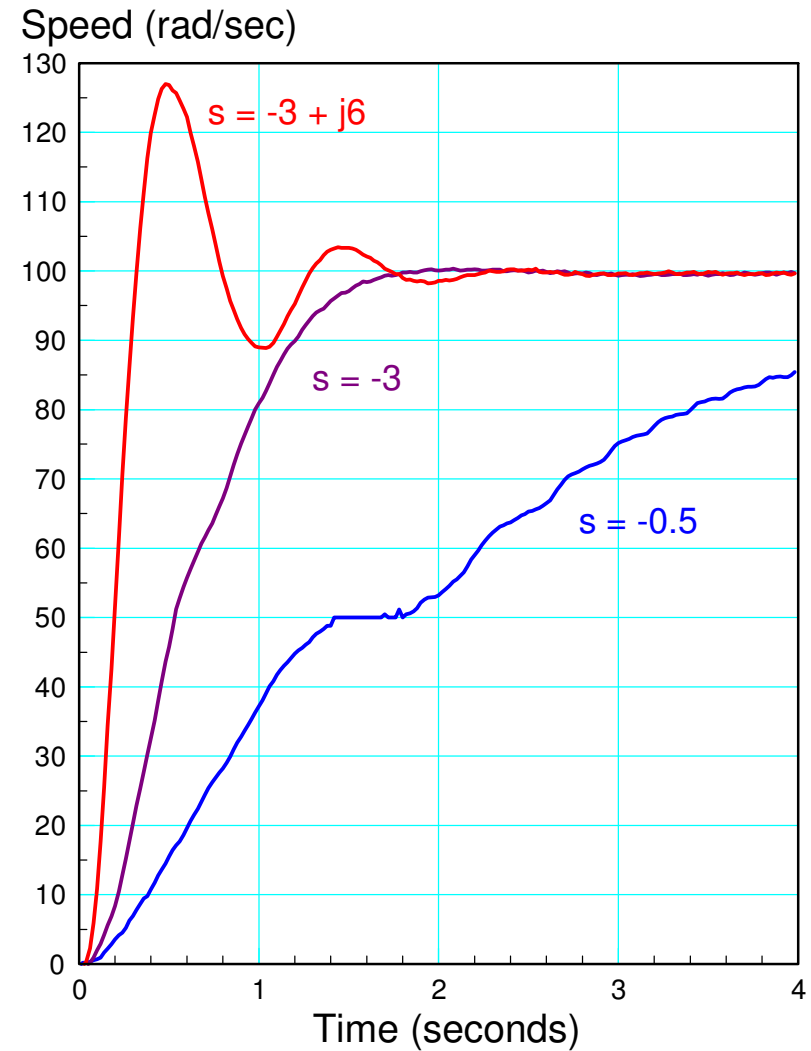
Experimental Results

- I Compensation

Steady-state speed = 100 rad/sec

- No Error for a step input
- Type-1 system

The response is what the root locus plot predicted



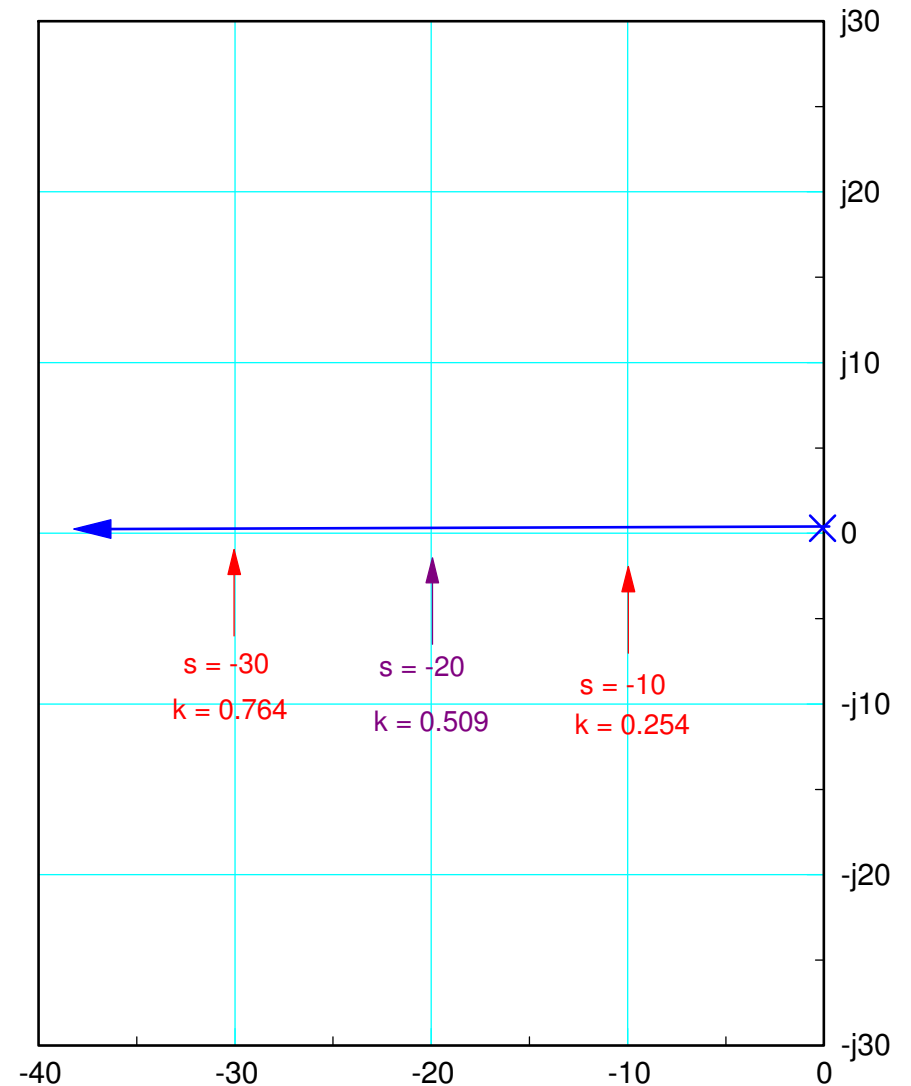
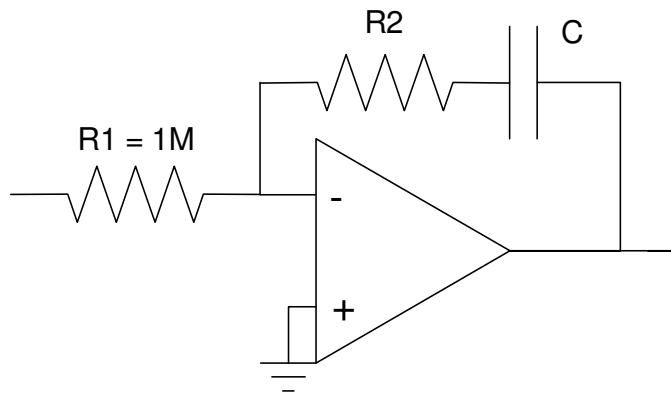
Speed Control: PI Control

$$K(s) = k \left(\frac{s+6}{s} \right)$$

$$GK = k \left(\frac{39.28}{s} \right)$$

$$\left(\frac{1}{R_2 C} \right) = 6$$

$$k = \left(\frac{R_2}{R_1} \right)$$



PI Control: Experimental Results

$s = -10$

$$K(s) = 0.254 \left(\frac{s+6}{s} \right)$$

- $R = 254k$, $C = 0.656\mu F$

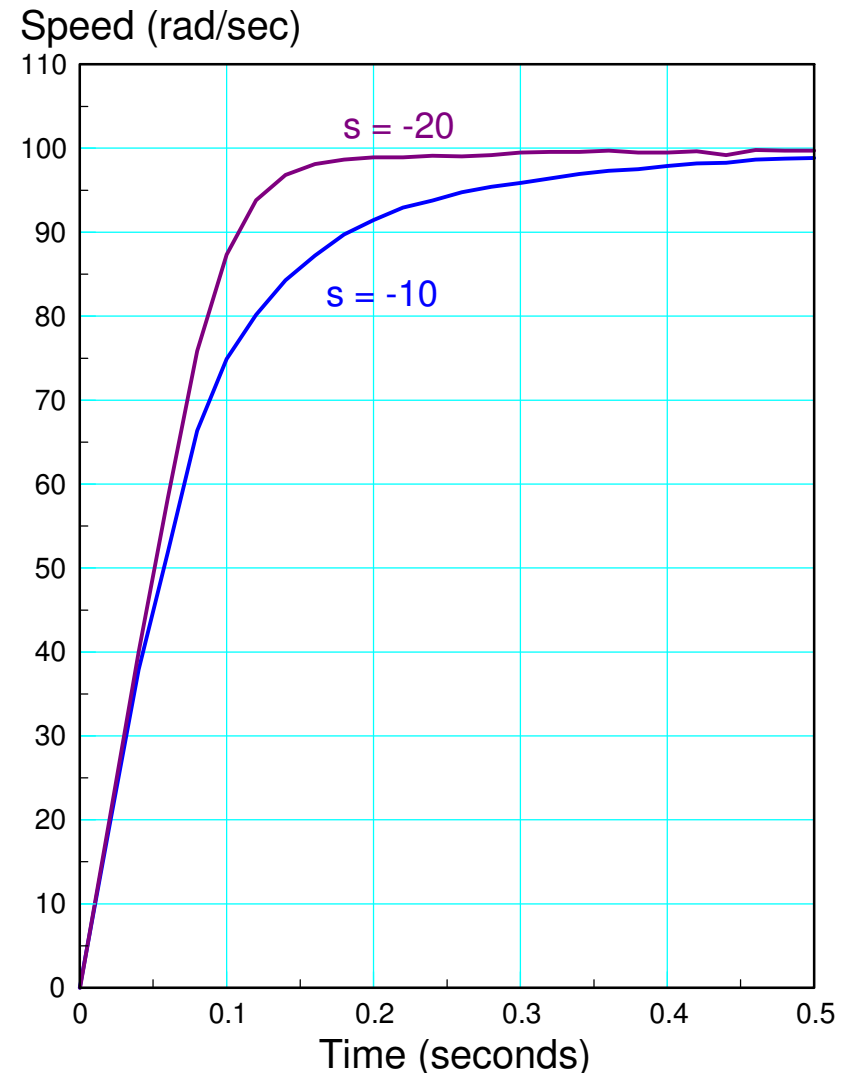
$s = -20$

$$K(s) = 0.509 \left(\frac{s+6}{s} \right)$$

- $R = 509k$, $C = 0.328\mu F$

$$k = 0.509$$

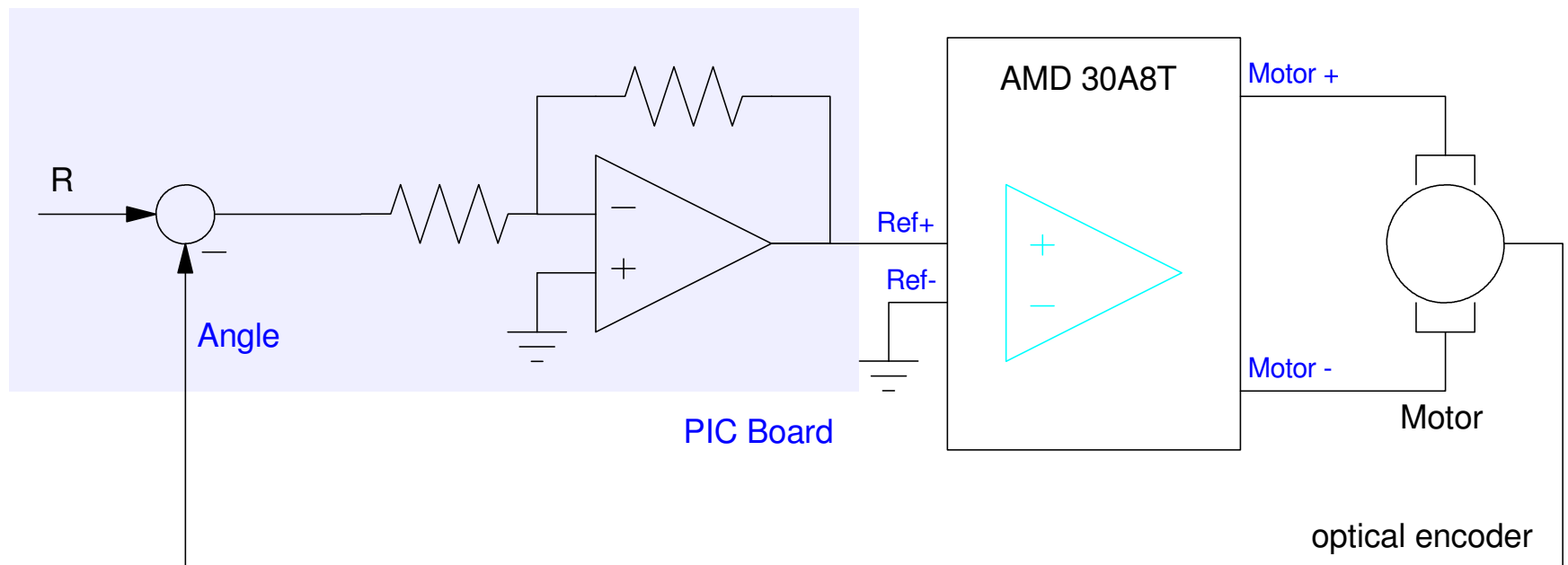
Again, the results are what the root locus plot predicts



Position Control:

- Change the sensor to an angle sensor and you have position control

$$\theta = \left(\frac{39.28}{s(s+6)} \right) V_a$$



Position Control: $K(s) = k$

$$s = -3.0$$

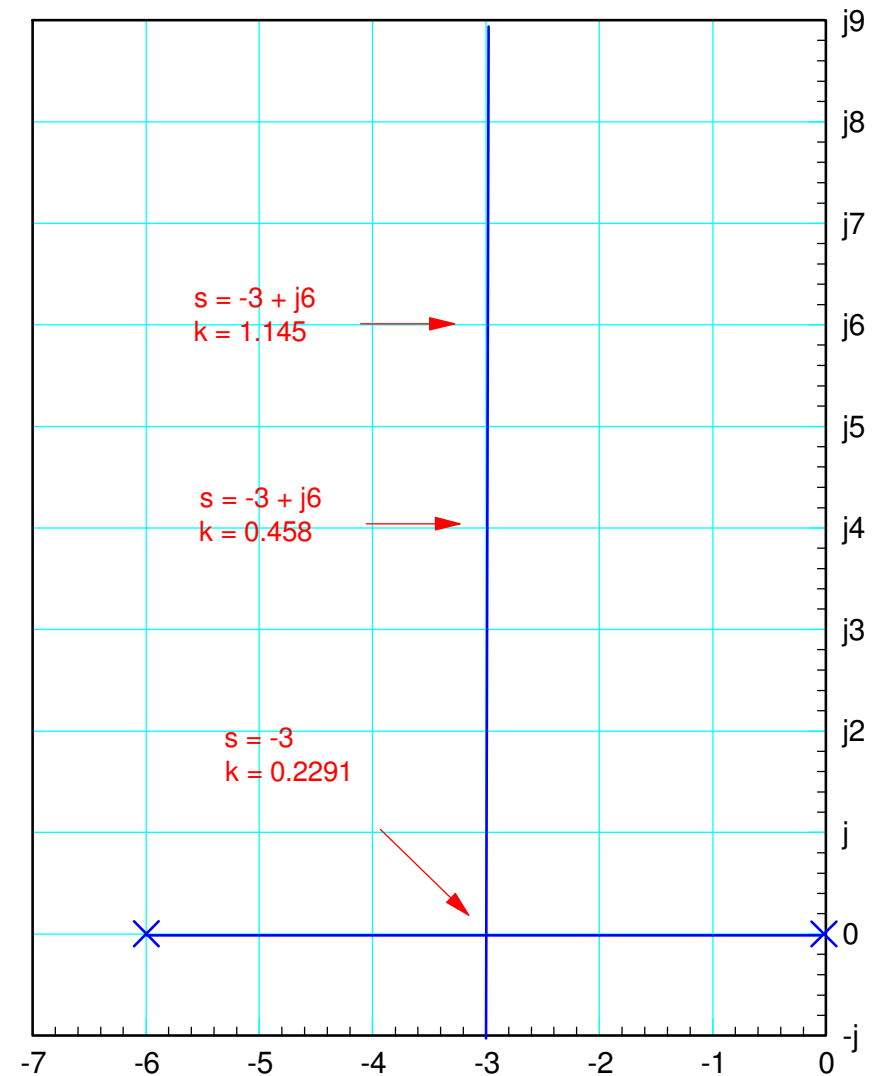
$$k = 0.2291$$

$$s = -3 + j3$$

$$k = 0.458$$

$$s = -3 + j6$$

$$k = 1.1456$$



Position Control: $K(s) = k$

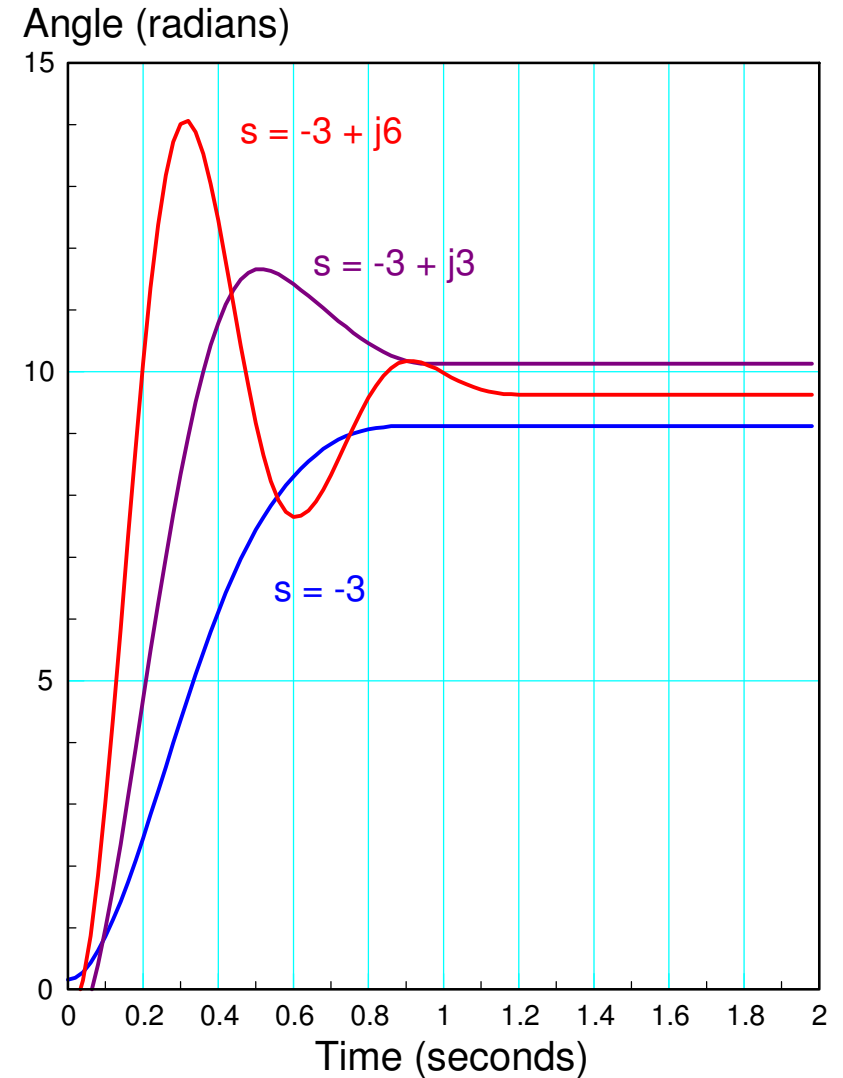
- Experimental Results

In theory, the steady-state error is zero

- Type-1 System

In practice, static friction causes a slight error

Otherwise, the response is what the root locus plot predicts

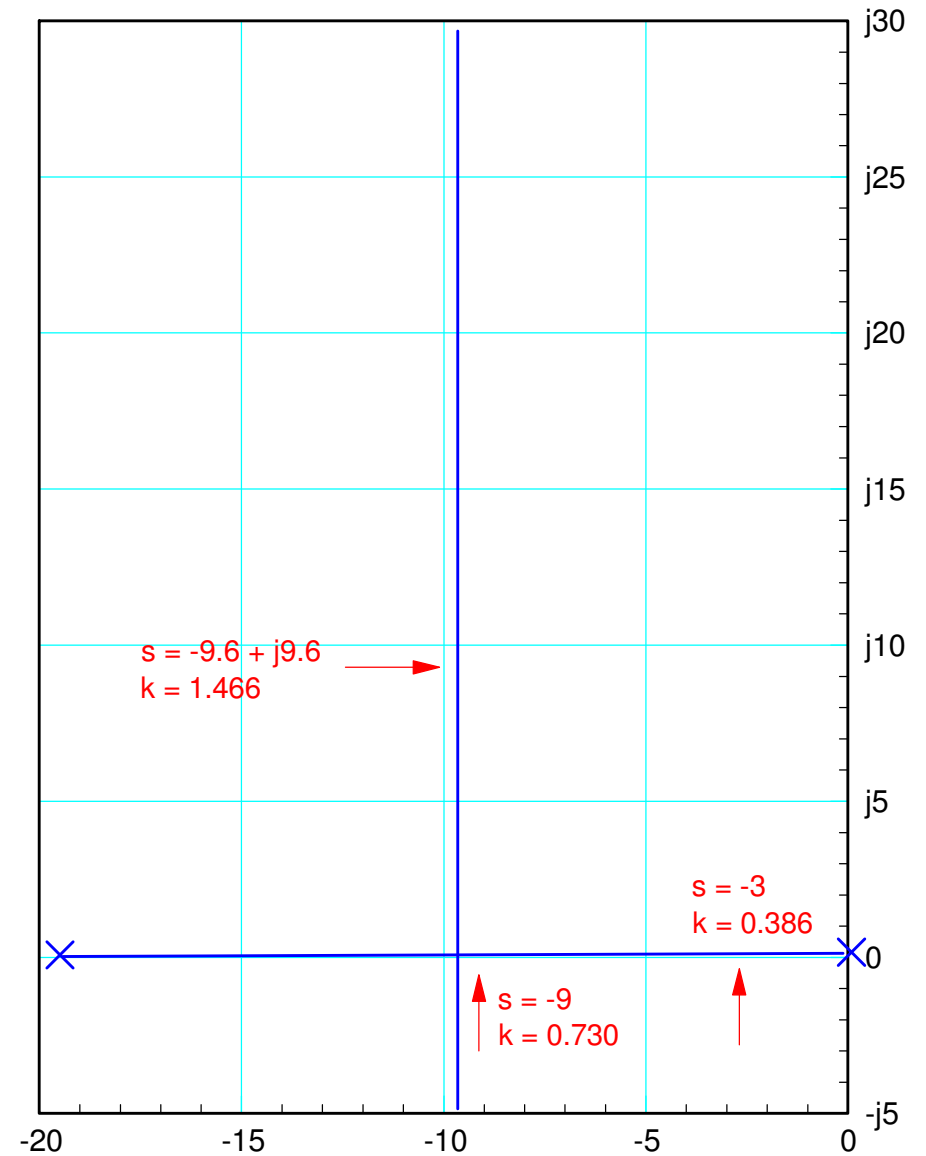
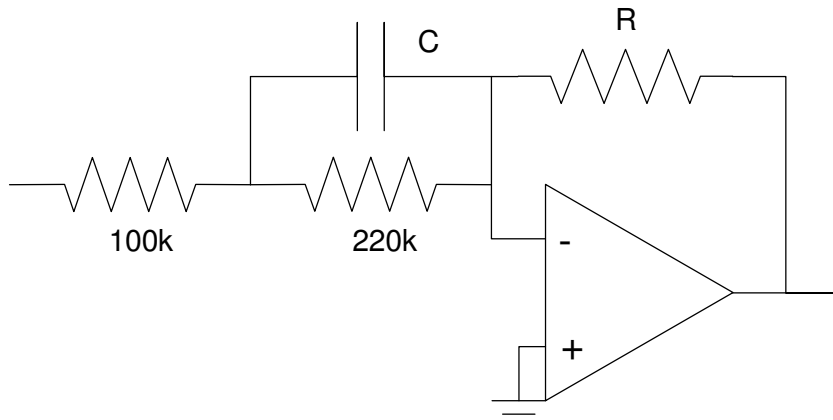


Lead Compensation

- Cancel the pole at -6
- Replace it with a pole at -19.2

$$K(s) = 3.2k \left(\frac{s+6}{s+19.2} \right)$$

$$\left(\frac{1}{220k \cdot C} \right) = 6 \quad C = 757nF$$



Lead Compensation:

$$s = -3$$

- $k = 0.386$, $R = 1.23M$

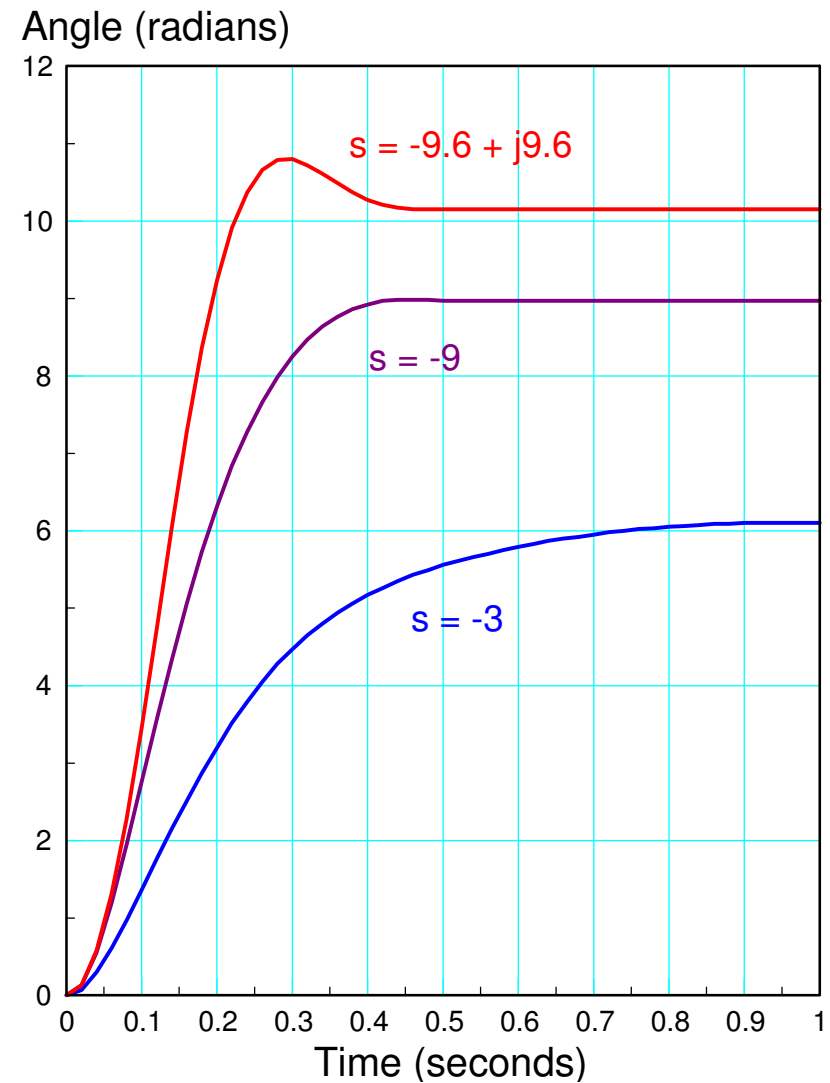
$$s = -9$$

- $k = 0.730$, $R = 2.336M$

$$s = -9.6 + j9.6$$

- $k = 1.466$, $R = 4.69M$

Again, the experimental results are what root locus plots predicted



Summary

Root locus really works

- It predicts how the system will behave as the gain changes
- The response is as the root locus plot predicts

