

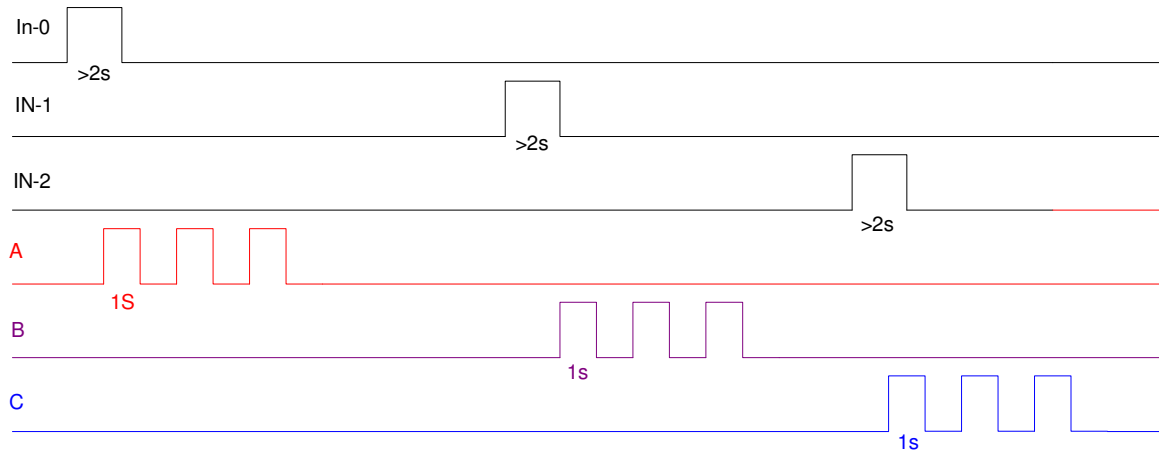
Homework #7: ECE 461/661

If-Statements in Ladder Logic, Gain Compensation, Lead Compensation

If-Statements in Ladder Logic

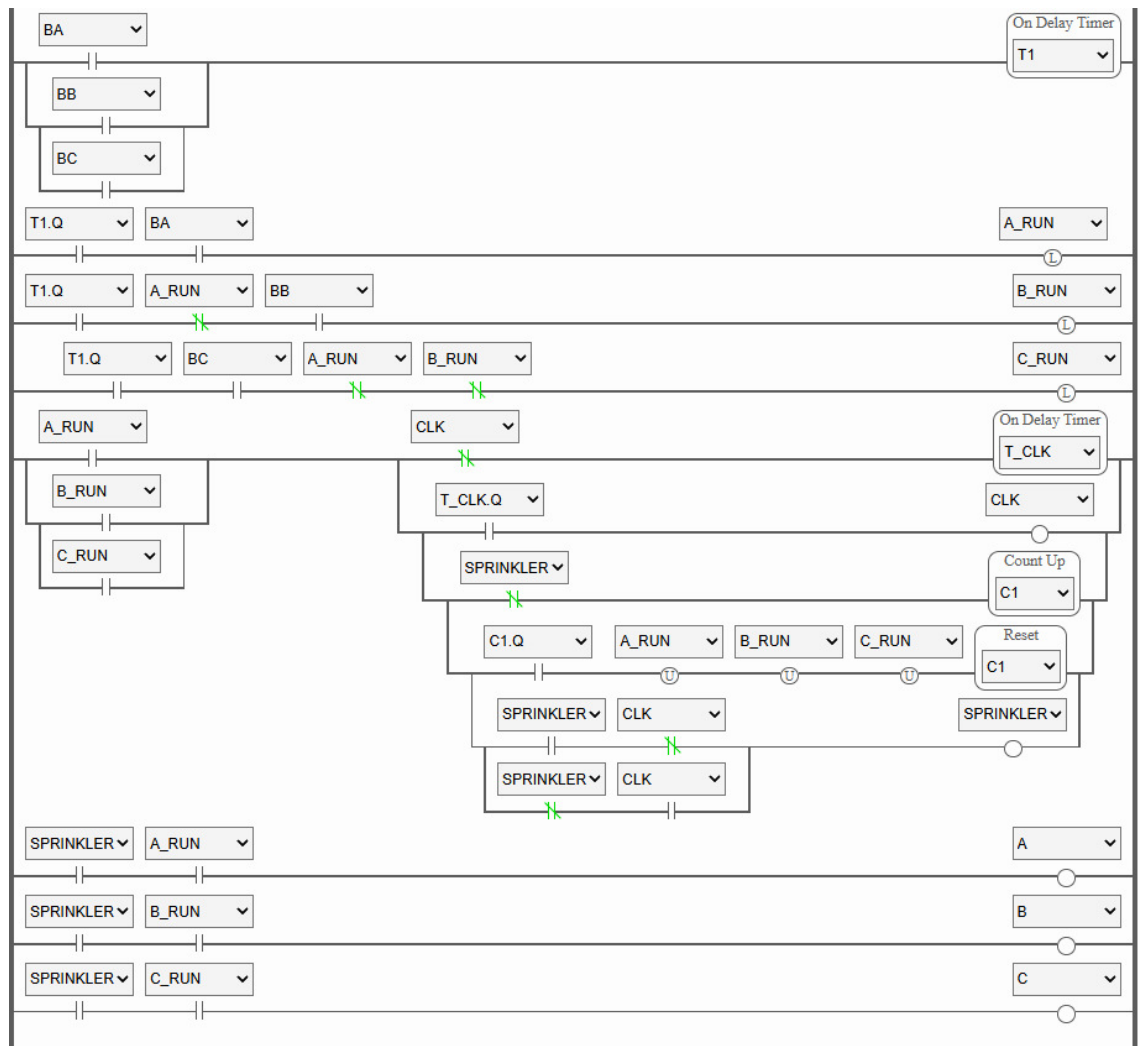
Write a ladder-logic program for an irrigation system

- If Button 0 (IN-0) is held down for 2 seconds, sprinkler A turns on and off three times then stops.
- If Button 1 (IN-1) is held down for 2 seconds, sprinkler B turns on and off three times then stops.
- If Button 2 (IN-2) is held down for 2 seconds, sprinkler C turns on and off three times then stops.



1) Write a ladder logic program using timer blocks to implement this watering system

- Your pick which program
- Code Composer Workbench or PLC Fiddle recommended



2) Verify your code's operation

Buttons held down for less than 2 seconds

- nothing happens

Button BA held down for 2 seconds

- A pulses three times
- One second per pulse
- Stops after three

Button BB held down for 2 seconds

- B pulses three times
- One second per pulse
- Stops after three

Button BC held down for 2 seconds

- C pulses three times
- One second per pulse
- Stops after three

3) Demonstrate your ladder logic program

- In-person
- Make a short video
- Over zoom

better in the video

Gain Compensation: $K(s) = k$

Problem 4-5) Assume

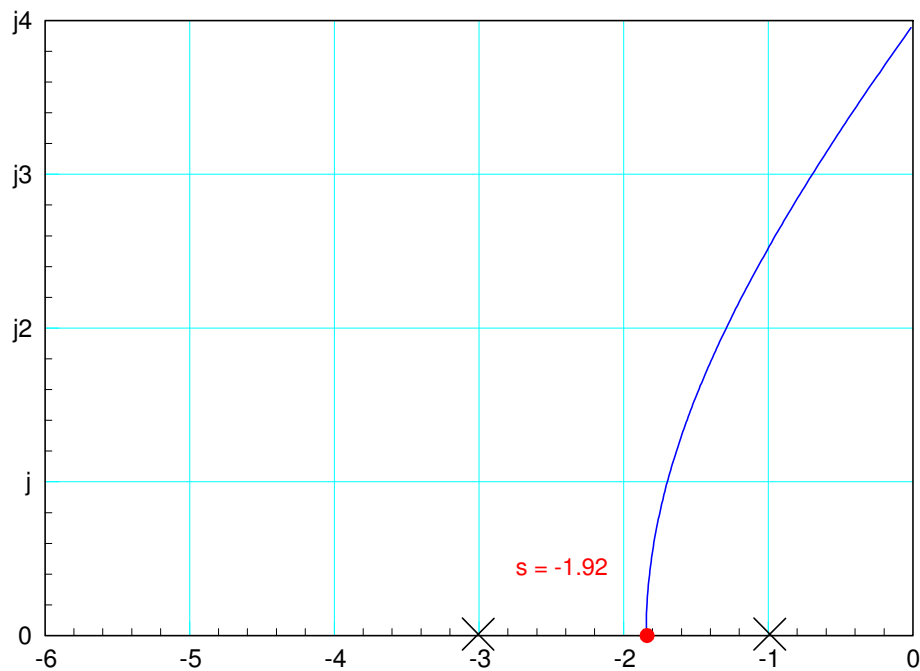
$$G(s) = \left(\frac{2}{(s+1)(s+3)(s+7)(s+10)} \right)$$

4) Find a gain compensator that results in the closed-loop poles being at the breakaway point. For this value of k

- Find the error constant, K_p
- Find the closed-loop poles
- Plot the step response of the closed-loop system

Step 1: Find your design point.

From the root locus plot (below) the breakaway point is about $s = -1.92$



Step 2: Find k

At any point on the root locus,

$$(GK)_s = -1$$

In Matlab

```
>> G = zpk([], [-1, -3, -7, -10], 2)
```

$$\frac{2}{(s+1)(s+3)(s+7)(s+10)}$$

```
>> s = -1.92;
>> evalfr(G,s)

ans =    -0.0490
```

comment: $G(s)$ is negative real. This value of s is on the root-locus

```
>> k = 1/abs(ans)

k =    20.3919

>> Gcl = minreal(G*k / (1 + G*k))

          40.7837
-----
(s+1.773) (s+1.92) (s+7.559) (s+9.749)
```

Comment:

- There is a closed-loop pole at $s = -1.92$. There should be: k was selected to place a root there
- This isn't *exactly* the breakaway point. There should be a repeated root if you're at the breakaway point. It's close - but slightly off.

Find the error constant, K_p

```
>> Kp = evalfr(G,0) * k

Kp =    0.1942
```

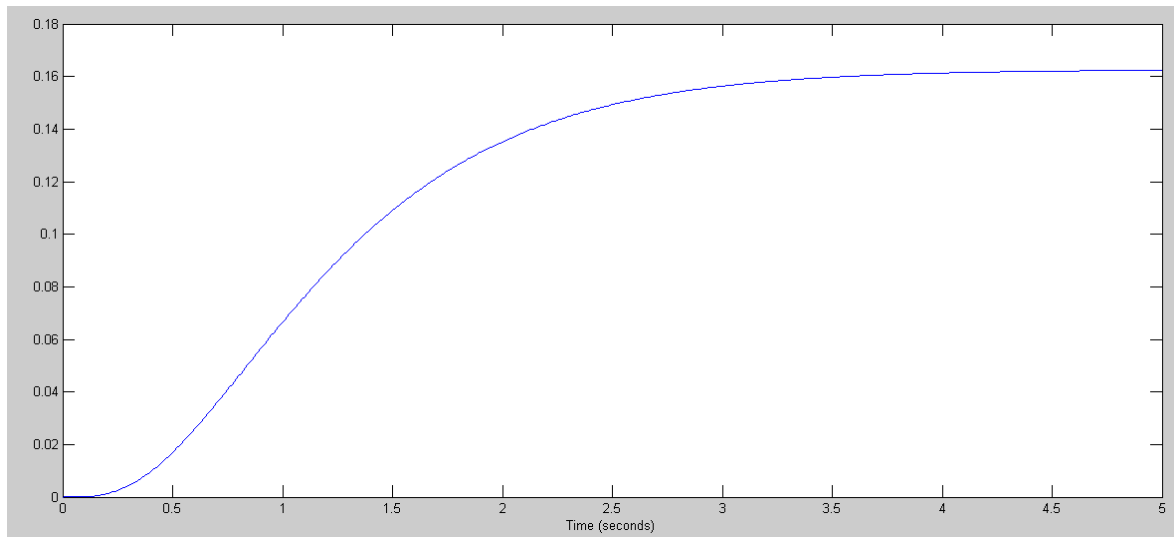
Find the closed-loop poles

```
>> eig(Gcl)

-1.7726
-1.9200
-7.5587
-9.7487
```

Plot the step response of the closed-loop system

```
>> t = [0:0.01:5]';  
>> y = step(Gcl, t);  
>> plot(t,y);  
>> xlabel('Time (seconds)')
```



Pretty much what you expect:

- No oscillations (dominant pole is real)
- $T_s = 2$ seconds (dominant pole is at -1.778)
- Doesn't track a constant set point (type-1 system with $K_p = 0.1942$)

5) Find a gain compensator that results in 20% overshoot for the step response. For this value of k

- Find the error constant, K_p
- Find the closed-loop poles
- Plot the step response of the closed-loop system

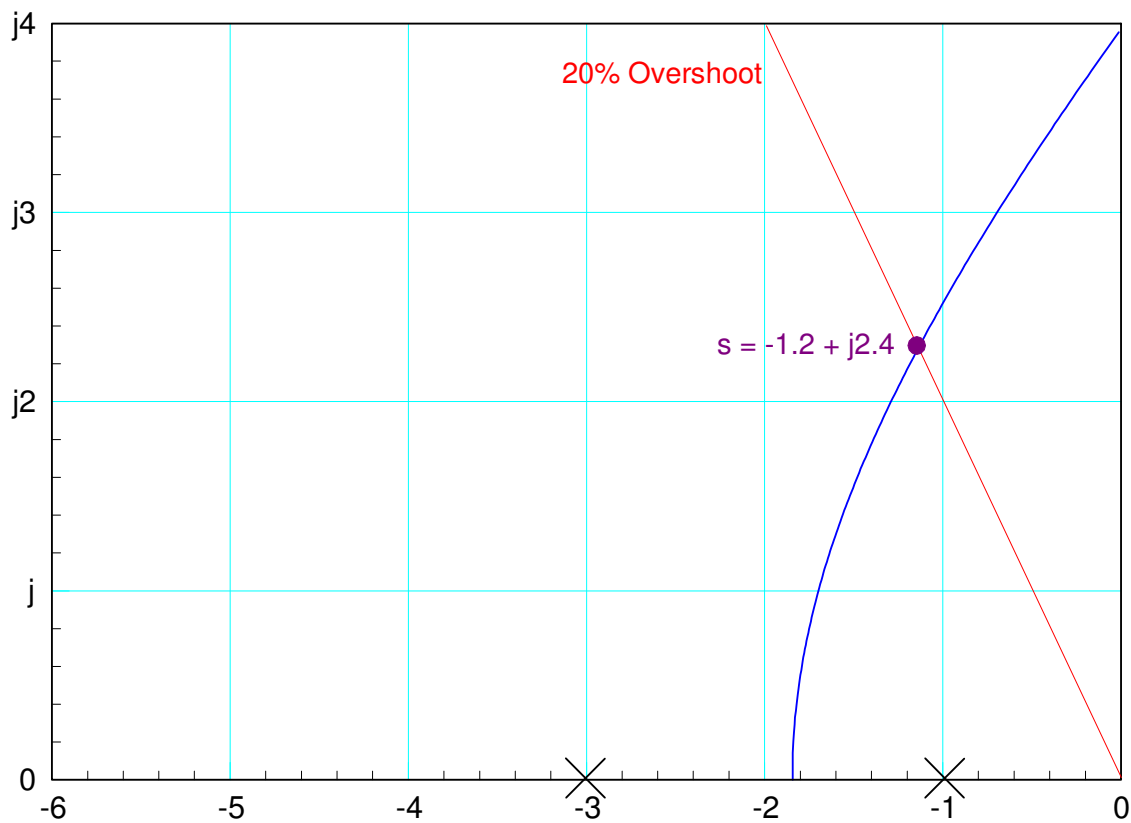
Step 1: Find your design point, s.

Sketch the root locus plot (below).

Add in the damping line for 20% overshoot

- $\zeta = 0.4559$
- $\theta = 62.877^\circ$
- $\tan(\theta) = 1.952$ *go over one, up 1.952*

From the root locus plot, $s = -1.2 + 2.4j$



Step 2: Find k

At any point on the root locus, $G^*k = -1$

```
>> G = zpk([], [-1, -3, -7, -10], 2)
```

$$\frac{2}{(s+1)(s+3)(s+7)(s+10)}$$

```
>> s = -1.2 + 2.4j;
```

```
>> evalfr(G, s)
```

```
ans = -0.0048 + 0.0005i
```

Comment: $G(s)$ should be negative real. It's not, meaning s isn't on the root locus. It's close, however.

```
>> k = 1/abs(ans)
```

```
k = 206.8307
```

```
>> Gcl = minreal(G*k / (1 + G*k))
```

$$\frac{413.6614}{(s^2 + 18.79s + 92.83)(s^2 + 2.205s + 6.718)}$$

```
>> eig(Gcl)
```

```
-1.1027 + 2.3457i  
-1.1027 - 2.3457i  
-9.3973 + 2.1267i  
-9.3973 - 2.1267i
```

The point that's *actually* on the root locus is $s = -1.1027 + 2.3457j$

Find the error constant, K_p

```
>> Kp = evalfr(G, 0) * k
```

```
Kp = 1.9698
```

Find the closed-loop poles

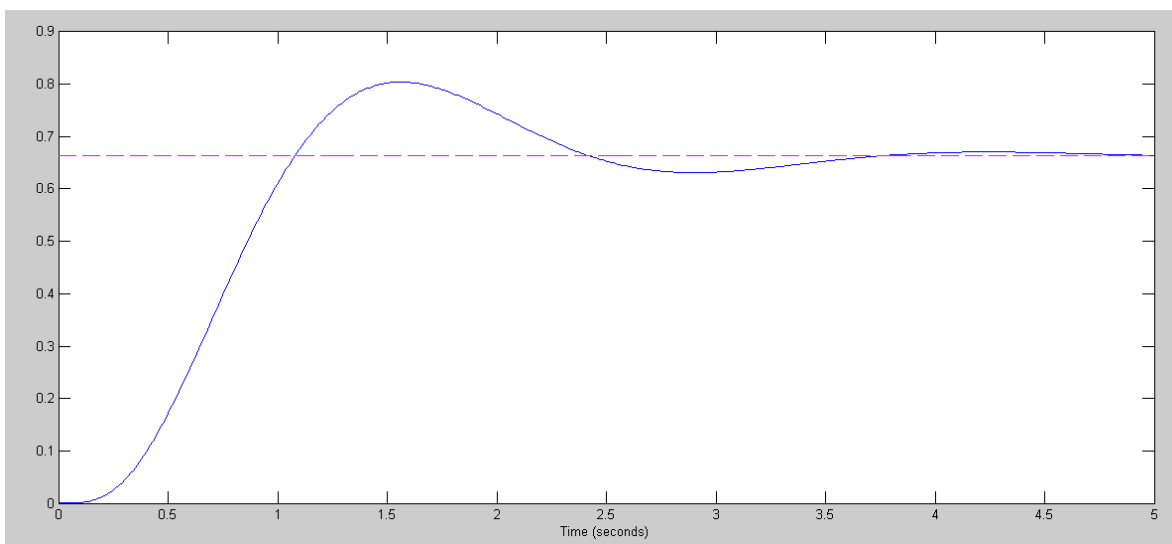
```
>> eig(Gcl)
```

```
-1.1027 + 2.3457i  
-1.1027 - 2.3457i  
-9.3973 + 2.1267i  
-9.3973 - 2.1267i
```

Plot the step response of the closed-loop system

```
>> t = [0:0.01:5]';  
>> y = step(Gcl, t);  
  
>> DC = evalfr(Gcl, 0)  
  
DC =    0.6633  
  
>> OS = max(y) / DC  
  
OS =    1.2122  
  
>> plot(t,y,'b',t,0*t+DC,'m--')  
>> xlabel('Time (seconds)')
```

The overshoot is actually 21.22% (supposed to be 20%)



Lead Compensation: $K(s) = k \left(\frac{s+a}{s+10a} \right)$

$$G(s) = \left(\frac{2}{(s+1)(s+3)(s+7)(s+10)} \right)$$

6) Design a lead compensator which results in 20% overshoot for the step response. For this $K(s)$,

- Find the error constant, K_p
- Find the closed-loop poles
- Plot the step response of the closed-loop system

Step 1: Pick the form for $K(s)$:

- Keep the pole at $s = -1$. This helps the tracking for constant set points (as close to $s=0$ as you get for this system)
- Cancel the next slowest pole at $s = -3$
- Replace it with a pole at $s = -30$

$$K(s) = k \left(\frac{s+3}{s+30} \right)$$

$$GK = \left(\frac{2k}{(s+1)(s+7)(s+10)(s+30)} \right)$$

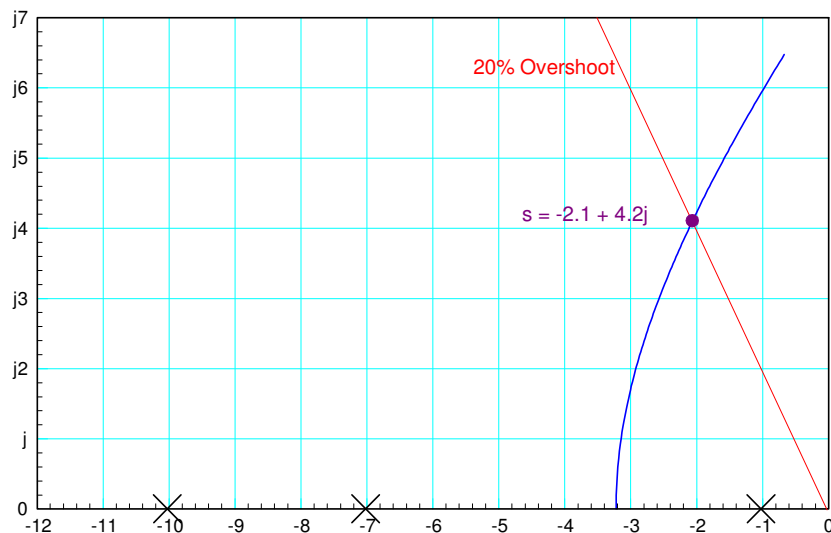
Step 2: Sketch the root locus plot

```
>> GK = zpk([], [-1, -7, -10, -30], 2)

          2
-----
(s+1) (s+7) (s+10) (s+30)

>> k = logspace(-2, 2, 200)';
>> R = rlocus(G, k);
>> hold on
>> plot([0, -2], [0, 1.9505*2], 'r--')
>> [real(R'), imag(R')]

```



Pick a design point on the root locus plot (20% overshoot)

```
>> s = -2.1 + 4.2j;
>> evalfr(GK, s)

ans = -2.8261e-004 +9.0579e-006i

>> k = 1 / abs(ans)

k = 3.5366e+003

>> K = zpk(-3,-30,k)

3536.5844 (s+3)
-----
(s+30)
```

Check the closed-loop system

```
>> Gcl = minreal(G*K / (1+G*K))

7073.1688
-----
(s+14.5) (s+29.43) (s^2 + 4.067s + 21.49)

>> eig(Gcl)

-14.5043
-29.4291
-2.0333 + 4.1661i      where the dominant pole really is...
-2.0333 - 4.1661i

>> t = [0:0.01:5]';
>> y = step(Gcl, t);
>> DC = evalfr(Gcl, 0)

DC = 0.7711

>> OS = max(y) / DC

OS = 1.2007
```

Find the error constant, Kp

```
>> Kp = evalfr(G*K, 0)

Kp = 3.3682
```

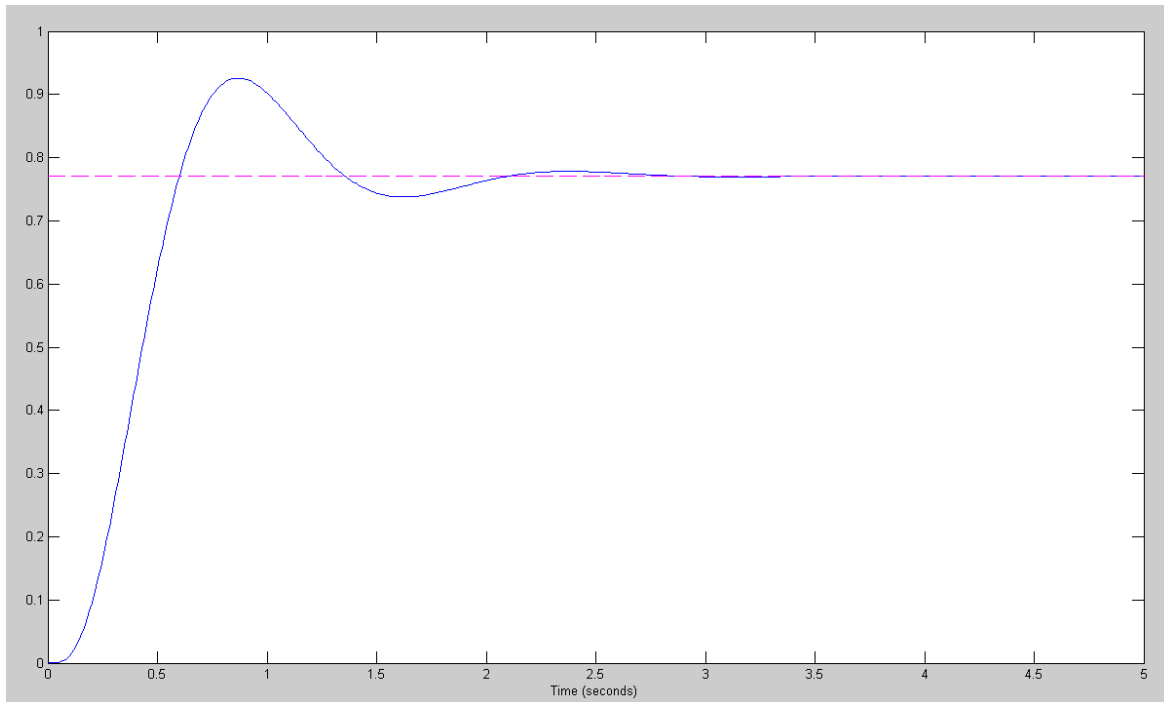
Find the closed-loop poles

```
>> eig(Gcl)

-14.5043
-29.4291
-2.0333 + 4.1661i
-2.0333 - 4.1661i
```

Plot the step response of the closed-loop system

```
>> t = [0:0.01:5]';  
>> y = step(Gcl, t);  
>> plot(t,y,'b',t,0*t+DC,'m--')  
>> xlabel('Time (seconds)')
```



Note: With a lead compensator, you get

- A faster system, and
- Better tracking (larger K_p)

7) Give an op-amp circuit to implement $K(s)$

$$K(s) = 3536.5844 \left(\frac{s+3}{s+30} \right)$$

3536 is a lot of gain for an op-amp. Instead of doing this all at once, express this as two gains

$$K(s) = 35.36 \left(\frac{s+3}{s+30} \right) \cdot 100$$

For the lead compensator, let

$$R_3 = 1M \text{ (arbitrary)}$$

Find R_1 by letting s go to infinity:

$$\left(35.36 \left(\frac{s+3}{s+30} \right) \right)_{s \rightarrow \infty} = 35.36 = \frac{1M}{R_1}$$

$$R_1 = 28.28k\Omega$$

Find R_2 by letting s go to zero

$$\left(35.36 \left(\frac{s+3}{s+30} \right) \right)_{s \rightarrow 0} = 3.536 = \frac{1M}{R_1 + R_2}$$

$$R_2 = 254.5k\Omega$$

Find C to place the zero at $s = -3$

$$\frac{1}{R_2 C} = 3$$

$$C = 1.310\mu F$$

