Meeting Design Specs using Bode Plots

When designing a compensator to meet design specs including:

- Steady-state error for a step input
- Phase Margin
- 0dB gain frequency

the procedure using Bode plots is about the same as it was for root-locus plots.

- Add a pole at s = 0 if needed (making the system type-1)
- Start cancelling zeros until the system is too fast
- For every zero you added, add a pole. Place these poles so that the phase adds up.

When using root-locus techniques, the phase adds up to 180 degrees at some spot on the s-plane.

When using Bode plot techniques, the phase adds up to give you your phase margin at some spot on the jw axis.

Example 1:

Problem: Given the following system

$$G(s) = \left(\frac{1000}{(s+1)(s+3)(s+6)(s+10)}\right)$$

Design a compensator which result in

- No error for a step input
- 20% overshoot for a step input, and
- A settling time of 4 second

Solution: First, convert to Bode Plot terminology

No overshoot means make this a type-1 system. Add a pole at s = 0.

20% overshoot means

- $\zeta = 0.4559$
- $M_m = 1.2322$
- $GK = 1\angle -132.12^0$
- Phase Margin = 47.88 degrees

A settling time of 4 seconds means

- s = -1 + j X
- s = -1 + j2 (from the damping ratio)
- The 0dB gain frequency is 2.00 rad/sec

In short, design K(s) so that the system is type-1 and

$$(GK)_{s=j2} = 1\angle -132.12^0$$

Start with

$$K(s) = \left(\frac{s+1}{s}\right)$$

At 4 rad/sec

$$GK = \left(\frac{1000}{s(s+3)(s+6)(s+10)}\right)_{s=j2} = 2.1508\angle -153.43^0$$

There is too much phase shift, so start cancelling zeros. If you cancel the zero at -3

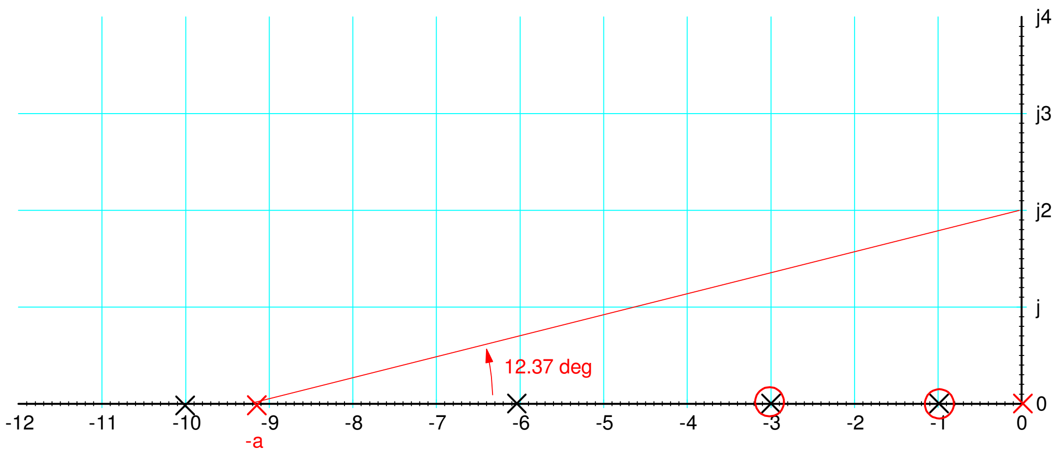
$$K(s) = \left(\frac{(s+1)(s+3)}{s}\right)$$

$$GK = \left(\frac{1000}{s(s+6)(s+10)}\right)_{s=j2} = 7.7522\angle -119.75^0$$

This is less than the desired phase shift of -132.12 degrees - which is good. When you add two zeros, you also need to add two poles.

$$K(s) = \left(\frac{(s+1)(s+3)}{s(s+a)}\right)$$

Pick 'a' so that the phase shift is -132.12 degrees (i.e. the angle of (s+a) makes up for the remainder: 12.37 degrees)



The pole at -a is chosen so that the angle to s = j2 is 12.37 degrees

This results in

$$GK = \left(\frac{1000k}{s(s+a)(s+6)(s+10)}\right)_{s=j2} = 1\angle -132.12^0$$

$$\angle\left(\frac{1}{s+a}\right)_{s=j2} = -12.37^0$$

$$a = \frac{2}{\tan\left(12.37^0\right)} = 9.115$$

and

$$K(s) = k\left(\frac{(s+1)(s+3)}{s(s+9.1154)}\right)$$

To find k,

$$GK = \left(\frac{1000}{s(s+6)(s+9.1154)(s+10)}\right)_{s=j2} = 0.8307\angle -132.12^0$$

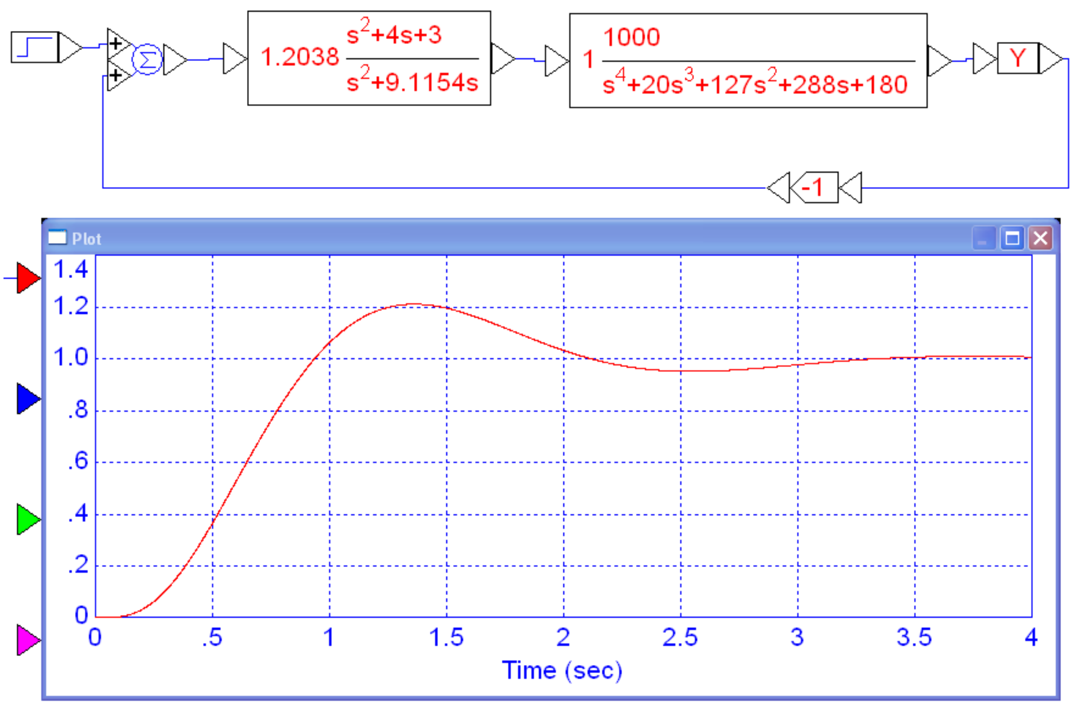
This should be 1.000, so add a gain, k,

$$k = \frac{1}{0.8307} = 1.2038$$

and

$$K(s) = 1.2038\left(\frac{(s+1)(s+3)}{s(s+9.1154)}\right)$$

Checking in VisSim:



Step Response for K(s) chosen for 20% overshoot and a 4 second settling time.

Note that the overshoot is a little high. This is typical with designs using Bode plots since

- Designing to a phase margin means you intersect the M-circle rather than beging tangent
- This results in the resonance being slightly too high
- Which result sin the overshoot being slightly too high.

Example 2: System with Delay

A nice feature of using Bode plots is delays don't cause any problems: you don't need to use any Pade approximations or stuff like that.

Repeat the previous design if a delay of 200ms is added to G(s):

$$G(s) = \left(\frac{1000}{(s+1)(s+3)(s+6)(s+10)}\right) \cdot e^{-0.2s}$$

Solution: Again, you want to make this a type-1 system with

$$(GK)_{s=j2} = 1\angle -132.12^0$$

Start with adding a pole at s = 0 to make it type-1

$$K(s) = \left(\frac{1}{s}\right)$$

Start cancelling zeros until the phase of GK at s = j2 is less than -132 degrees. This requires three zeros

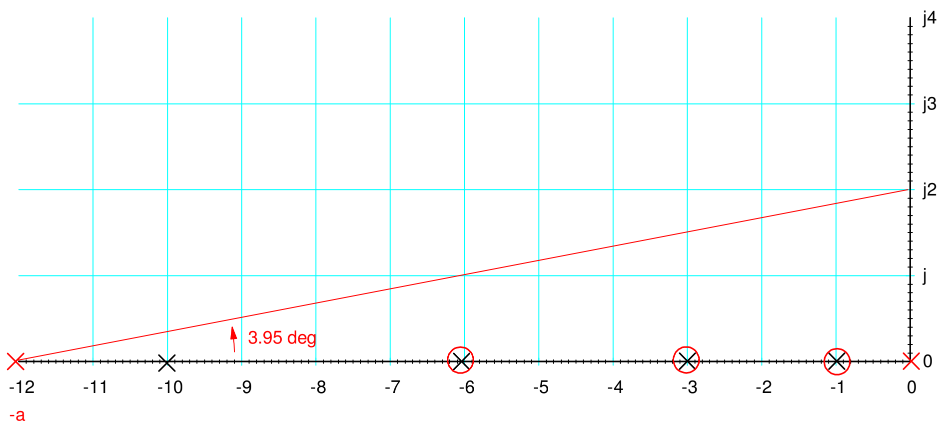
$$K(s) = \left(\frac{(s+1)(s+3)(s+6)}{s}\right)$$

$$GK = \left(\left(\frac{1000}{s(s+10)}\right) \cdot e^{-0.2s}\right)_{s=j2} = 49.029\angle -124.22^0$$

With three zeros, you need to add three poles. One goes to the origin, the other two place at -a:

$$K(s) = \left(\frac{(s+1)(s+3)(s+6)}{s(s+a)^2}\right)$$

To make the phase add up to -132.12 degrees



a is chosen so that the angle from a to j2 is 3.9459 degres

$$\angle(s+a)^2 = 7.8918^0$$

$$\angle(s+a) = 3.9459^0$$

$$a = \frac{2}{\tan\left(3.94^0\right)} = 28.995$$

Then

$$K(s) = k\left(\frac{(s+1)(s+3)(s+6)}{s(s+28.995)^2}\right)$$

To find k

$$GK = \left(\left(\frac{1000}{s(s+10)(s+28.995)^2}\right) \cdot e^{-0.2s}\right)_{s=j2} = 0.0508\angle -132.12^0$$

This should be one, so add a gain

$$k = \frac{1}{0.0508} = 17.2288$$

and

$$K(s) = 17.2288\left(\frac{(s+1)(s+3)(s+6)}{s(s+28.995)^2}\right)$$

Checking in VisSim