

ECE 461/661 - Test #1: Name _____

Fall 2026

1) Let $Y(A, B, C, D)$ be defined by the following Karnaugh map.

a) Find Y as a function of $\{A, B, C, D\}$

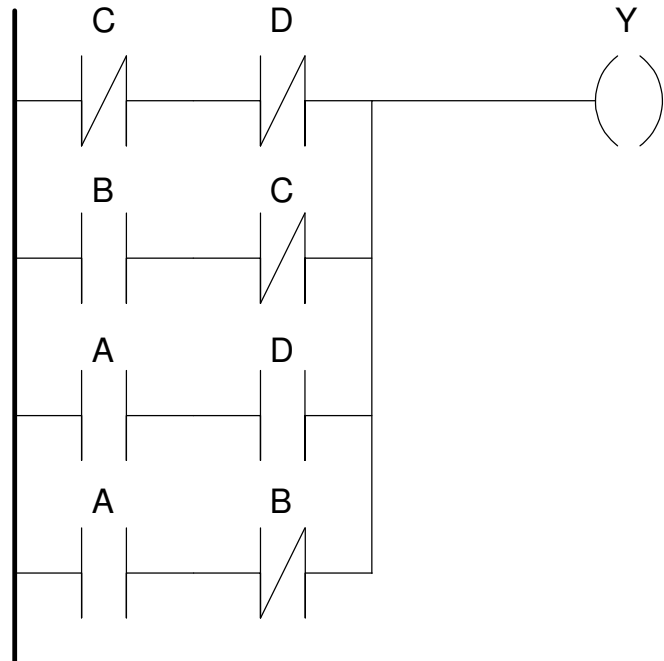
b) Give a ladder logic program to implement Y

- Drawing on paper is sufficient

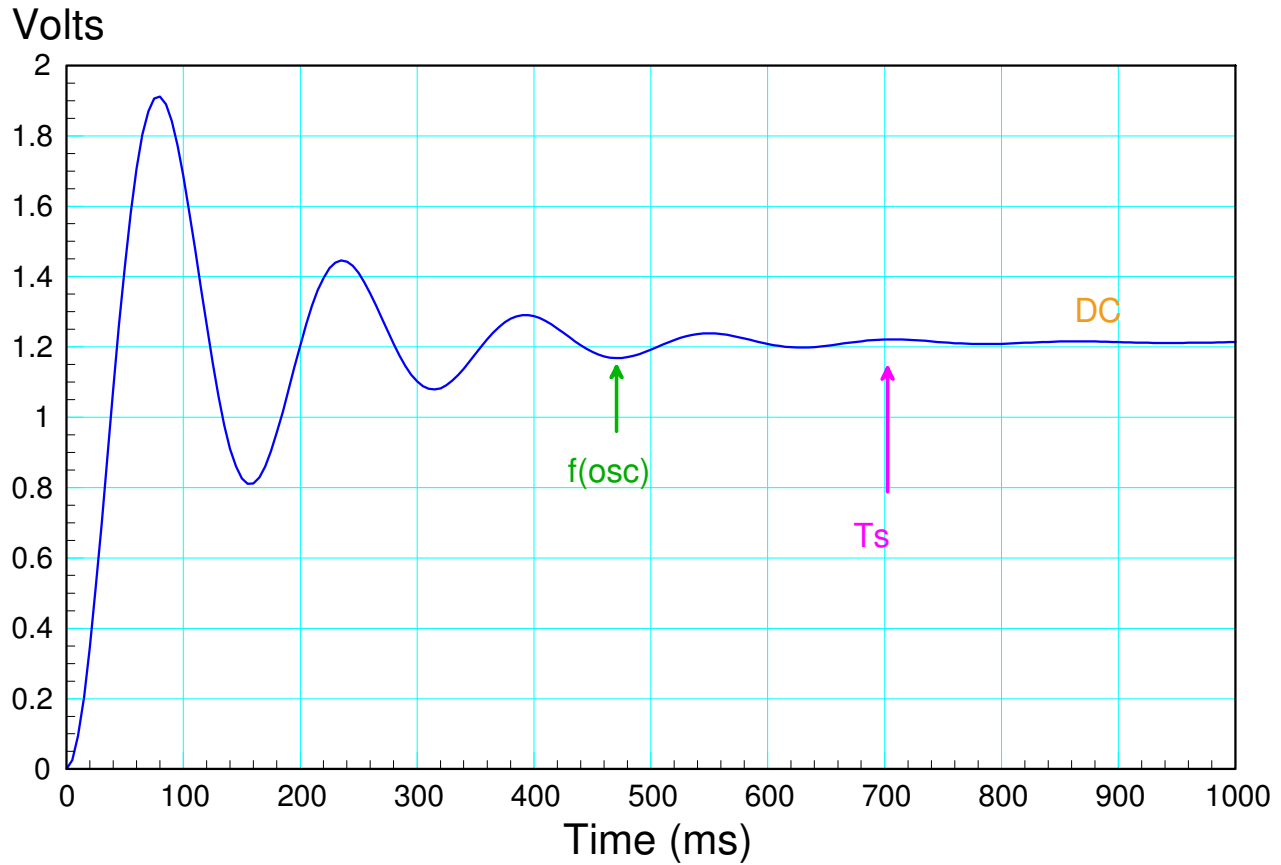
a)

$$Y = \overline{C}\overline{D} + B\overline{C} + AD + A\overline{B}$$

		CD			
		00	01	11	10
Y	00	1	0	0	0
	01	1	1	0	0
	11	1	1	1	0
	10	1	1	1	1



2) Give the transfer function for a system with the following response to a unit step input:



This is a second-order system

$$G(s) \approx \left(\frac{k}{(s+a+jb)(s+a-jb)} \right)$$

2% settling time is about 700ms

$$a = \frac{4}{700ms} = 5.71$$

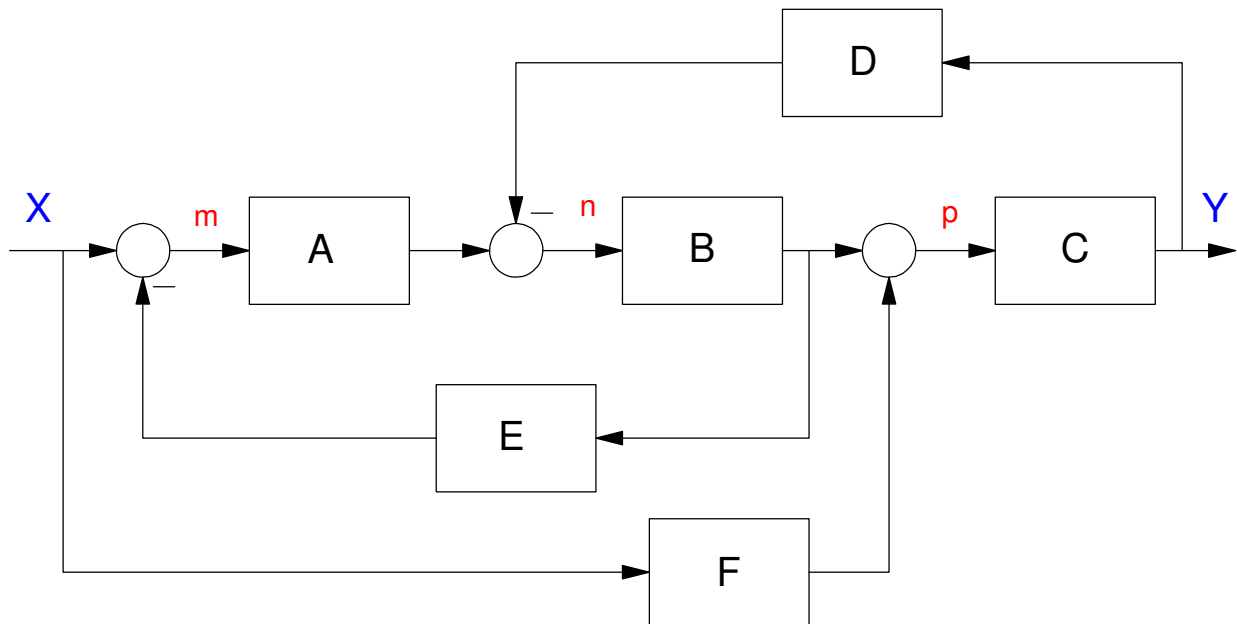
The frequency of oscillation is

$$b = \left(\frac{3 \text{ cycles}}{470ms} \right) 2\pi = 40.10 \frac{rad}{sec}$$

Pick k to make the DC gain equal to 1.2

$$G(s) \approx \left(\frac{1968}{(s+5.71+j40.10)(s+5.71-j40.10)} \right)$$

3) Find the transfer function from X to Y



Shortcut

$$Y = \left(\frac{ABC+FC}{1+ABE+BCD} \right) X$$

Long way

$$m = X - EBn$$

$$n = Am - DY$$

$$p = Bn + FX$$

$$Y = Cp$$

Substitute

$$n = A(X - EBn) - DY \qquad n = \left(\frac{AX - DY}{1 + AEB} \right)$$

$$p = B \left(\frac{AX - DY}{1 + AEB} \right) + FX$$

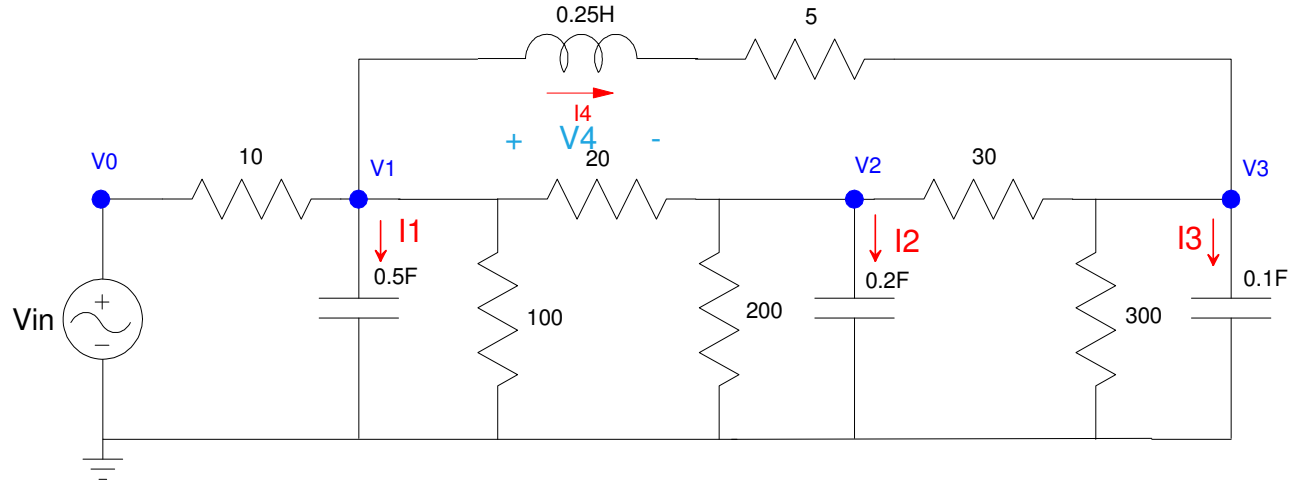
$$Y = Cp = C \left(B \left(\frac{AX - DY}{1 + AEB} \right) + FX \right)$$

$$(1 + AEB)Y = CB(AX - DY) + (1 + AEB)CFX$$

$$Y = \left(\frac{CBA + CF + AEBCF}{1 + AEB + CBD} \right) X$$

4) For the following RLC circuit:

- Write the dynamics of this system as four coupled differential equations in terms of $\{V_{in}, V_1, V_2, V_3, I_4\}$
- You don't need to solve or put in state-space form (that's a different problem on the test)



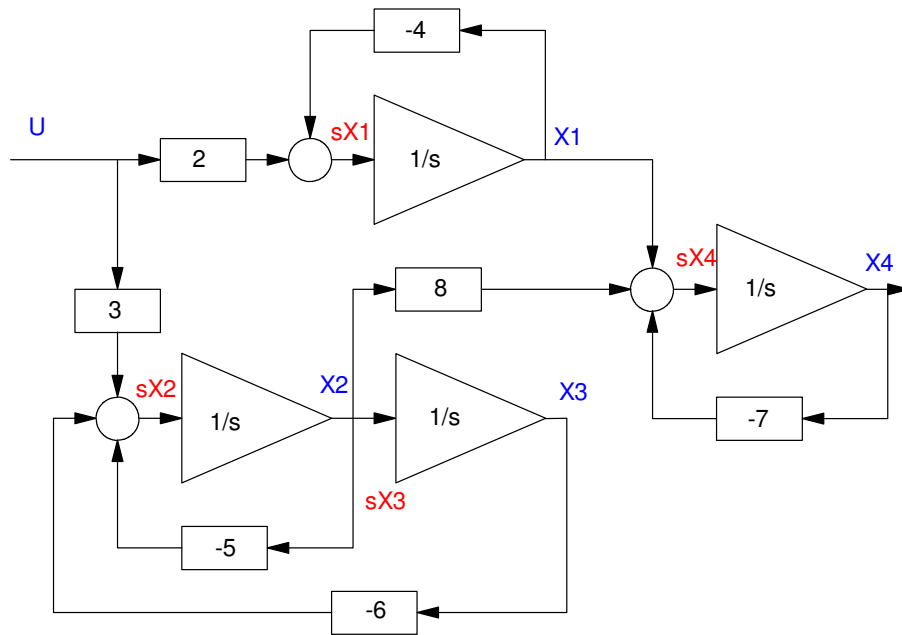
$$I_1 = 0.5sV_1 = \left(\frac{V_0 - V_1}{10}\right) - I_4 + \left(\frac{V_2 - V_1}{20}\right) - \left(\frac{V_1}{100}\right)$$

$$I_2 = 0.2sV_2 = \left(\frac{V_1 - V_2}{20}\right) + \left(\frac{V_3 - V_2}{30}\right) - \left(\frac{V_2}{200}\right)$$

$$I_3 = 0.1sV_3 = I_4 + \left(\frac{V_2 - V_3}{30}\right) - \left(\frac{V_3}{300}\right)$$

$$V_4 = 0.25sI_4 = V_1 - 5I_4 - V_3$$

5) Express the following dynamics in state-space form. Only the A and B matrices are needed:



Write the dynamics

$$sX_1 = 2U - 4x_1$$

$$sX_2 = 3U - 5x_2 - 6x_3$$

$$sX_3 = x_2$$

$$sX_4 = x_1 + 8x_2 - 7x_4$$

in matrix form

$$\begin{bmatrix} sX_1 \\ sX_2 \\ sX_3 \\ sX_4 \end{bmatrix} = \begin{bmatrix} -4 & 0 & 0 & 0 \\ 0 & -5 & -6 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 8 & 0 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} U$$