Error Constants and Steady-State Error

ECE 461/661 Controls Systems

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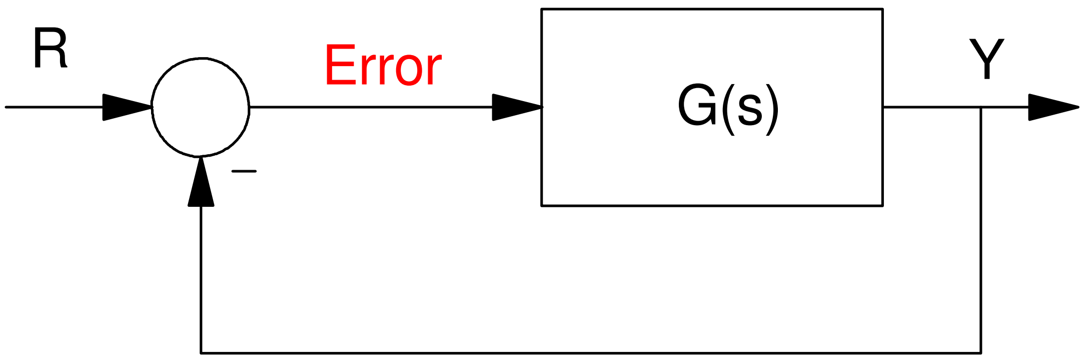
Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Unity Feedback

- Compare the desired output (R = Reference) to the actual output (Y)
- Creates an error driven device

Example: Cruise control on a car

- If R > Y (E > 0), accelerate
- If R < Y (E < 0), decelerate
- If R = Y (E = 0), do nothing



Error Constants

What is the steady state error?

- Zero is the goal

What is the steaty-state error for

- R = 1 (unit step) models tracking a constant set point
- R = t (unit ramp) models tracking a set point with constant velocity
- $R = t^2$ (unit parabola) models tracking a set point with constant acceleration

Error constants attempt to assign a single number to a system

- Bigger is better
- Large error constants mean small error

Definitions:

Type N System: The plant, G(s), has N poles at s=0.

Error Constants: Kp, Kv, Ka. The DC gain of a plant:

Kp: The DC gain of a type-0 system

- $K_p = \lim_{s \to 0} (G(s))$

Kv: The DC gain of a type-1 system:

- $K_v = \lim_{s \to 0} (s \cdot G(s))$ or $\lim_{s \to 0} (G(s)) = \frac{K_v}{s}$

Ka: The DC gain of a type-2 system:

- $K_a = \lim_{s \to 0} (s^2 \cdot G(s))$ or $\lim_{s \to 0} (G(s)) = \frac{K_z}{s^2}$

Steady-State Error: The error as $t \to \infty$ for a unity feedback system

Unit Inputs

Unit Step: $R(s) = \frac{1}{s}$

- Historically modeled trying to point an antiaircraft gun at a German bomber in WWII.
- Models tracking a constant setpoint, (room = 72F, speed = 55mph, etc)

Unit Ramp: $R(s) = \frac{1}{s^2}$

- Historically modeled trying to track a German bomber moving at a constant speed across the sky.
- Models tracking a setpoint which is rising or falling at a constant rate.

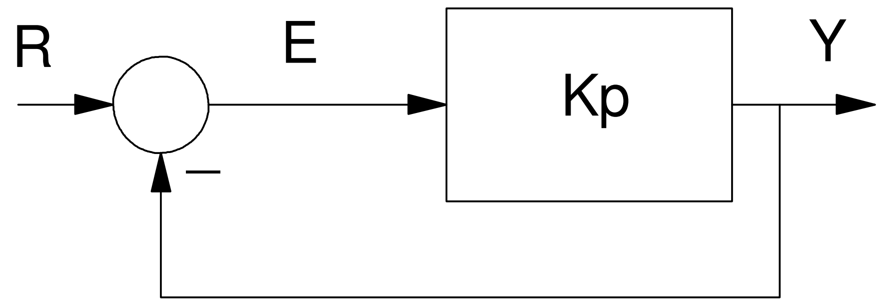
Unit Parabola: $R(s) = \frac{1}{s^3}$

- Historically, models German bombers as they fly over you (the angle of the antiaircraft gun whips around at the zenith).
- I'm not sure what this models.

Type-0 Systems:

i) if R(s) is a unit step:

$$E = \left(\frac{1}{K_p+1}\right)\left(\frac{1}{s}\right) \Rightarrow e(t) = \left(\frac{1}{K_p+1}\right)$$



ii) if R(s) is a unit ramp:

$$E(s) = \left(\frac{1}{K_p+1}\right)\left(\frac{1}{s^2}\right) \Rightarrow e(t) = \left(\frac{1}{K_p+1}\right)t$$

error goes to infinity

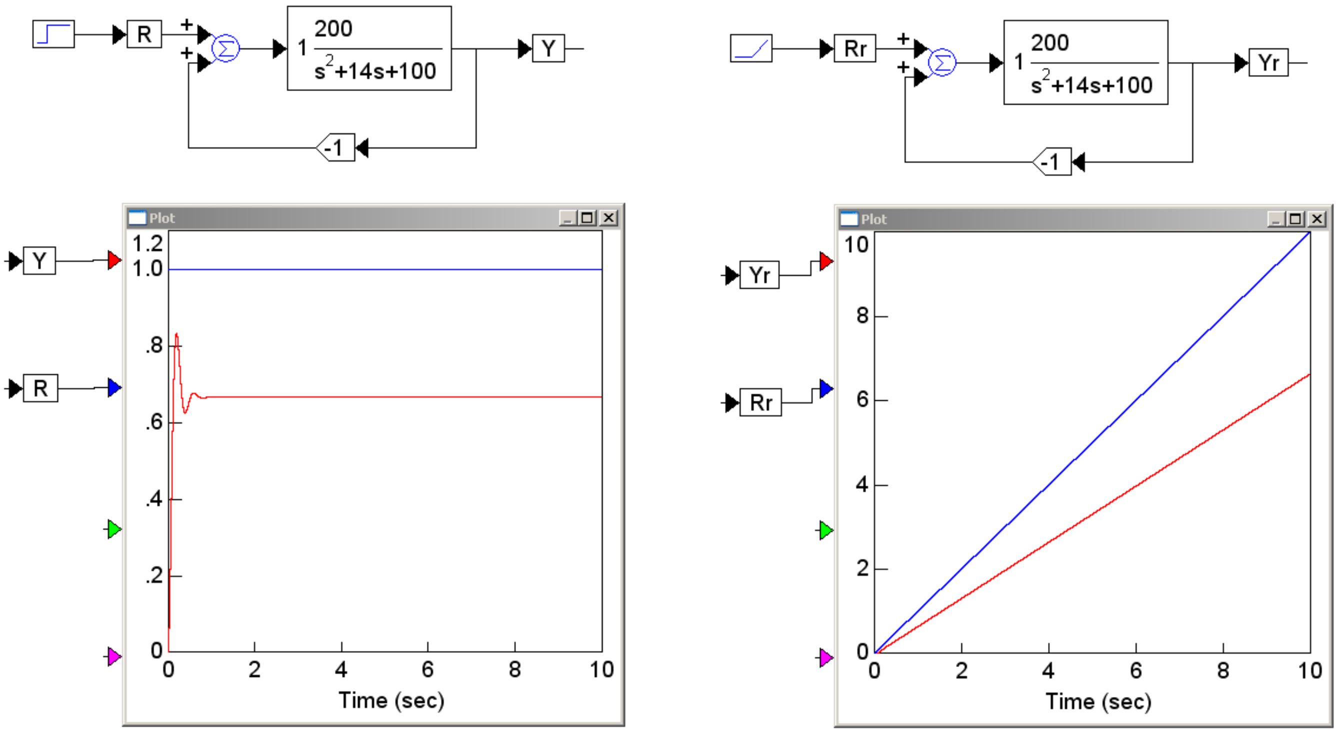
Example:

$$G(s) = \left(\frac{200}{s^2+14s+100}\right)$$

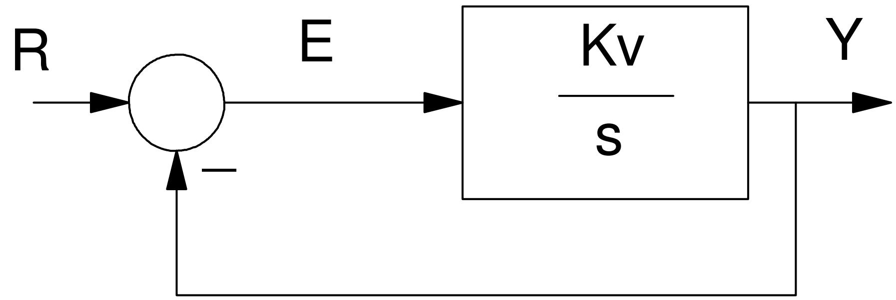
$$K_p = G(s)_{s\to 0} = 2$$

$$E_{step} = \frac{1}{K_p+1} = \frac{1}{3}$$

$$E_{ramp} = \infty$$



Type-1 Systems:



i) R = a unit step:

$$E = \left(\frac{s}{K_v+s}\right)\left(\frac{1}{s}\right) = \left(\frac{0}{s}\right) + \left(\frac{1}{K_v+s}\right)$$

$$e(t) = 0 + transient$$

ii) R = a unit ramp:

$$E = \left(\frac{s}{K_v+s}\right)\left(\frac{1}{s^2}\right) = \left(\frac{1}{K_v+s}\right)\left(\frac{1}{s}\right) = \left(\frac{\frac{1}{K_v}}{s}\right) + \left(\frac{c}{s+K_v}\right)$$

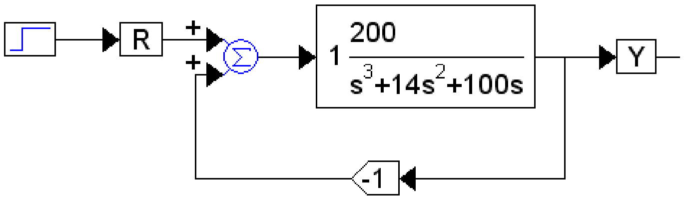
$$e(t) = \left(\frac{1}{K_v}\right) + transients$$

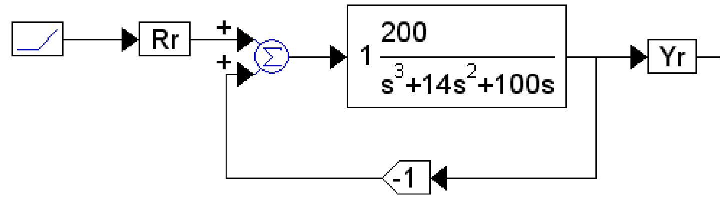
iii) R = a unit parabola:

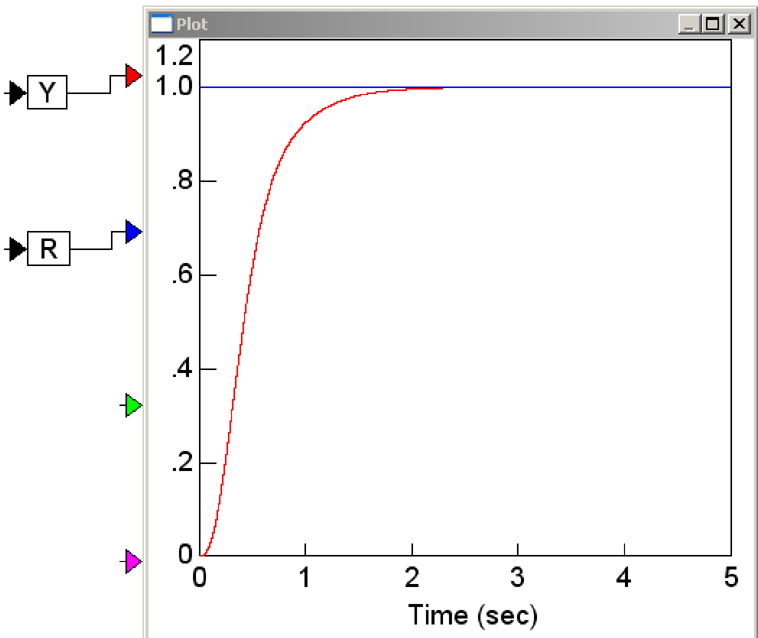
$$E = \left(\frac{s}{K_v+s}\right)\left(\frac{1}{s^3}\right) = \left(\frac{1}{K_v+s}\right)\left(\frac{1}{s^2}\right) = \left(\frac{\frac{1}{K_v}}{s^2}\right) + \left(\frac{c}{s}\right) + \left(\frac{d}{s+K_v}\right)$$

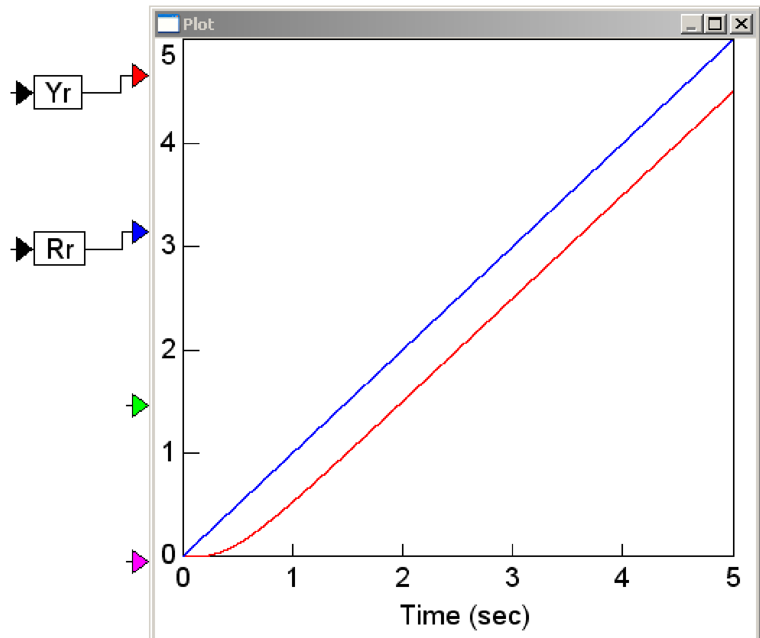
$$e = \left(\frac{1}{K_v}\right)t + \ldots$$

Example: $G(s) = \left(\frac{200}{s\left(s^2+14s+100\right)}\right)$ $G(s)_{s\to 0} = \frac{2}{s}$ $K_p = \infty$ $K_v = 2$

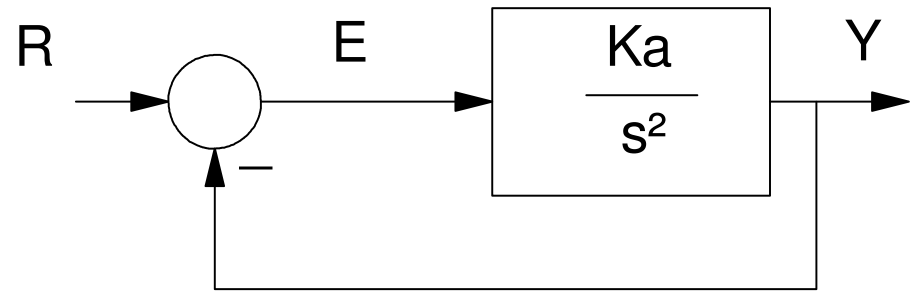








Type-2 systems:



i) If R is a unit step:

$$E = \left(\frac{1}{1+\frac{K_a}{s^2}}\right)\frac{1}{s} = \left(\frac{s^2}{s^2+K_a}\right)\frac{1}{s} = 0 + \left(\frac{c}{s^2+K_a}\right)$$

$$e(t) = 0 + (transients)$$

ii) if R is a unit ramp:

$$E = \left(\frac{s^2}{s^2+K_a}\right)\frac{1}{s^2} = 0 + \left(\frac{1}{s^2+K_a}\right)$$

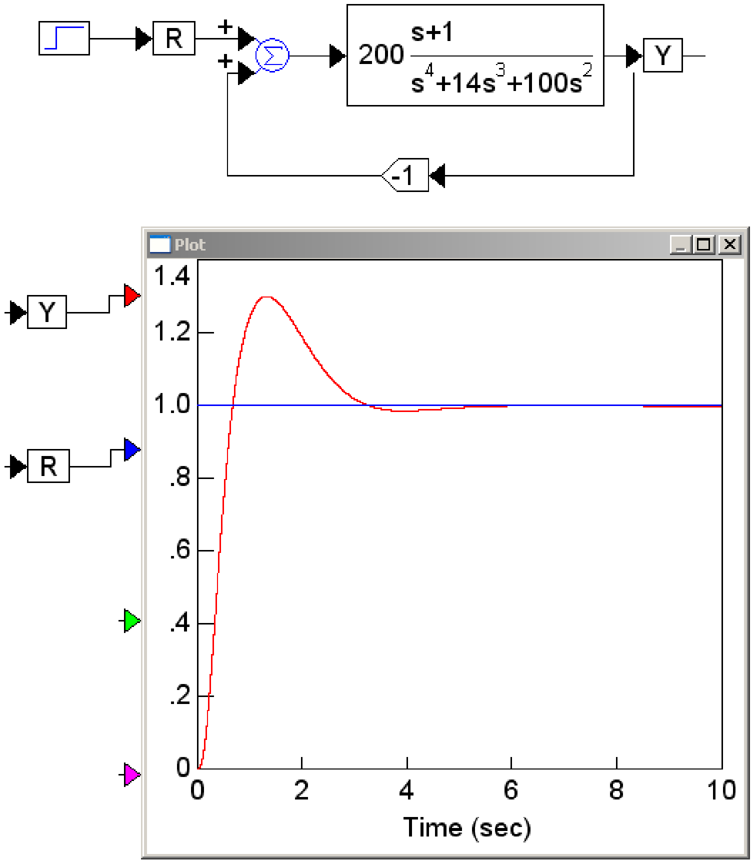
$$e(t) = 0 + (transients)$$

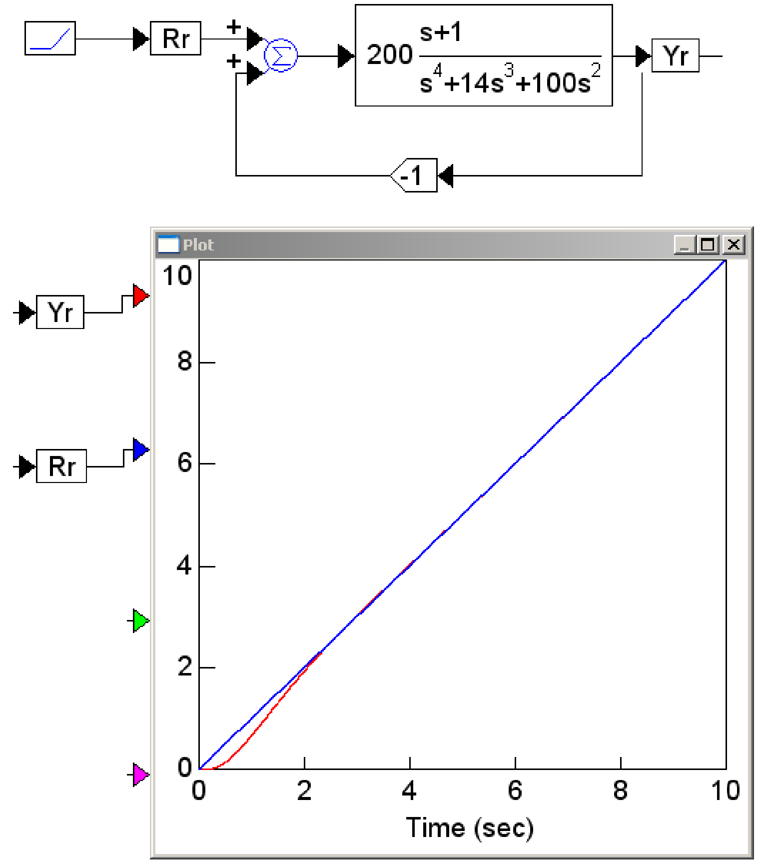
iii) Unit parabola:

$$E = \left(\frac{s^2}{s^2+K_a}\right)\frac{1}{s^3} = \left(\frac{\frac{1}{K_a}}{s}\right) + \left(\frac{c}{s^2+K_a}\right)$$

$$e(t) = \left(\frac{1}{K_a}\right) + transients$$

Example: $G(s)=\left(\dfrac{200(s+1)}{s^2\left(s^2+14s+100\right)}\right)$ $\qquad G(s)_{s\to 0}=\dfrac{2}{s^2}=\dfrac{K_a}{s^2}$





Handout

Determine the system type, the error constants, and the steady-state error for a step input:

G(s)	System Type	Kp	Kv	Steady-state error for a unit step input
$\left(\dfrac{200}{(s+3)(s+6)}\right)$				
$\left(\dfrac{2000}{(s+3+j10)(s+3-j10)(s+20)}\right)$				
$\left(\dfrac{20}{s(s+3)(s+6)}\right)$				
$\left(\dfrac{200}{(s-1)(s+5)(s+10)}\right)$				

Summary:

Computation of Error Constants			
	Kp	Kv	Ka
Type 0	$\lim_{s\to 0} (G(s))$	0	0
Type 1	∞	$\lim_{s\to 0} (s \cdot G(s))$	0
Type 2	∞	∞	$\lim_{s\to 0} (s^2 \cdot G(s))$

Steady-State Error			
	Unit Step	Unit Ramp	Unit Parabola
Type 0	$\left(\frac{1}{K_p+1}\right)$	∞	∞
Type 1	0	$\left(\frac{1}{K_v}\right)$	∞
Type 2	0	0	$\left(\frac{1}{K_a}\right)$