Gain Compensation using Root Locus

ECE 461/661 Controls Systems

Jake Glower - Lecture #22

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

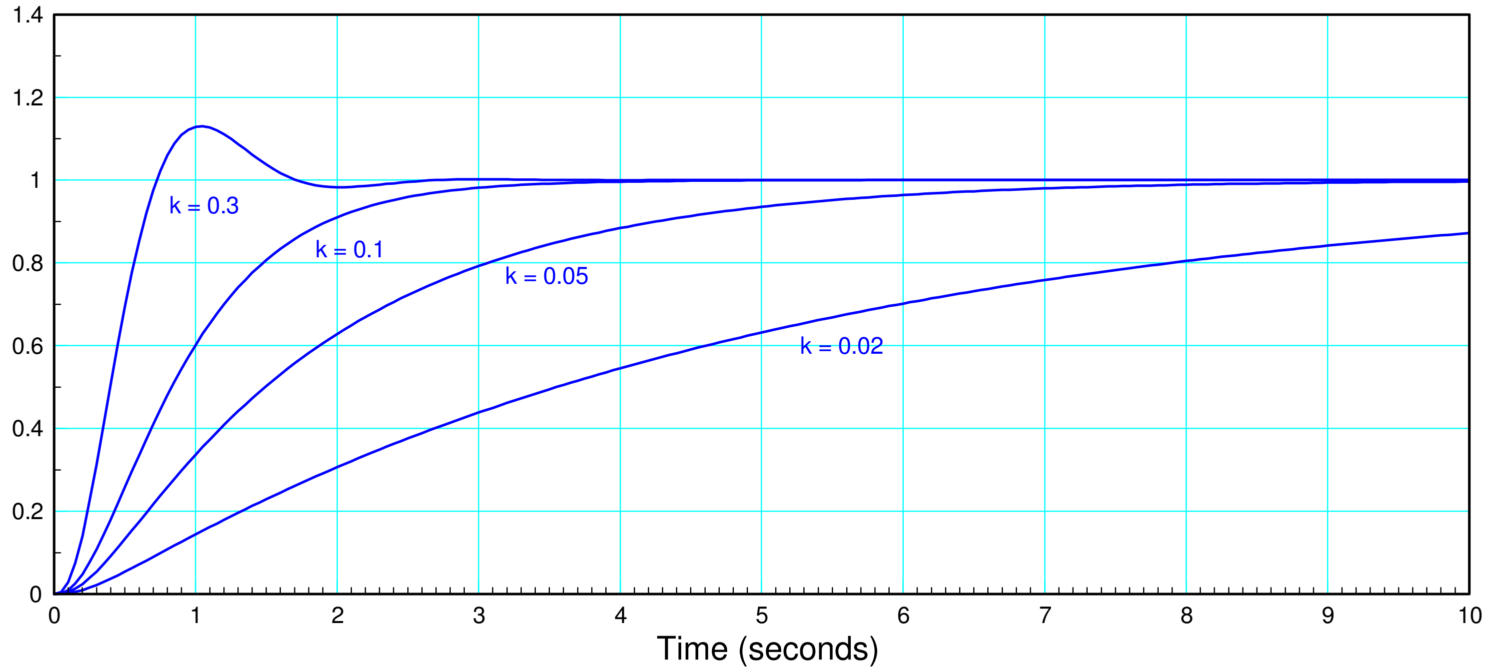
Gain Compensation

Constraint:

- K(s) = k

Goal: Find the "best" value for k

- As large as possible
- Without too much overshoot



Finding k from a root locus plot

Assume $G(s) = \left(\frac{20}{s(s+2)(s+5)} \right)$

Find k for s = -0.1:

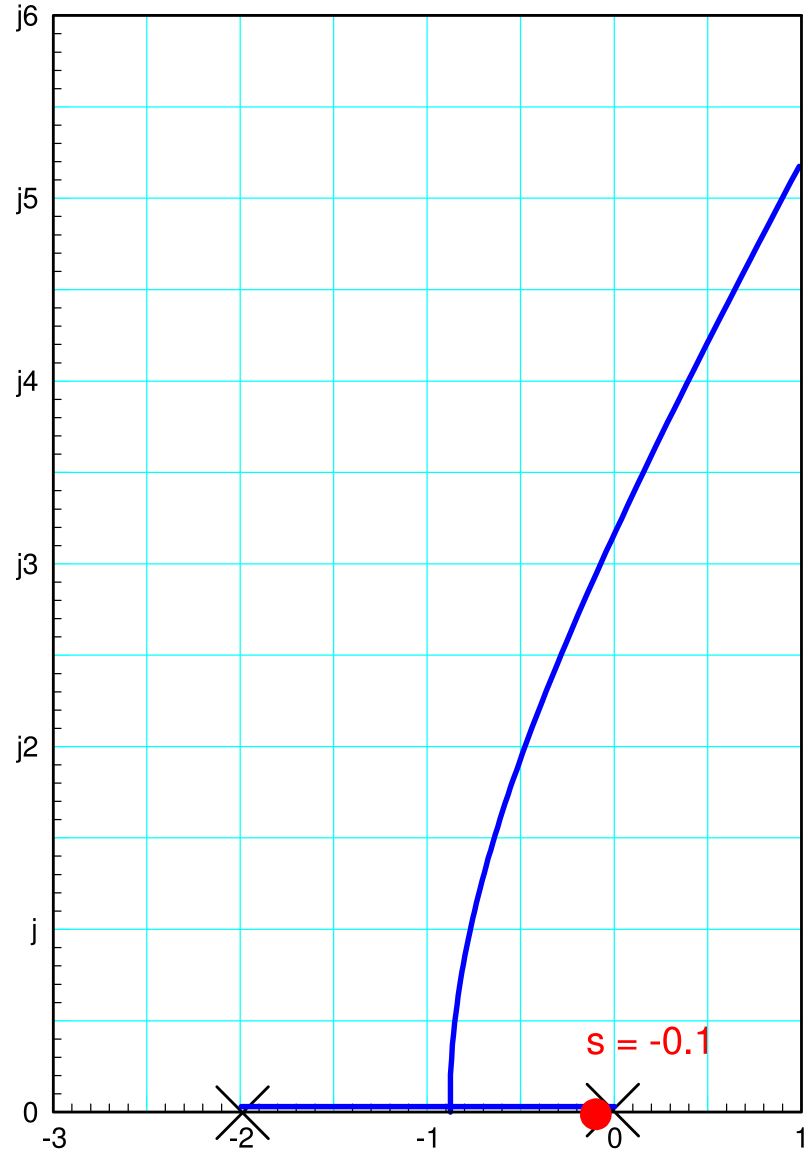
$$(Gk)_{s=-0.1} = -1$$

$$(-21.482) \cdot k = 1\angle 180^0$$

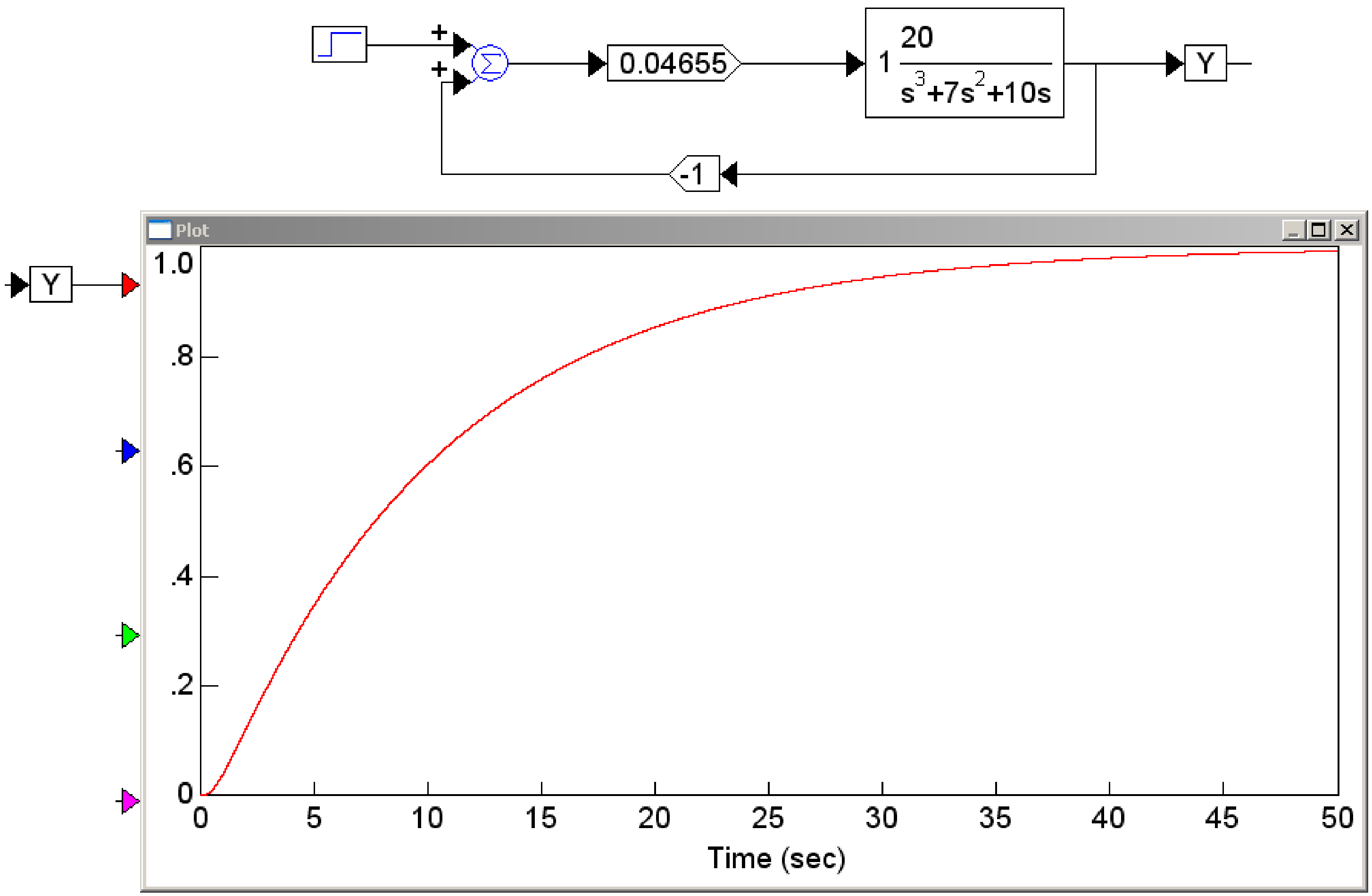
$$k = \frac{1}{21.482} = 0.04655$$

Result:

- No error for a step
  - Type-1 system
- No overshoot
  - real dominant pole
- $T_{2\%}$= 40 seconds
  - 4 / 0.1



Verifying this in VisSim:



Place the closed-loop dominant pole at s = -0.5

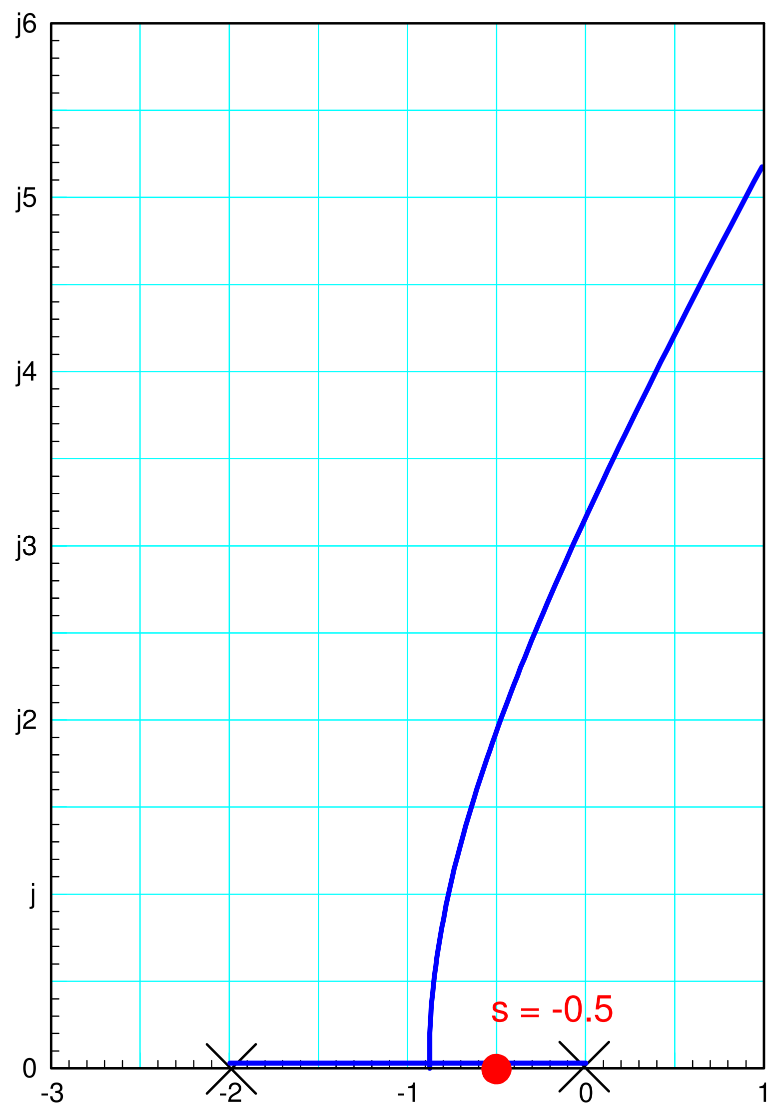
$$\left(\frac{20k}{s(s+2)(s+5)}\right)_{s=-0.5} = 1\angle 180^0$$

$$-5.9259k = 1\angle 180^0$$

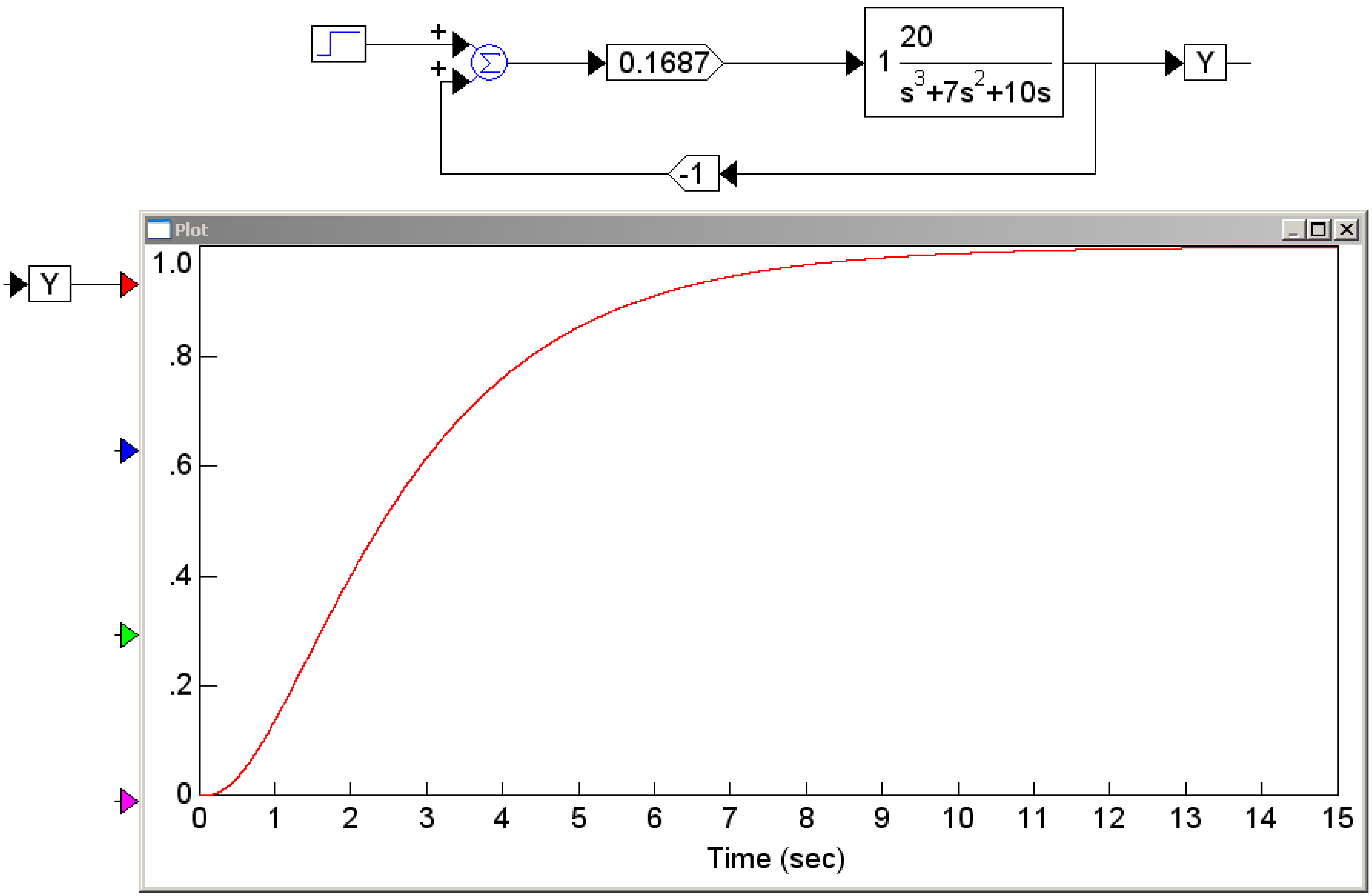
$$k = 0.1687$$

Resulting step response

- No error for a step
  - Type-1 system
- No overshoot
  - real dominant pole
- $T_{2\%}$= 8 seconds
  - Ts = 4 / 0.5

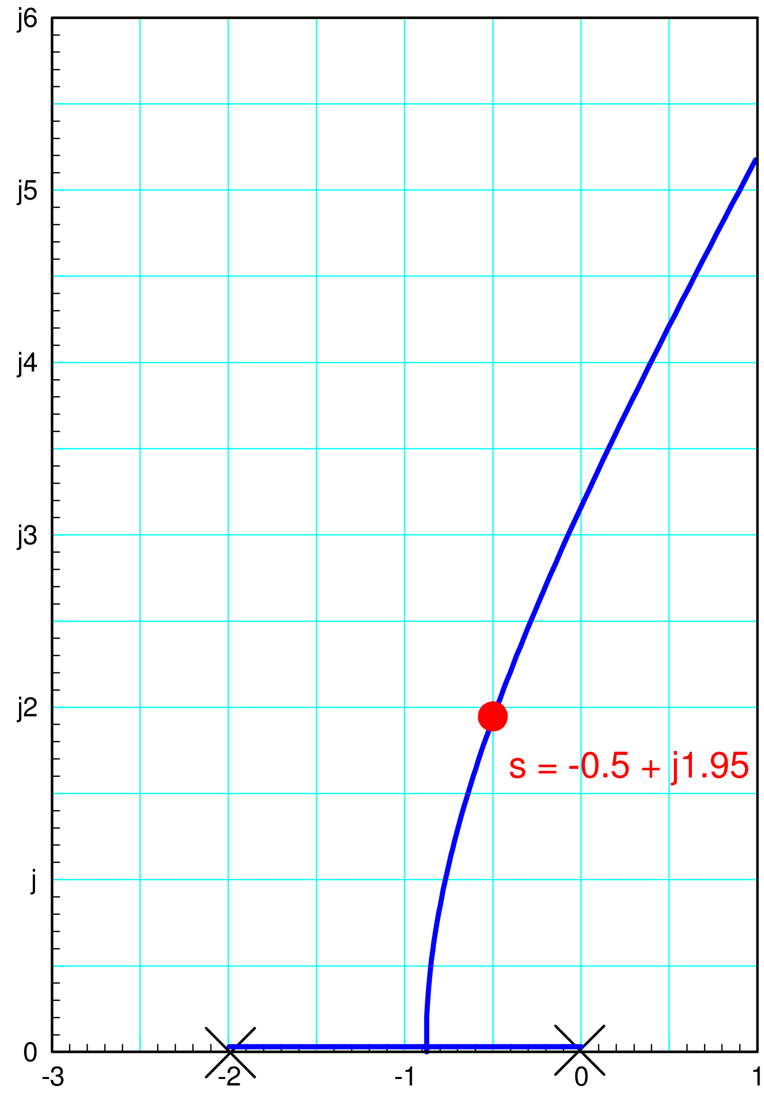


Checking with VisSim:

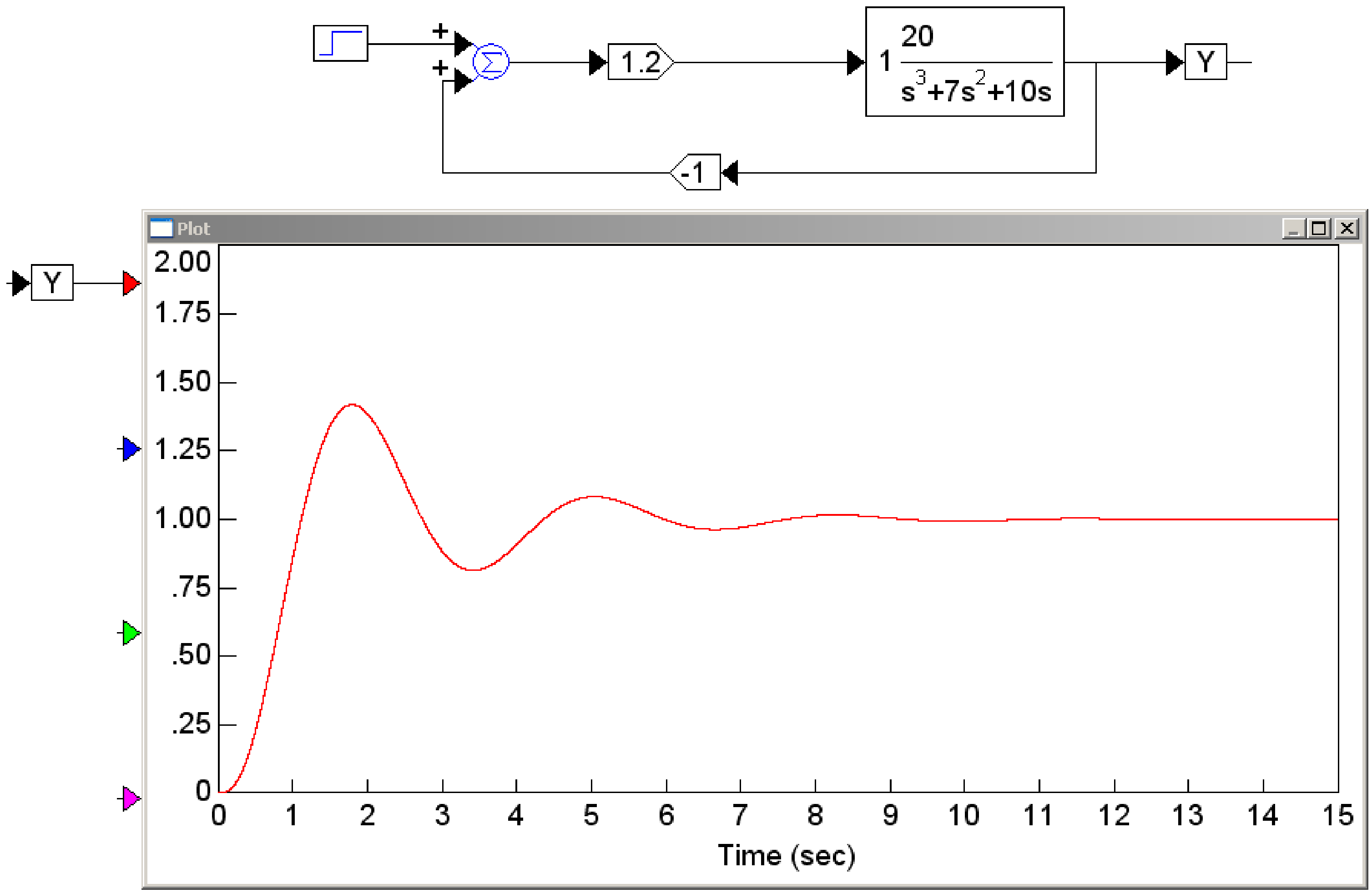


Place the closed-loop dominant pole at s = -0.5 + j1.9365

```
 x = evalfr(G, -0.5 + j*1.9365)

  -0.8333 + 0.0000i

 k = -1/x

   1.2000 + 0.0000i

 k = 1.2;

 Gcl = minreal(G*k / (1+G*k));

zpk(Gcl)

        24
-------------------
(s+6) (s^2 + s + 4)
```



Verifying in VisSim:



Gain Compensation

Find the "best" gain (k)

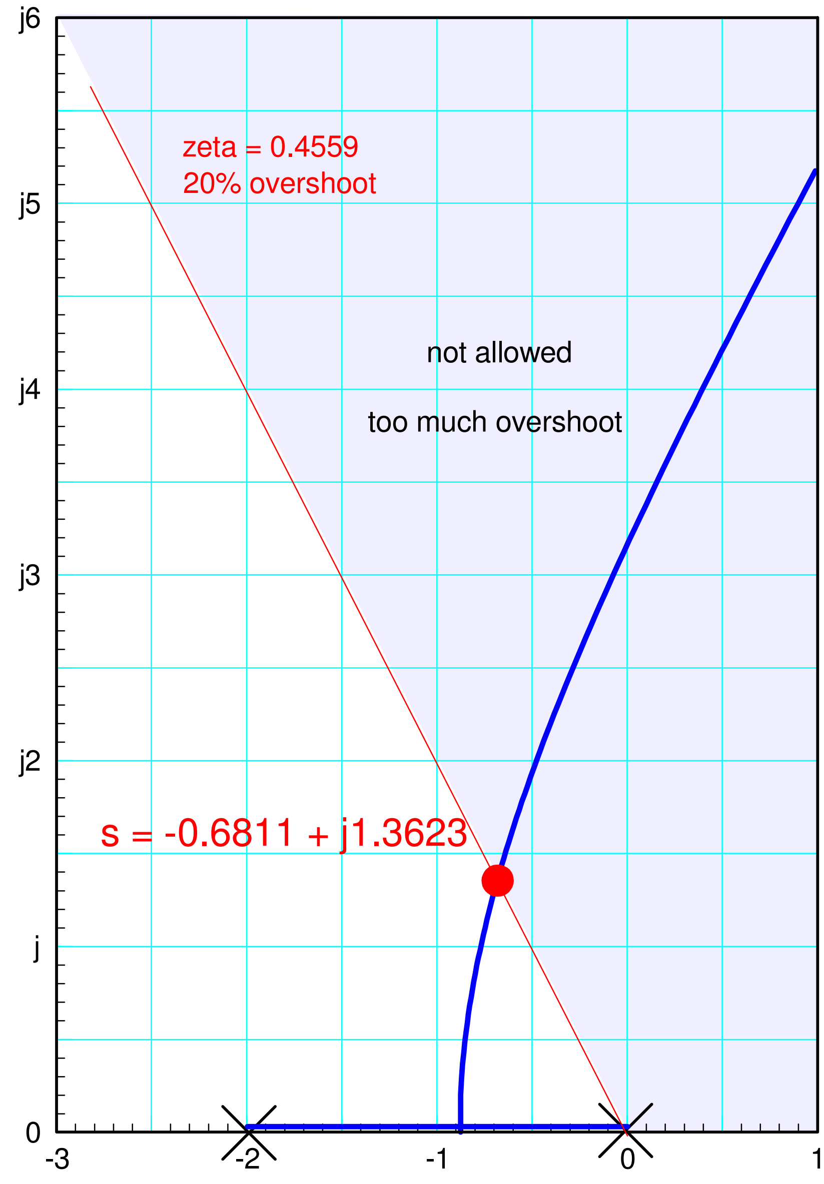
- k is as large as possible, but
- The overshoot for a step input is 20% or less.

Pick k so that the damping ratio is 0.4559 (20% overshoot)

Find the spot on the root locus which intersects the 0.4559 damping line

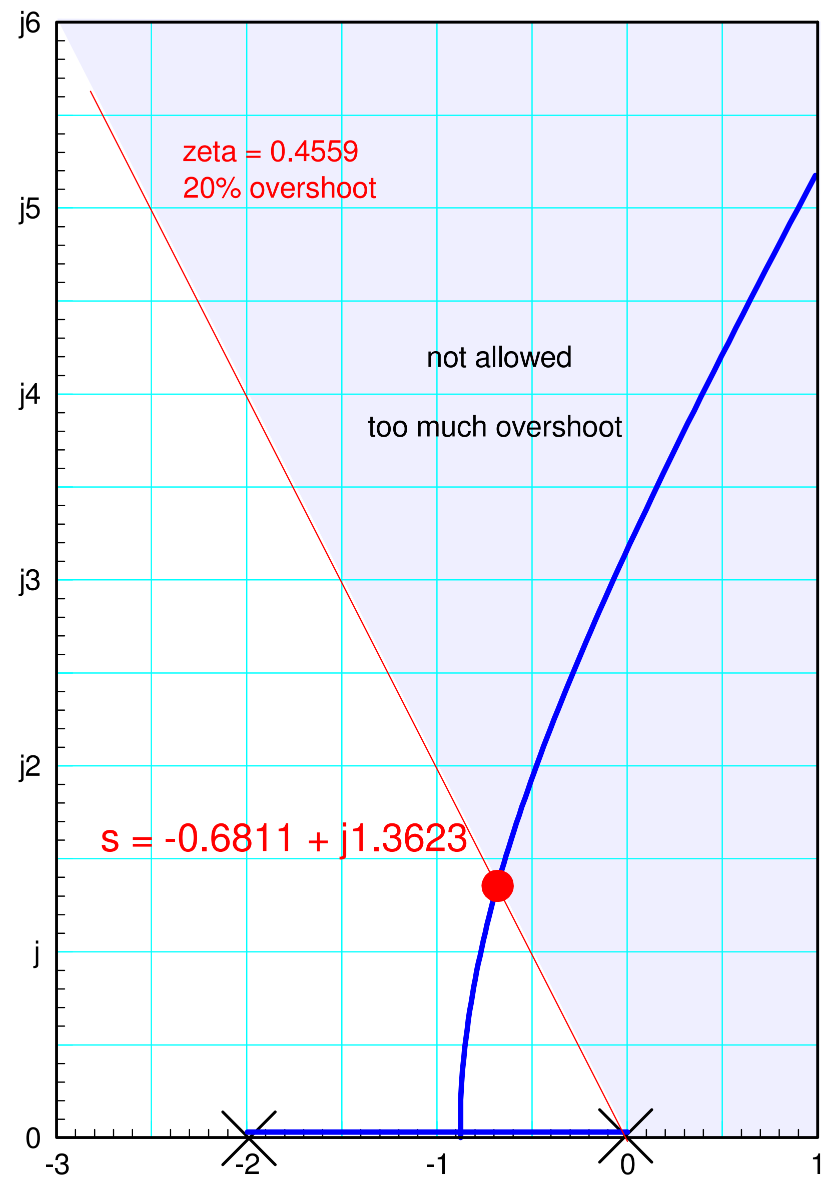
The solution is

$$s = -0.6811 + j1.3623$$

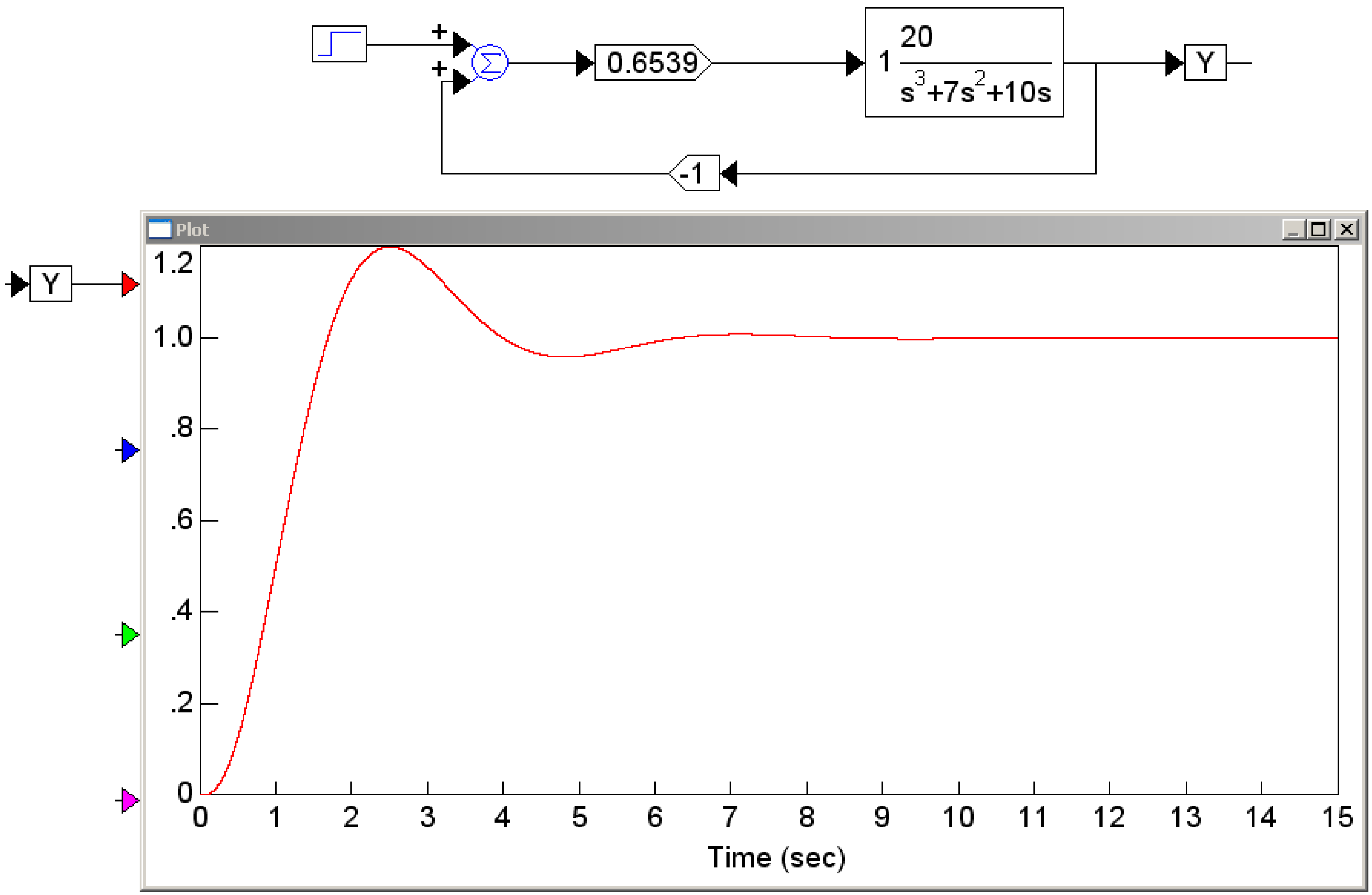


To find k:

```
s = -0.6811 + j*1.3623;
x = evalfr(G, s)

 -1.5292 - 0.0001i

k = 1 / abs(x)

   0.6539

Gcl = minreal(G*k / (1+G*k));
zpk(Gcl)

              13.0786
-------------------------------
(s+5.638) (s^2 + 1.362s + 2.32)
```



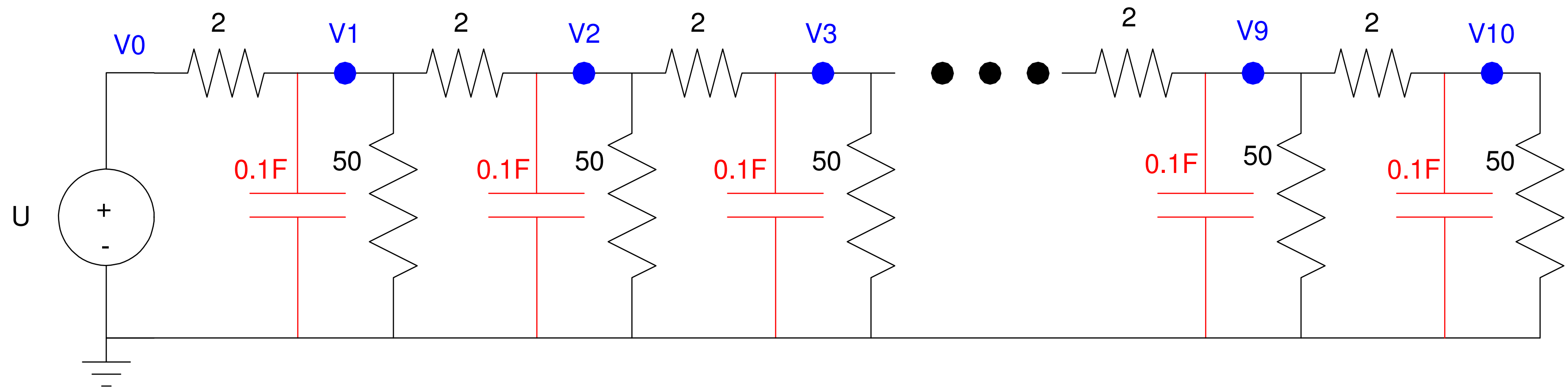
Checking in VisSim:



Example 2: Heat Equation

Control the tip temperature of the following 10th-order RC filter (heat.m) so that

- a) The system is as fast as possible with no overshoot, or
- b) There is 20% overshoot (or less) in the step response



Temperature along a metal bar modeled as a 10th order RC filter

Step 1: Model the system

- 10th-order RC filter in state-space form

$$\frac{10000000000}{(s+39.21)\,(s+36.62)\,(s+32.57)\,(s+27.41)\,(s+21.59)\,(s+15.65)\,(s+10.1)\,(s+5.439)\,(s+2.081)\,(s+0.3234)}$$

Keep the 5 dominant poles, match the DC gain

$$\frac{361.2378}{(s+15.65)\,(s+10.1)\,(s+5.439)\,(s+2.081)\,(s+0.3234)}$$

Sketch the root locus

only the portion near the origin is shown

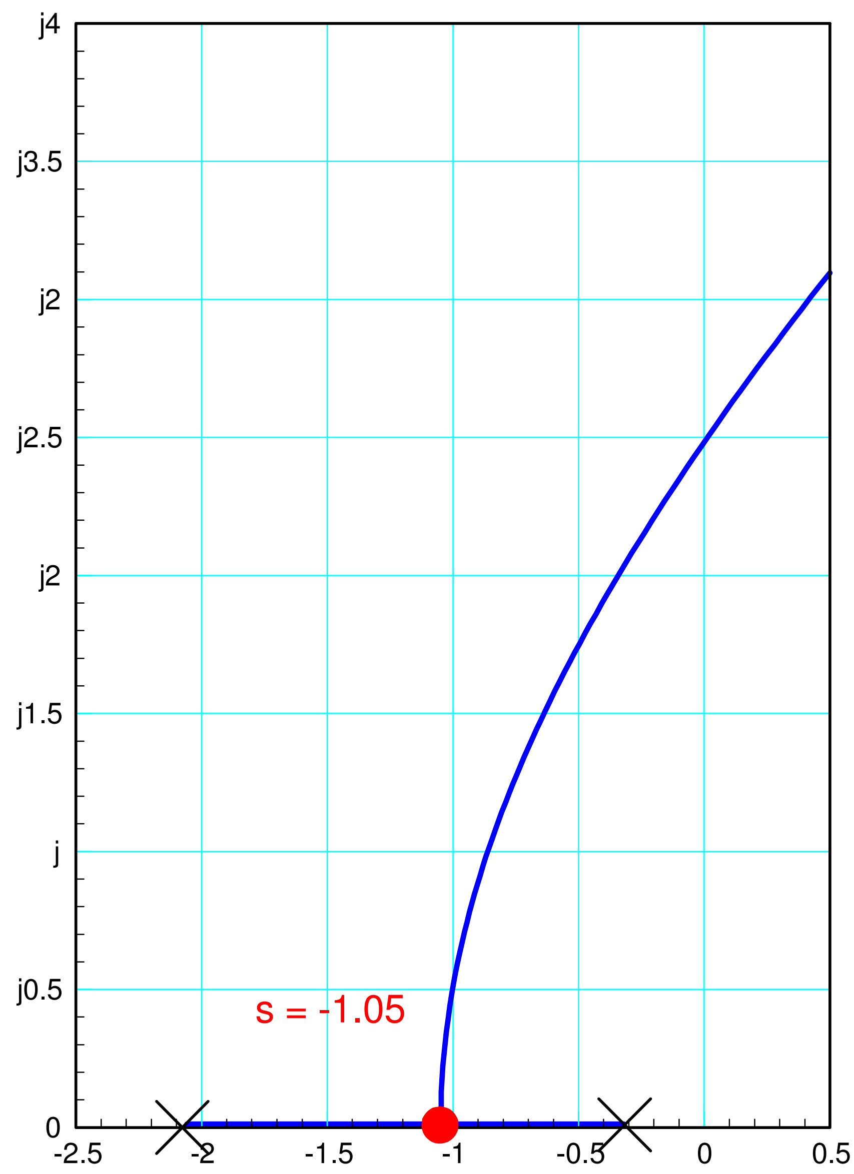
Pick a point on the root locus

a) s = -1.05

- As fast as possible
- With no overshoot

$$(GK)_{s=-1.05} = 1\angle 180^0$$

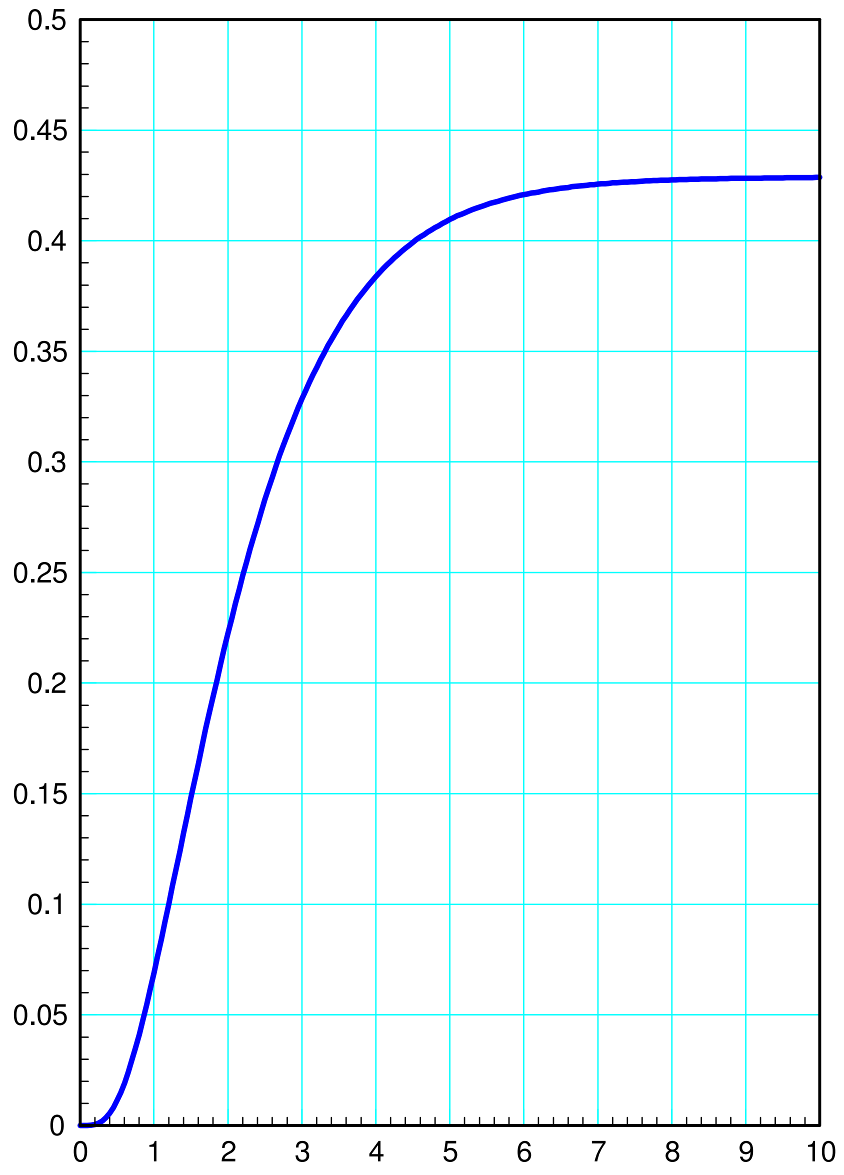
```
evalfr(G5, -1.05)

  -0.8318

k = 1/abs(ans)

   1.2022
```



Checking the step response in Matlab:

- Note the dominant pole is where we placed it

```
Gcl = minreal(G5*k / (1+G5*k));
eig(Gcl)

 -15.6859
  -9.8721
  -5.9352
  -1.0500
  -1.0494

t = [0:0.01:5]';
y = step(Gcl, t);
plot(t,y);
```



b) Find k so that

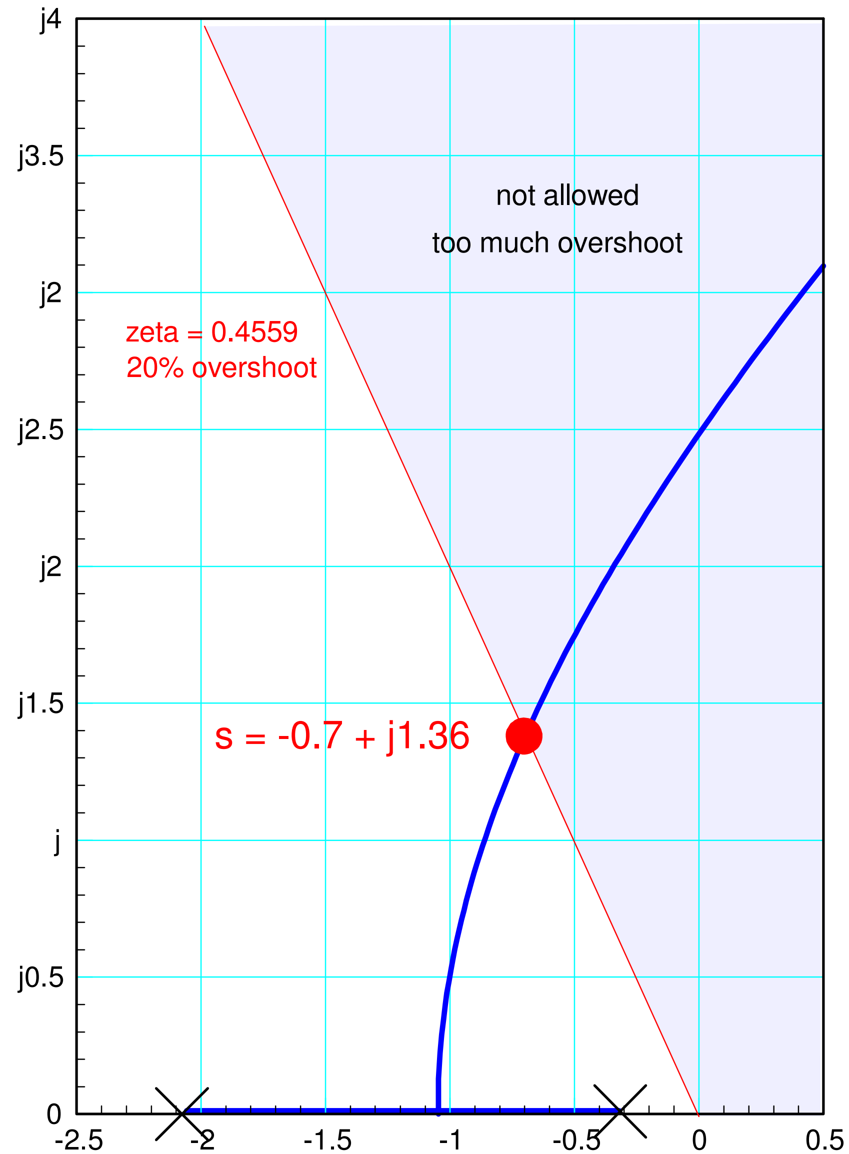
- The feedback gain, k, is as large as possible, but
- There is 20% overshoot (or less) in the step response

Pick s:

$$s = -0.7 + j1.36$$

$$(GK)_{s=-0.7+j1.36} = -1$$

```
s = -0.7 + j*1.36;
evalfr(G5,s)

 -0.1879 - 0.0017i

k = 1/abs(ans)

   5.3217
```



Checking in Matlab

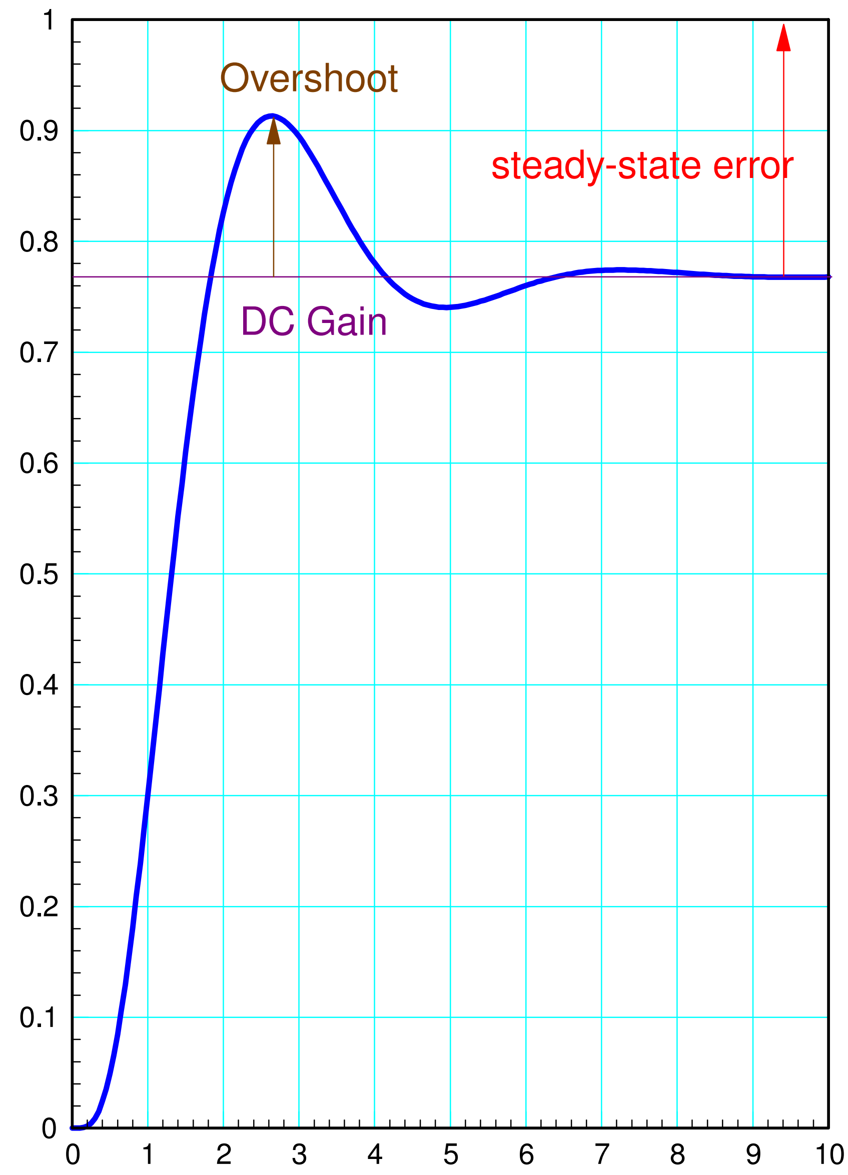
```
Gcl = minreal(G5*k / (1+G5*k));
y = step(Gcl, t);
plot(t,y);
```

Actual overshoot is 18.79%

```
DC = evalfr(Gcl,0)

    0.7687

OS = (max(y) - DC) / DC

    0.1879
```

Steady-state error is less

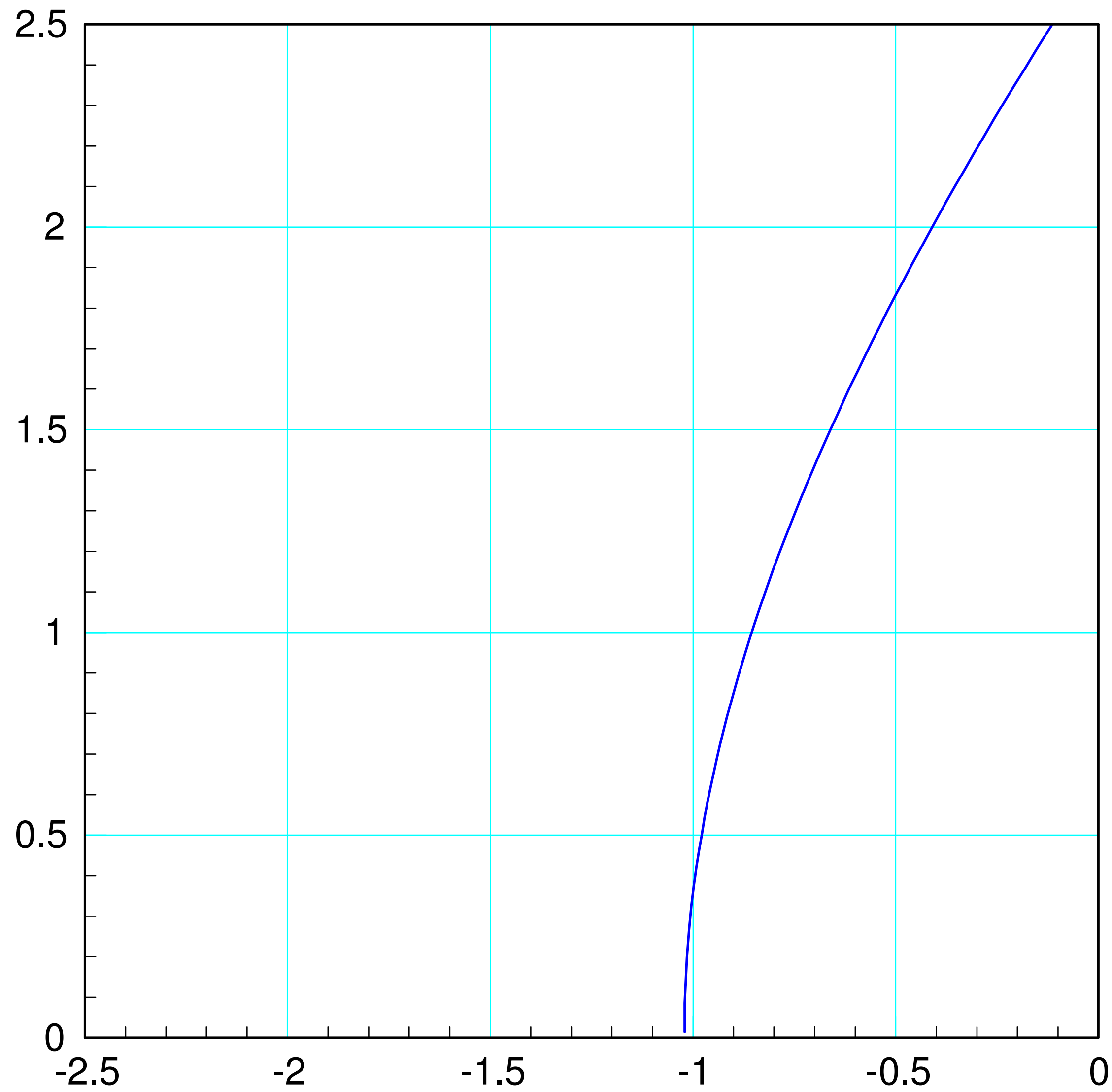
- Gain is higher



Handout:

$$G(s) = \left(\frac{200}{(s+0.3)(s+2)(s+5)(s+10)}\right)$$

Find k for 20% overshoot



Summary: Gain Compensation

The root locus plot tells you how the poles shift as the gain changes

The 'best' gain is usually

- The largest gain,
- That results in acceptable overshoot

That gain can be found from

$$\left|G(s)\cdot k\right|_s = 1$$