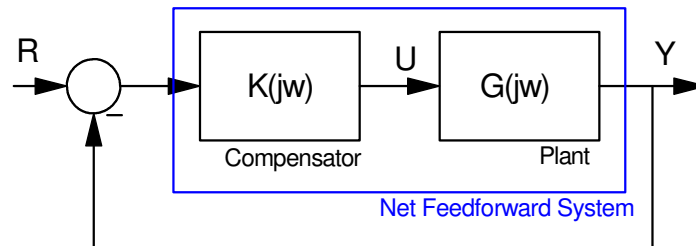


Gain Compensation and Nichols Charts

Stability & Feedback

Consider the following feedback system.



If this models the speaker system at a rock concert, a problem can arise if the gain becomes too large. If this happens, the system will go unstable and a loud screech will be heard. Essentially, the system has gone unstable - meaning the gain has gone to infinity at some frequency (the frequency of the screech).

For this system to be useful (at least not annoying), it needs to be stable. The problem is to find the condition where the system is stable.

Assume that at some frequency the signal adds in phase. In this case, the loop gain will be 'a' where 'a' is a positive constant. Since the signal feeds back upon itself, the overall gain will be

$$\left(\frac{GK}{1+GK} \right) = a + a^2 + a^3 + a^4 + \dots$$

If 'a' is less than one, this converges. If 'a' is greater or equal to one, this adds up to infinity. Essentially this is what happens at a rock concert.

- At some frequency, the signal adds to itself.
- When you crank up the gain, the gain at this frequency increases as well.
- When the loop gain at this frequency equals one, the signal builds upon itself, creating a loud screeching noise.

To fix this problem, you back off on the gain until the screech stops. This is analogous to the stability condition using frequency domain techniques:

For a system to be stable, the gain must be less than one when the net phase around the loop is zero degrees.

Since negative feedback is used, the feedback adds 180 degrees phase shift. For the net phase to be zero degrees, the phase of GK must be 180 degrees as well. Hence,

For a system to be stable, the gain GK must be less than one when the phase of GK is 180 degrees.

Since stability is rather important, a term, Gain Margin, is used to tell you how stable the system is

Gain Margin: How much you can increase the gain before the system just goes unstable.

This is similar to a safety margin: a 6dB gain margin means that the gain could be off by 6dB without causing the system to go unstable.

In addition to being stable, you would like the system to behave well as well. From a root-locus standpoint, 'behave well' means that the dominant poles are a certain distance from the $j\omega$ axis, as defined by the damping ratio. From a frequency-domain standpoint, 'behave well' means that the resonance isn't just finite, but it is limited to some number, such as 6dB as well. From the second-order approximations specifying the resonance also specifies the damping ratio

$$M_m = \begin{cases} 1 & \zeta > 0.7 \\ \frac{1}{2\zeta\sqrt{1-\zeta^2}} & 0 < \zeta < 0.7 \end{cases}$$

- so it's just another way of saying the same thing.

Since the closed-loop system goes unstable when $G(j\omega) = -1 = 1\angle 180^\circ$, you can think of this constraint on M_m as being a constraint on how close $G(j\omega)$ can get to -1. The problem is that 'close' is defined by the closed-loop gain: $\left(\frac{G}{1+G}\right)$, which is a nonlinear function of G . As a result, some graphical tools are defined for defining 'close'.

In each of these tools, two sets of coordinates are used

- One set defines $G(j\omega)$
- The other defines $\frac{G}{1+G}$

By plotting $G(j\omega)$ one set of coordinates, you can read off $\left(\frac{G}{1+G}\right)$ by looking at the other. The various tools (Nichols Charts, Nyquist Diagrams, Inverse Nyquist Diagrams) are simply different ways to plot $G(j\omega)$. Their goal in all cases is to show you what 'close to -1' means.

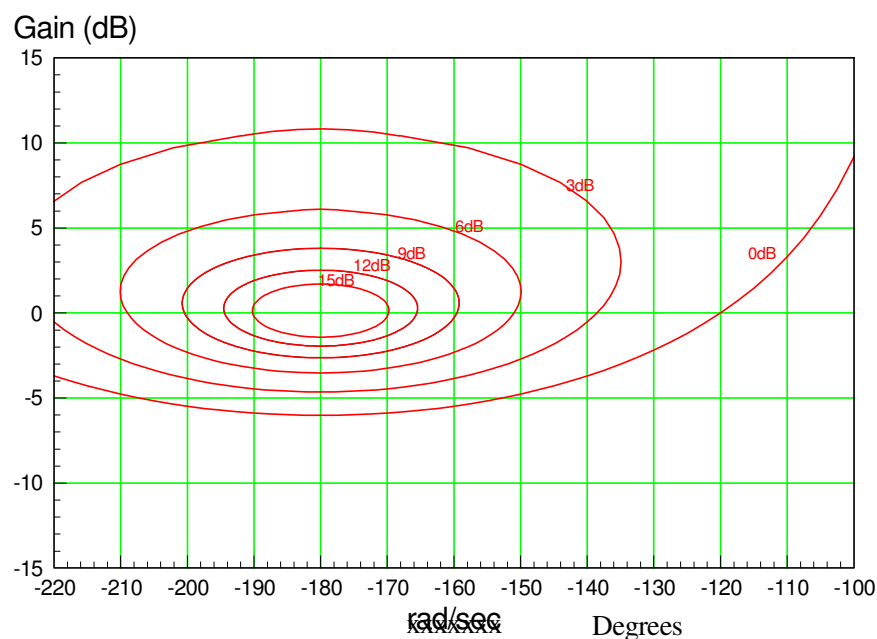
Gain Compensation Using Nichols Charts:

A Nichols Chart is a plot where $G(j\omega)$ is graphed with

- The X-axis plotting the phase of $G(j\omega)$, and
- The Y-axis plotting the gain of $G(j\omega)$ in dB.

In addition, M-circles are drawn on top of this plot. These M-circles define all points where the closed-loop gain $\left(\frac{G}{1+G}\right)$ will be constant. This tells you that all points on the same M-circle are the same distance from -1.

Example: The following is a Nichols Chart. The ovals (red) M-circles tell you all points which are equal distance to -1.



Nichols Chart: M-circles plot points where $\left|\frac{G}{1+G}\right|$ is constant

To use a Nichols chart

- Plot $G(j\omega)$ on the Nichols chart using the outer coordinates.
- Adjust the gain of $G(j\omega)$ by sliding it up and down. Adjust it so that the system is stable (The gain is less than 0dB at 180 degrees) and it is a specified distance from -1.

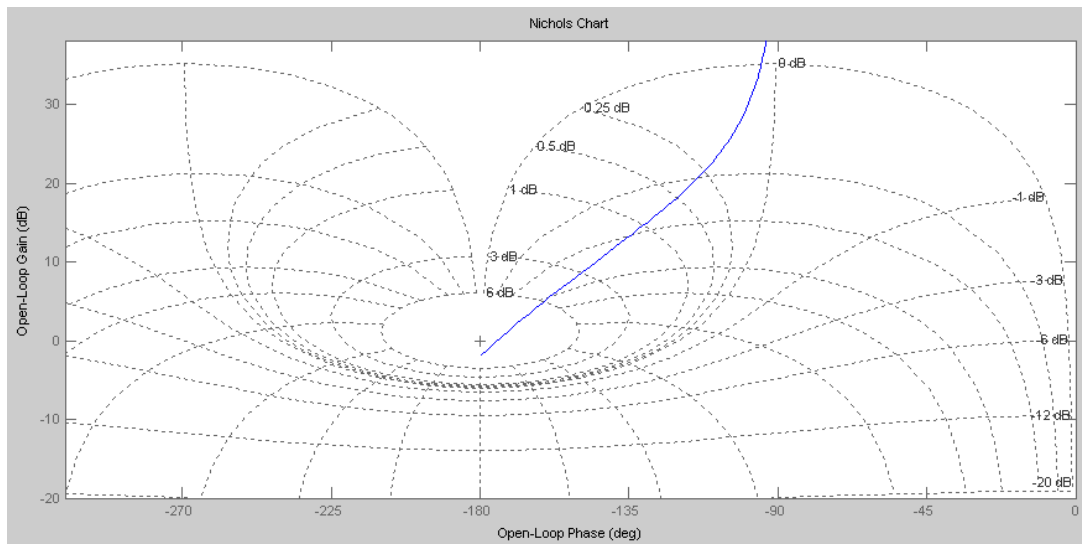
Example: Find a gain, k , so that

$$G(s) = \left(\frac{2000}{s(s+5)(s+20)} \right)$$

- Has a resonance of 6dB or less, and
- K is as large as possible.,

Solution: Plot $G(j\omega)$ on a Nichols chart along with the 6dB M-circle. In Matlab:

```
G = zpk([], [0, -5, -20], 2000);
w = logspace(-1, 1, 100)';
nichols(G, w);
grid on
```



Nichols Chart of $G(s)$ using the Matlab command *nichols()*

One problem with Matlab's *nichols* function is it doesn't allow you to draw an arbitrary M-circle. You can get around this by computing it yourself.

```
function [] = Nichols2(Gw, Mm)

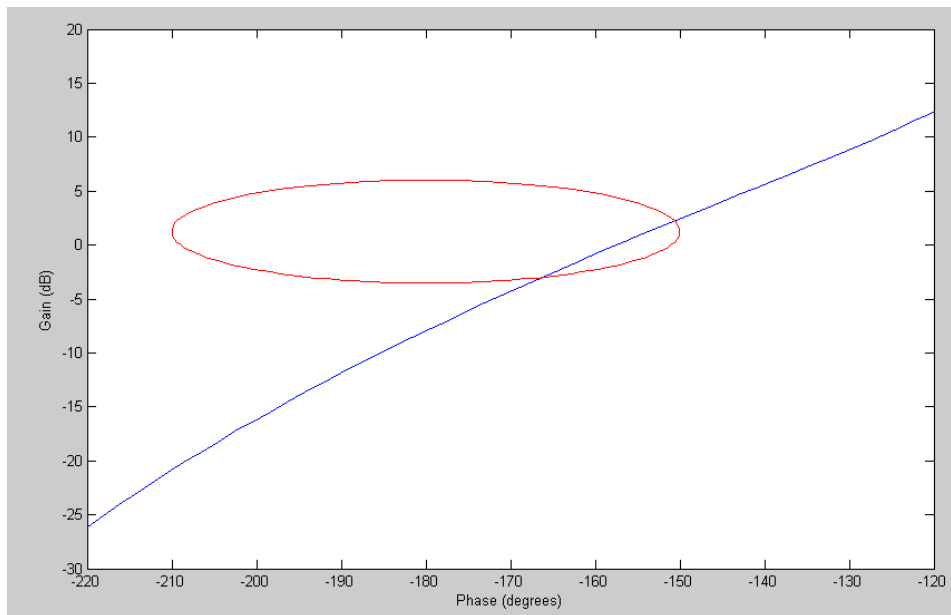
    Gwp = unwrap(angle(Gw))*180/pi;
    Gwm = 20*log10(abs(Gw));

    % M-Circle
    phase = [0:0.01:1]' * 2*pi;
    Mcl = Mm * exp(j*phase);
    Mol = Mcl ./ (1 - Mcl);
    Mp = unwrap(angle(Mol))*180/pi - 360;
    Mm = 20*log10(abs(Mol));
    plot(Gwp, Gwm, 'b', Mp, Mm, 'r');
    xlabel('Phase (degrees)');
    ylabel('Gain (dB)');
    xlim([-220, -120]);
    ylim([-30, 20]);
end
```

From Matlab

```
G = zpk([], [0, -5, -20], 1000);
w = logspace(-2, 2, 200)';
Gw = Bode2(G, w);

Nichols2(Gw, 2);
```



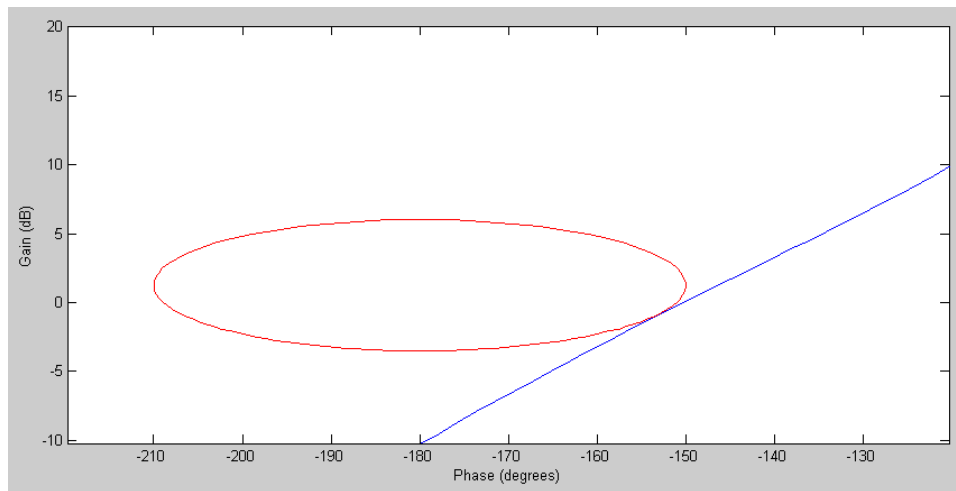
Nichols Chart of $G(s)$ along with the $M_m = 2$ M-circle

To make the maximum closed-loop gain equal to 2

- Slide the $G(j\omega)$ curve down until it is tangent to the M-circle, and
- The gain when the phase is 180 degrees is less than 0dB (i.e. the system is stable).

Using trial and error, you get $K = -8.38\text{dB}$

```
w, 2);
Nichols2(Gw*0.9, 2);
Nichols2(Gw*0.8, 2);
Nichols2(Gw*0.78, 2);
Nichols2(Gw*0.77, 2);
```



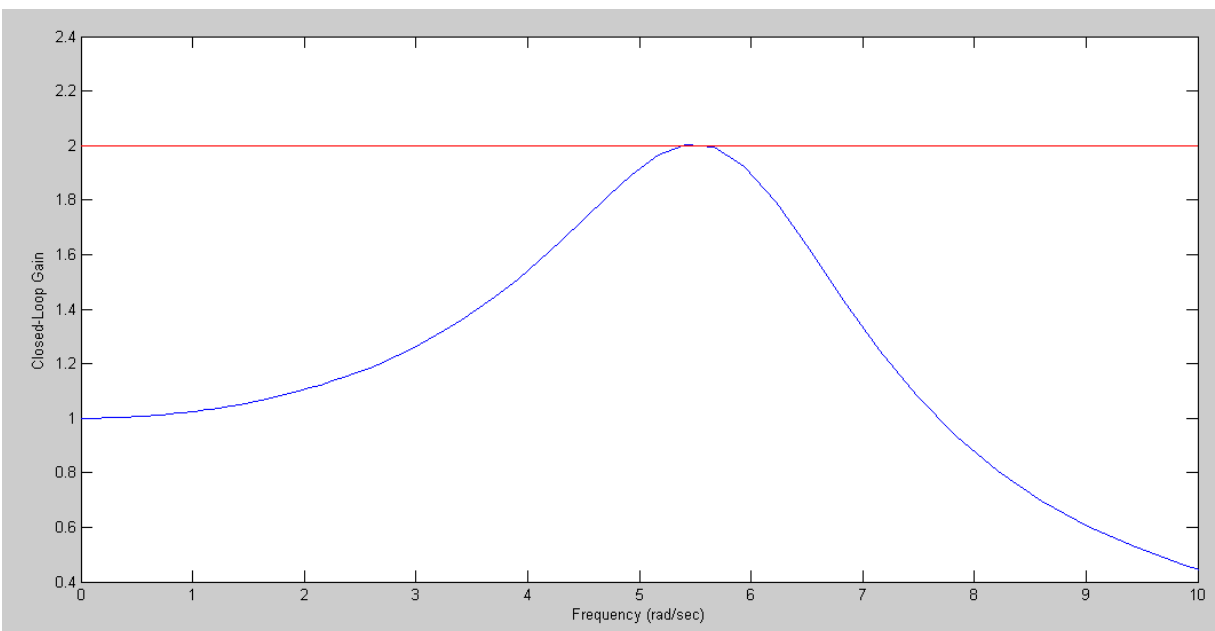
$G*K$ with $K = 0.77 = -2.27\text{dB}$ along with the $M_m = 2$ M-circle

Hence,

$$K = 0.77$$

You can check your answer by plotting the closed-loop gain with this value of K :

```
k = 0.77;
Gw_cl = Gw*k ./ (1 + Gw*k) ;
plot(w, abs(Gw_cl));
xlabel('Frequency (rad/sec)');
ylabel('Closed-Loop Gain');
```



Closed-Loop Gain of $G*K$. K was chosen so that the maximum gain was 2.000

Gain Compensation When G(s) Isn't Specified

Note that Nichols charts are nice in that you really don't need to know the transfer function of the system. All you need is the gain vs. frequency. Even then, all you need is the gain at (or near) the frequency where the resonance will be.

For example, design a gain compensator for a system with the following gain vs. frequency so that the $M_m = 2$:

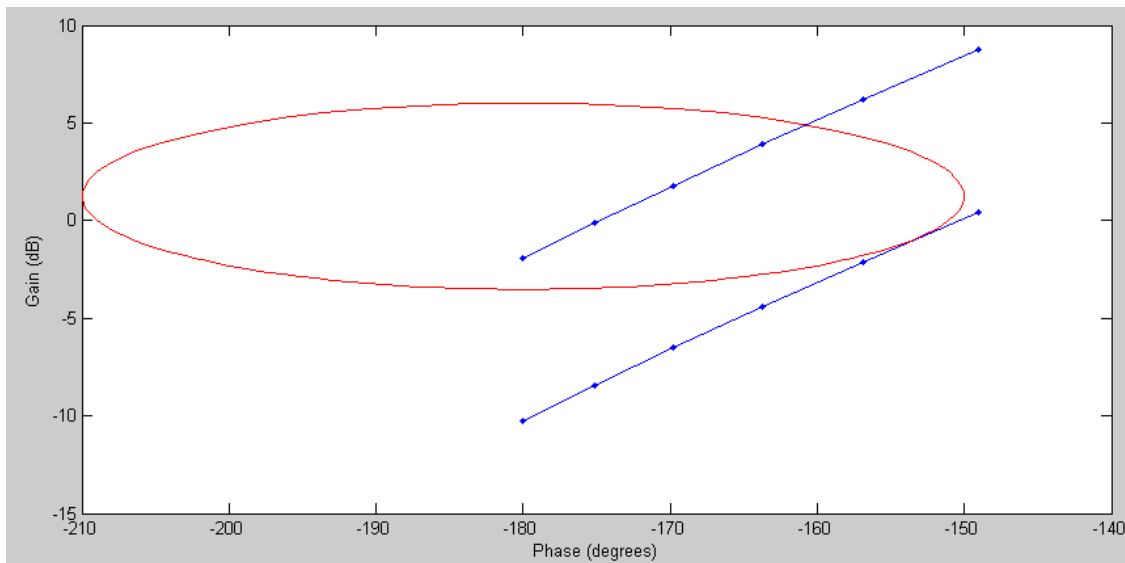
Freq	5	6	7	8	9	10
Gain (dB)	8.77	6.21	3.9	1.8	-0.14	-1.94
Phase (deg)	-149.04	-156.89	-163.75	-169.8	-175.17	-180

Solution: Transfer this data to a Nichols chart along with the M-circle (computed before). Adjust the gain of $G*k$ so that the data is tangent to the M-circle

```

dB = [8.77    6.21  3.9   1.8   -0.14 -1.94]';
P = [-149.04 -156.89 -163.75 -169.8 -175.17 -180]';
Gw = 10.^(dB/20) .* exp(j*pi*P/180);
Nichols2(Gw, 2);
Nichols2(Gw*0.3, 2);
Nichols2(Gw*0.35, 2);
Nichols2(Gw*0.38, 2);
    
```

This results in $k = 0.38$



Nichols Chart of $G(j\omega)$ (top blue curve) and with $K = -8.32\text{dB}$ (bottom blue curve) along with the $M_m = 2$ M-circle.

All you need to design a gain compensator is a few choice gain and phase measurements.

Nyquist Diagram

A Nyquist diagram is similar to a Nichols chart. In this case, you plot

- x-axis: $\text{real}(G(j\omega))$
- y-axis: $\text{imag}(G(j\omega))$
- m-circles: constant closed-loop gain

The result is similar to a Nichols chart:

- Adjust the gain until you are stable (gain when phase is 180 degrees is less than one), and
- Tangent to the desired M-circle

Nyquist diagrams are a little tricky to use since the closed-loop gain is

$$G_{cl} = \left(\frac{G}{1+G} \right) = \left(\frac{\text{distance to the origin}}{\text{distance to -1}} \right)$$

The resonance (maximum closed-loop gain) can be hard to follow since both terms change as you move along $G(j\omega)$

Matlab Code for a Nyquist Diagram:

```
function [] = Nyquist2(Gw, Mm)

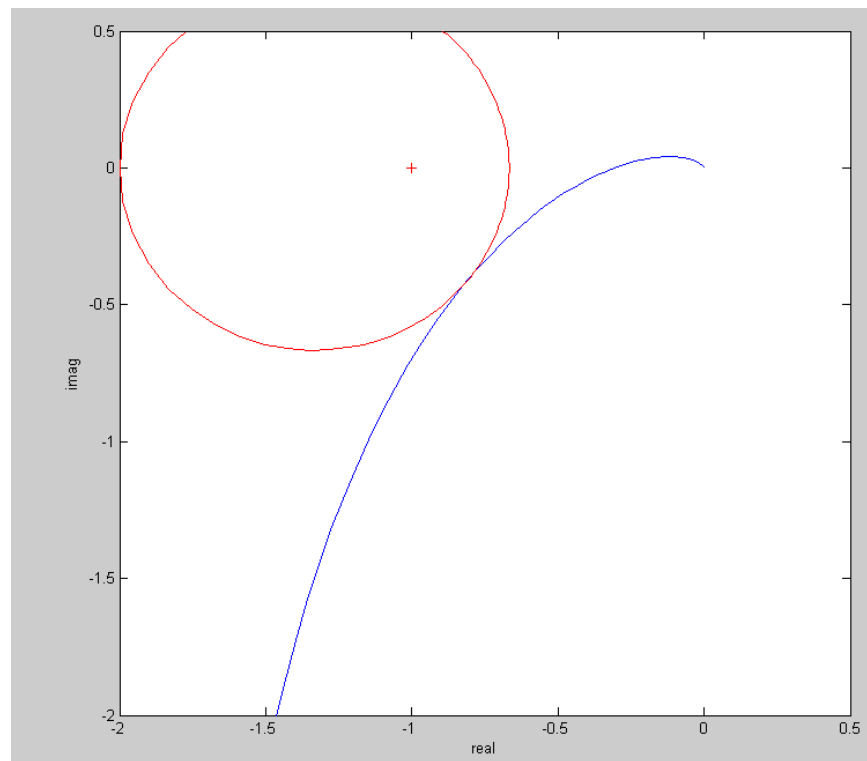
    Gwp = unwrap(angle(Gw))*180/pi;
    Gwm = 20*log10(abs(Gw));

    % M-Circle
    phase = [0:0.01:1]' * 2*pi;
    Mcl = Mm * exp(j*phase);
    Mol = Mcl ./ (1 - Mcl);

    plot(real(Gw), imag(Gw), 'b', real(Mol), imag(Mol), 'r', -1, 0, 'r+');
    xlabel('real');
    ylabel('imag');
    xlim([-2, 0.5]);
    ylim([-2, 0.5]);
    end
```

Calling sequence

```
w = logspace(-2, 2, 200)';
G = zpk([], [0, -5, -20], 1000);
Gw = Bode2(G, w);
Nyquist2(Gw, 2);
Nyquist2(Gw*0.9, 2);
Nyquist2(Gw*0.77, 2);
```

Nyquist diagram along with $Gk(j\omega)$ when $k = 0.77$

Inverse Nyquist Diagram

An Inverse Nyquist diagram is similar to a Nyquist diagram, except you plot the real and imaginary parts of $1/G$

- x-axis: $\text{real}(1/G(j\omega))$
- y-axis: $\text{imag}(1/G(j\omega))$
- m-circles: constant closed-loop gain

The result is similar to a Nyquist diagram except that the resonance is easy to find

$$G_{cl}^{-1} = \left(\frac{G}{1+G} \right)^{-1} = \left(\frac{1+G}{G} \right) = \left(\frac{1}{G} + 1 \right)$$

$$G_{cl} = \frac{1}{\text{closest distance to } -1}$$

In Matlab:

```
function [] = InverseNyquist2(Gw, Mm)

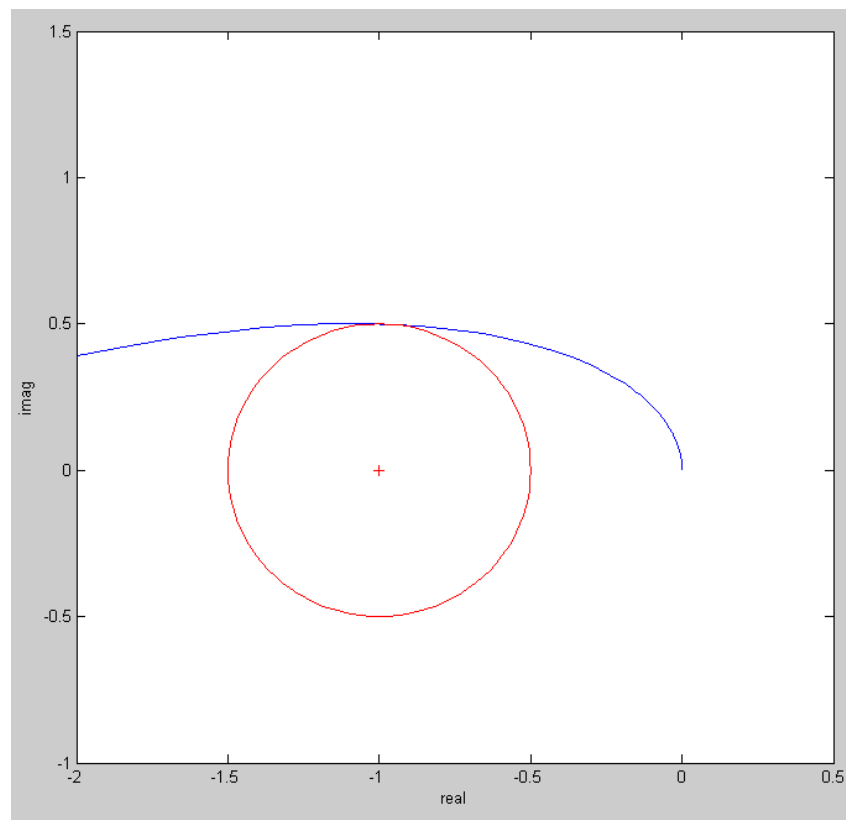
    Gwp = unwrap(angle(Gw))*180/pi;
    Gwm = 20*log10(abs(Gw));

    % M-Circle
    phase = [0:0.01:1]' * 2*pi;
    Mcl = Mm * exp(j*phase);
    Mol = Mcl ./ (1 - Mcl);

    plot(real(1./Gw), imag(1./Gw), 'b', real(1./Mol),
    imag(1./Mol), 'r', -1, 0, 'r+');
    xlabel('real');
    ylabel('imag');
    xlim([-2, 0.5]);
    ylim([-1, 1.5]);
end
```

The calling sequence is

```
w = logspace(-2, 2, 200)';
G = zpk([], [0, -5, -20], 1000);
Gw = Bode2(G, w);
InverseNyquist2(Gw, 2);
InverseNyquist2(Gw*0.9, 2);
InverseNyquist2(Gw*0.77, 2);
```



Inverse Nyquist Diagram with $M_m = 2$ M-circle. Note that the M-circle is centered at -1