Modeling a DC Servo Motor

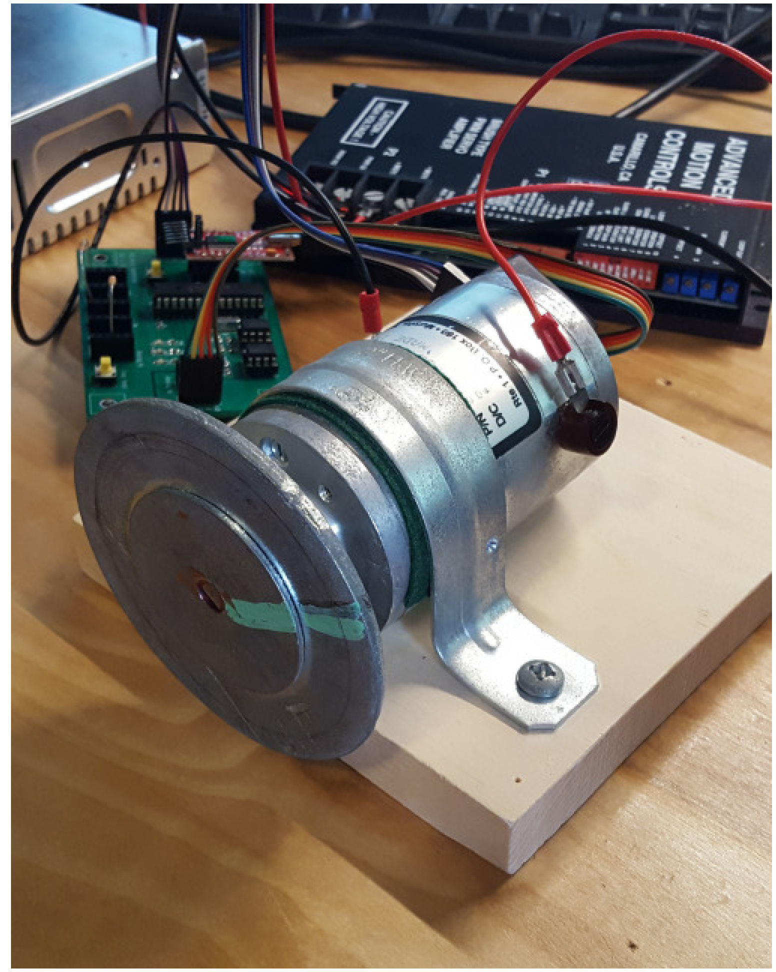
ECE 461/661 Controls Systems

Jake Glower - Lecture #17

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Problem: Determine the model for a DC servo motor

- Clifton 000-053479-002
- $\theta = \left(\frac{1}{s}\right)\left(\frac{K_t}{(Js+D)(L_a s+R_a)+K_t^2}\right)V_a$



Unfortunately, the dynamics are not printed on the motor (common problem)

Finding the Dynamics (Option #1)

Find the data sheets:

- www.ServoSystems.com from 2002
- Missing some information.
- Often times you can't find the data sheets...

CLIFTON PRECISION SERVO MOTOR MODEL JDH-2250-HF-2C-E

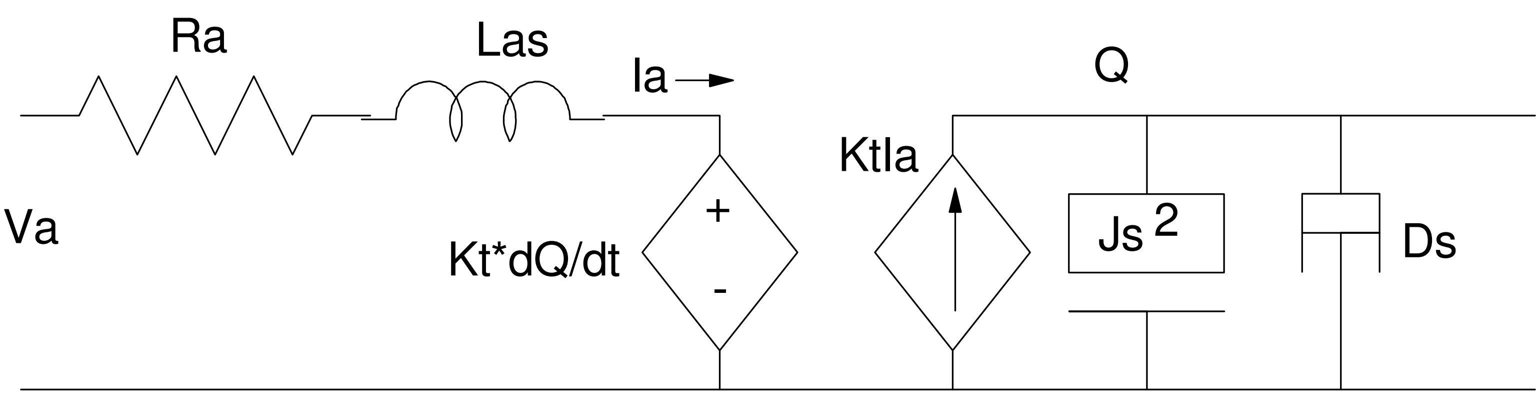
- Torque Constant: 15.76 oz-in. / A
- Back EMF: 11.65 VDC / KRPM
- Peak Torque: 125 oz-in.
- Cont. Torque: 16.5 oz-in.
- Encoder: 250 counts / rev.
- Channels A, B in quadrature, 5 VDC input (no index)
- Body Dimensions: 2.25" dia. x 4.35" L (includes encoder)
- Shaft Dimensions: 8 mm x 1.0" L w/flat

Stock No. DM-683
Special Price . . . . $59.00 ea.

Option #2: Take some measurements

You only need five parameters to define the DC servo motor

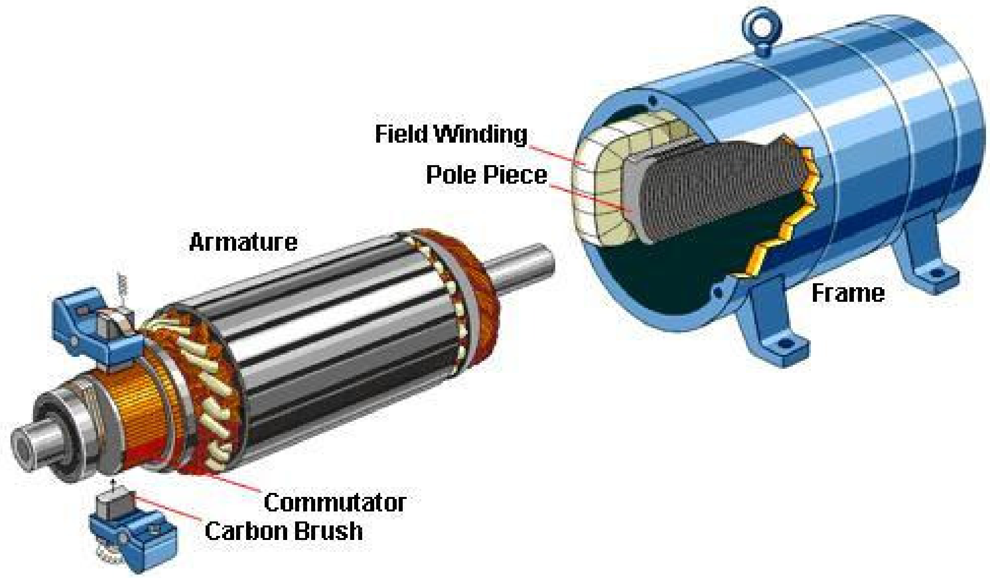
- Ra: Armature Resistance
- La: Armature Inductance
- Kt: Torque Constant
- J: Rotor Inertia
- D: Rotor friction



Armature Construction

The armature is essentially an electromagnet

- Multiple windings of copper wire around an iron core
- This makes a resistor
- This also makes an inductor



Ra: Armature Resistance

- Ra = 26.5 Ohms

Turn off the motor

- The back EMF will mess up the resistance measurement

Disconnect from the power supply

- The power supply will mess up the resistance measurement

Measure R

- Varies as you rotate the motor
- Each winding will have slightly different resistance

La: Armature Inductance

- La = 12.698mH

Turn off the motor

Disconnect from the power supply

Measure L

- Requires an inductance meter
- Usually this is small and can be assumed to be zero

Kt: Torque Constant

Option #1: Find the data sheets

- Old posting from ServoSystems c. 1998
- Torque Constant = 15.76 oz-in / A

$$K_t = \left(15.76\frac{oz\cdot in}{A}\right)\left(\frac{1lb}{16oz}\right)\left(\frac{4.45N}{lb}\right)\left(\frac{0.0254m}{in}\right) = 0.11133\frac{Nm}{A}$$

- Torque Constant = 11.65V/krpm

$$K_t = \left(11.65\,\frac{V\cdot\min}{krev}\right)\left(\frac{1krev}{1000rev}\right)\left(\frac{60\sec}{1\min}\right)\left(\frac{1rev}{2\pi rad}\right)$$

$$K_t = 0.111249\,\frac{V}{rad/\sec}$$

Note

- Both Kt's are the same if using mks units

Kt: Torque Constant (cont'd)

Option #2:

Apply a constant voltage (5.00V)

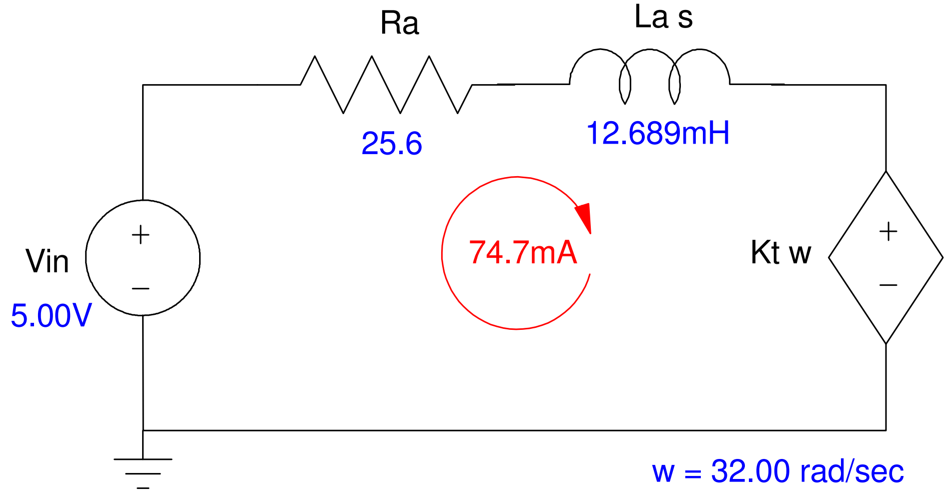
Measure the resulting speed (32.00 rad/sec)

Measure the current draw (74.7mA)

$$V_a = I_a R_a + K_t \omega$$

$$5.00V = 74.7mA \cdot 26.5\Omega + K_t \cdot 32.00\frac{rad}{\sec}$$

$$K_t = 0.094389\,\frac{V}{rad/\sec}$$



Close to the data sheets:

Kt = 0.111249 V / rad/sec

D: Motor Friction

- No-Load Speed = 32.00 rad/sec @ 5.00V @ 74.7mA

Energy must balance

Power In = Power Out

$$VI = T\omega = (D\omega)\omega = D\omega^2$$

$$(5.00V)(74.7mA) = D \cdot \left(32.00\frac{rad}{\sec}\right)^2$$

Assuming other losses are zero

$$D = 0.000364\frac{Nm}{rad/\sec}$$

J: Rotor Inertia (calculations)

Flywheel: 91mm dia x 5mm thick flywheel, solid iron

$$m = \left(7.847\frac{gm}{cc}\right)\left(\pi \cdot (4.55cm)^2\right)(0.5cm) = 255gm = 0.255kg$$

$$J = \tfrac{1}{2}mr^2 = \tfrac{1}{2}(0.255kg)(0.0455m)^2 = 0.000264\,kg\,m^3$$

Core: 20mm dia x 40mm long, solid iron

$$m = \left(7.847\frac{gm}{cc}\right)\left(\pi \cdot (1cm)^2\right)(4cm) = 98.6gm = 0.0986kg$$

$$J = \tfrac{1}{2}mr^2 = \tfrac{1}{2}(0.0986kg)(0.01m)^2 = 0.00000493\,kg\,m^3$$

Total:

J = 0.000 268 9 kg m^2

Option #2 to find D and J

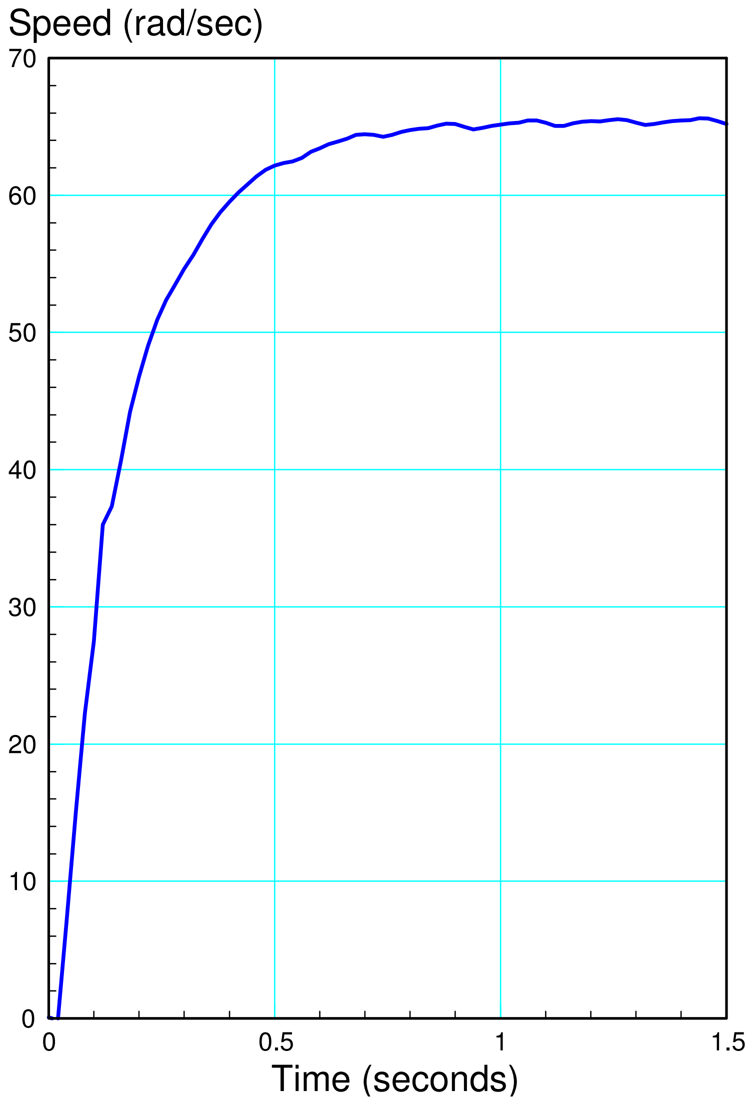
- Experimental

Apply a step input (+10V step)

Measure the speed vs. time

From the step response, determine the transfer function

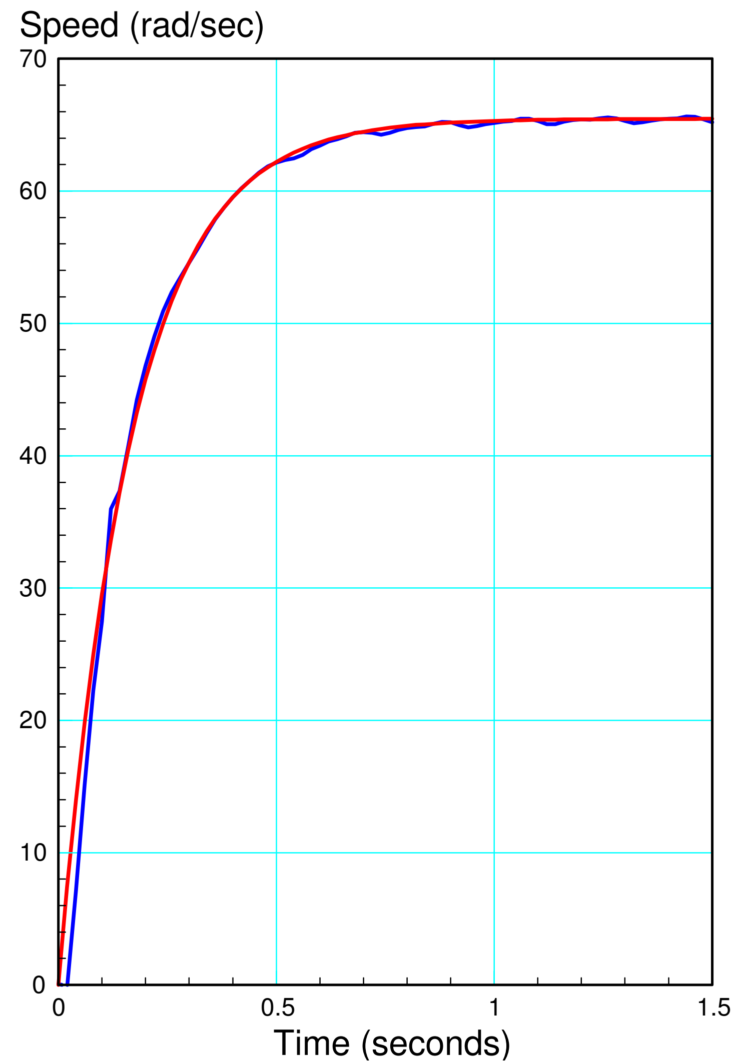
- First order
- DC gain = 6.40 (10V input)
- Settling time = 4/6 second



J: Rotor Inertia (cont'd)

- Blue = Actual Motor
- Red = 1st-order model

$$\omega \approx \left(\frac{39.28}{s+6}\right) V_a$$



Matching Terms

$$\omega = \left(\frac{K_t}{(Js+D)(Ls+R)+K_t^2}\right) V_a$$

Assuming L = 0

$$\omega = \left(\frac{K_t}{JRs+DR+K_t^2}\right) V_a$$

$$\omega = \left(\frac{\left(\frac{K_t}{JR}\right)}{s+\left(\frac{DR+K_t^2}{JR}\right)}\right) V_a$$

Matching Terms

$$\left(\frac{39.28}{s+6}\right) = \left(\frac{\left(\frac{K_t}{JR}\right)}{s+\left(\frac{DR+K_t^2}{JR}\right)}\right)$$

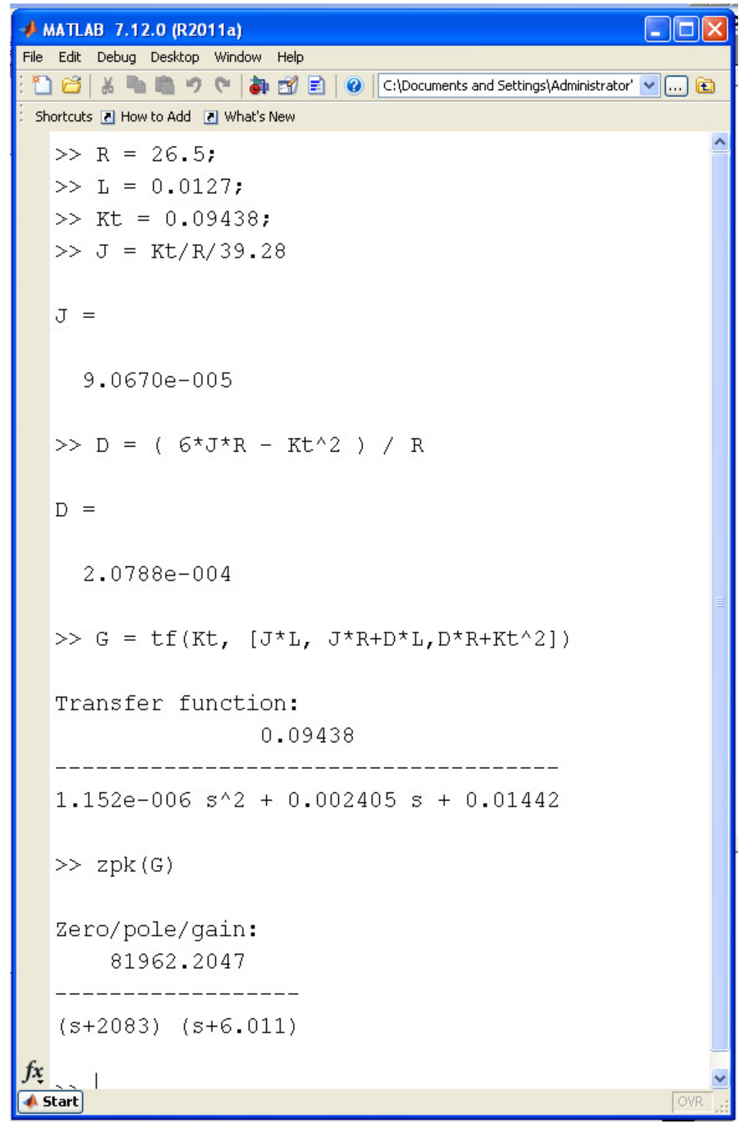
$$\left(\frac{K_t}{JR}\right) = 39.28$$

$$J = 0.00009068\ kg\ m^2$$

$$\left(\frac{DR+K_t^2}{JR}\right) = 6$$

$$D = 0.000207\ \frac{Nm}{rad/\sec}$$

Experimental results are slightly different



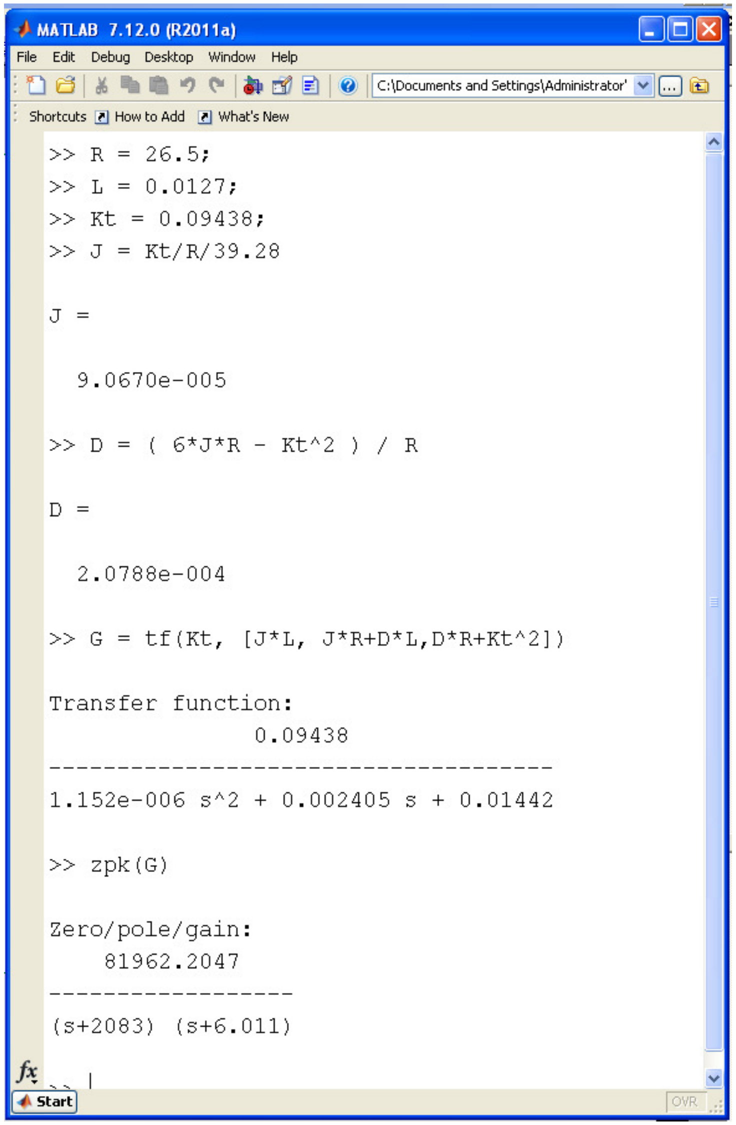
Net Result:

- Ra = 26.5 Ohms
- La = 12.7 mH
- Kt = 0.09438 V / rad/sec
- J = 0.000 090 68 kg m2
- D = 0.000 207 Nm / rad/sec

$$\omega = \left(\frac{K_t}{(Js+D)(Ls+R)+K_t^2} \right) V_a$$

$$\omega = \left(\frac{81962}{(s+6.01)(s+2083)} \right) V_a$$

$$\omega \approx \left(\frac{39.28}{s+6} \right) V_a$$

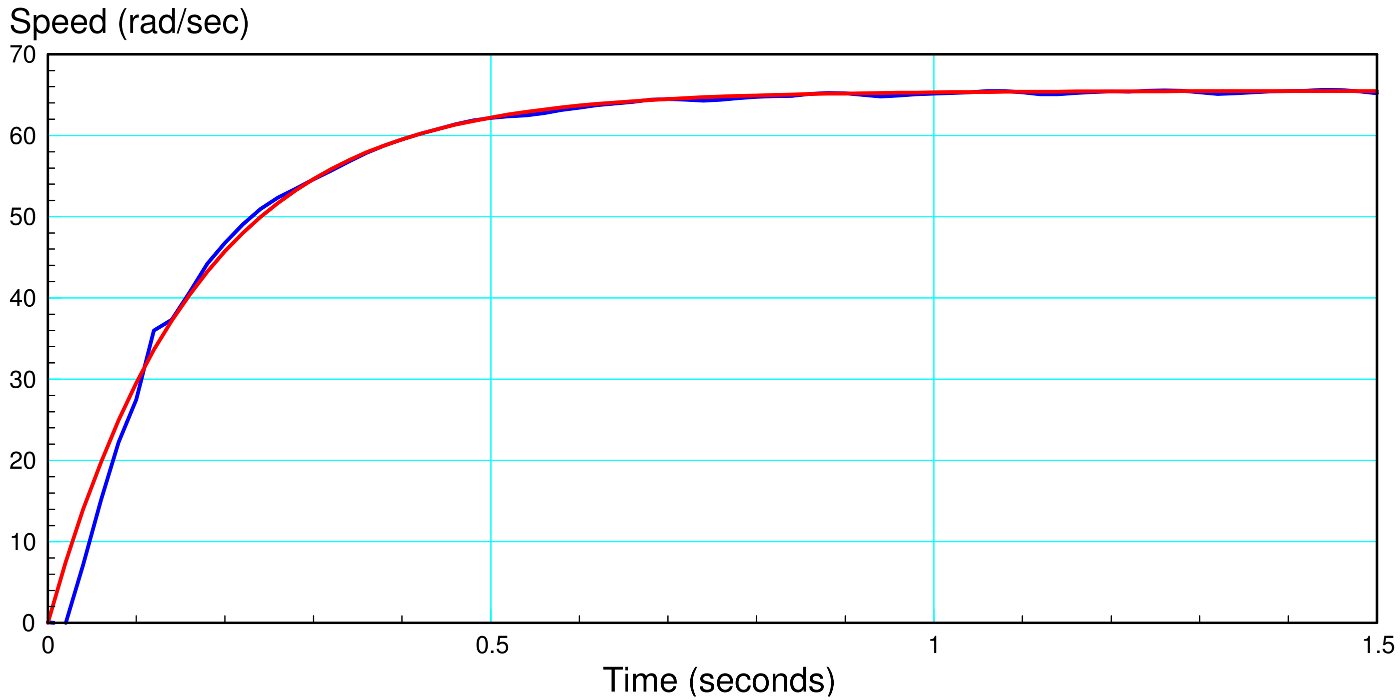


Significance:

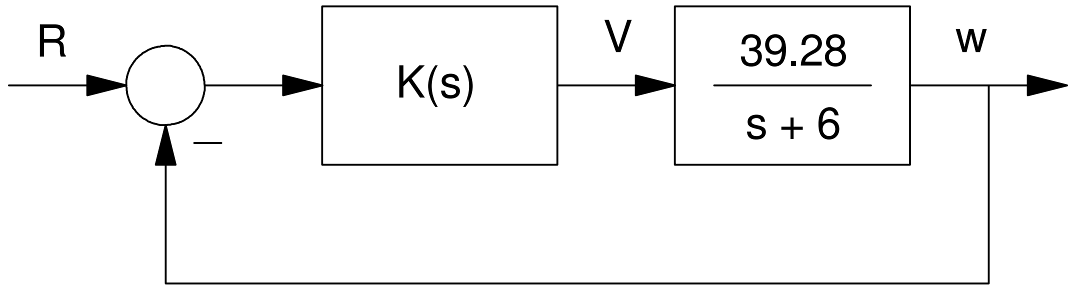
1. The mathematical model

$$\omega \approx \left(\frac{39.28}{s+6}\right) V_a$$

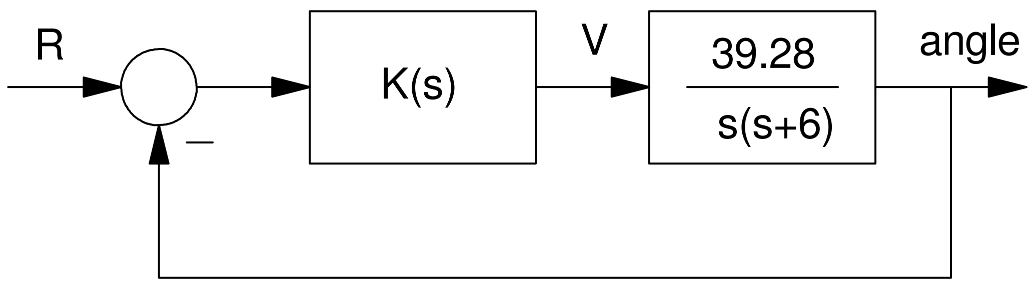
is a good approximation for the motor's actual dynamics.



2. Controllers which work for this model should work on the actual motor



Speed Control



Position Control

3) With just a few tests, you can determine the dynamics of a DC servo motor