

Homework #5: ECE 461/661

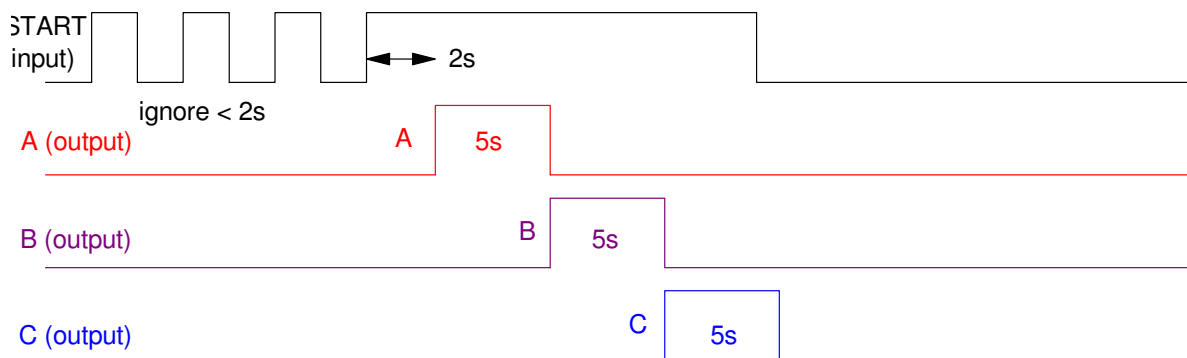
Timer Blocks, Error Constants, and Routh Criteria

Ladder Logic & Timer Blocks

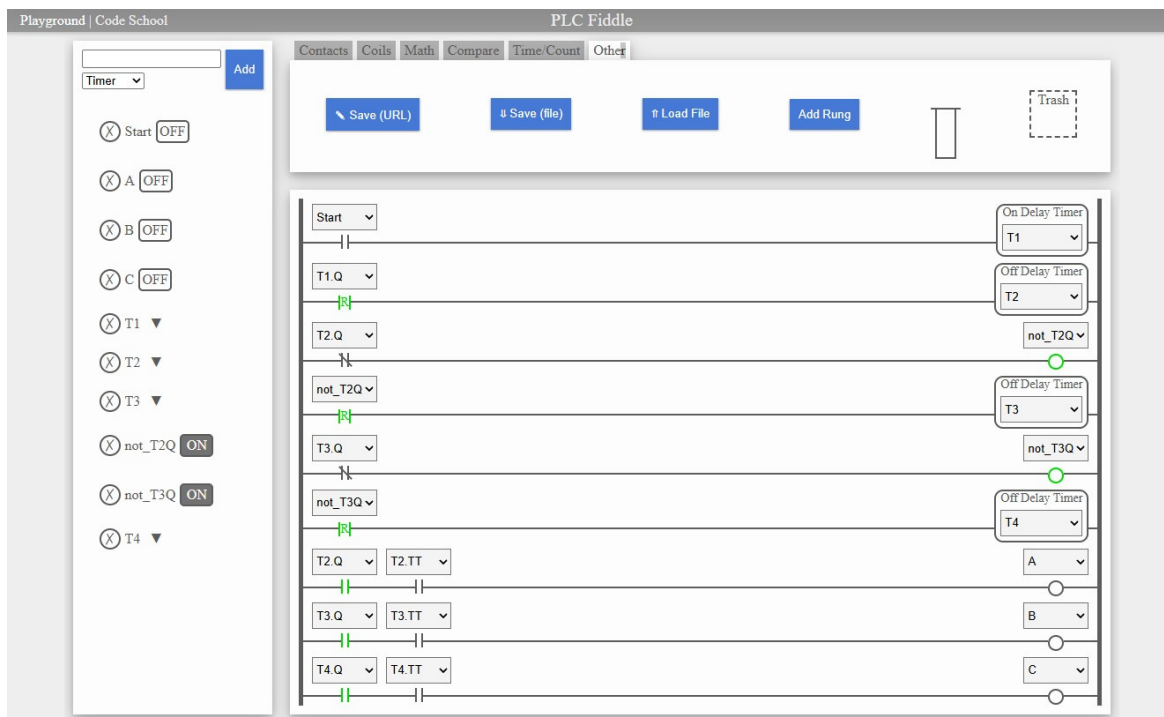
Write a ladder-logic program which uses timer blocks to implement a watering system

- In order to start operation, the START button has to be held down for 2 seconds.
- Once started, sprinkler A turns on for 5 seconds then turns off.
- When A turns off, sprinkler B turns on for 5 seconds then turns off.
- When B turns off, sprinkler C turns on for 5 seconds then turns off.

Once C turns off, the process ends and it waits for the START button to be pressed.

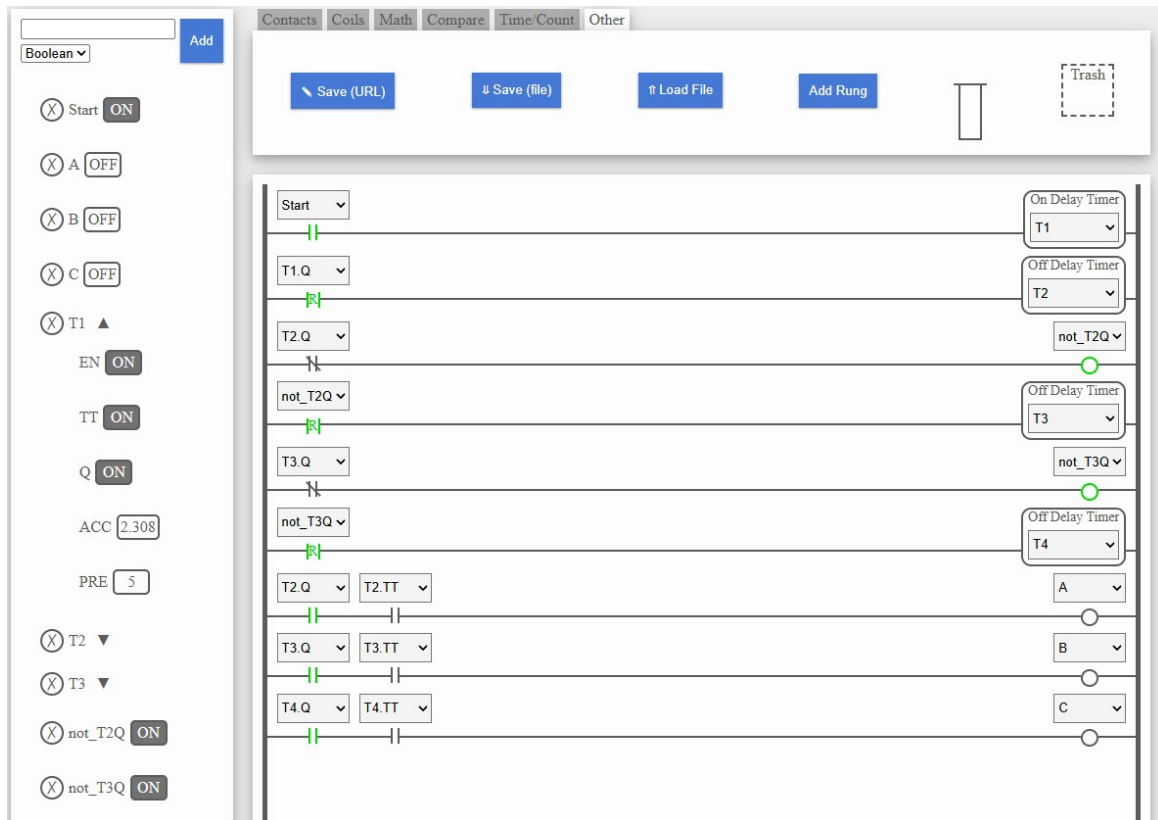


1) Write a ladder logic program using timer blocks to implement this watering system



2) Verify your code's operation

- Button presses less than 2 seconds are ignored
- Once started, the process continues turning on all three outputs, one at a time



3) Demonstrate your ladder logic program

- In-person
- Make a short video
- Over zoom

Shows off better in the video

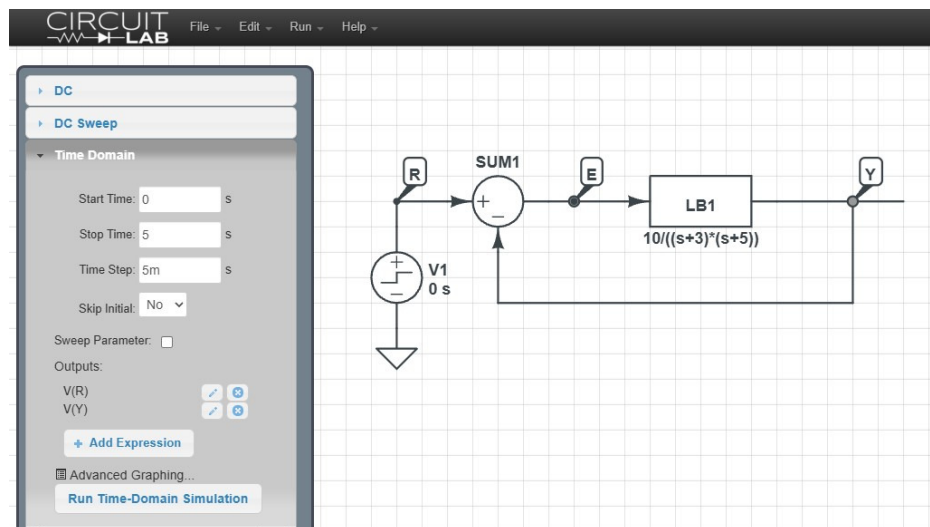
Error Constants

4) Determine the system type, the error constants, and the steady-state error for the following systems

G(s)	System Type	K _p	K _v	Steady-State Error for a Step Input
$\left(\frac{10}{(s+3)(s+5)}\right)$	0	10/15	0	0.600
$\left(\frac{10}{s(s+3)(s+5)}\right)$	1	inf	10/15	0
$\left(\frac{10(s+1)}{s^2(s+3)(s+5)}\right)$	2	inf	inf	0
$\left(\frac{10}{(s-1)(s+5)}\right)$	0	-2	0	-1.000

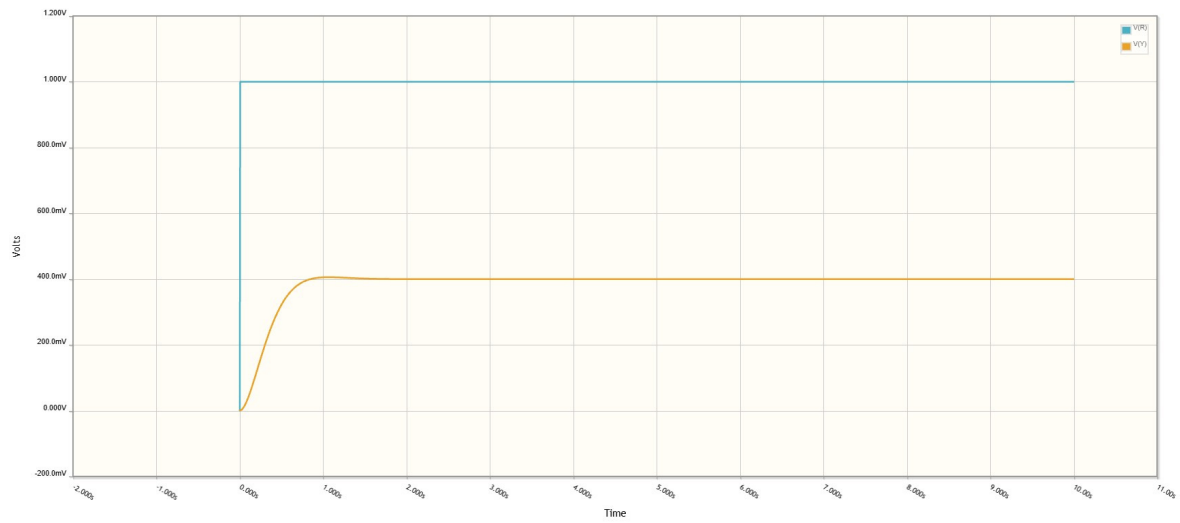
Checking in CircuitLab (not required).

- Build the feedback system
- Run a time-domain simulation
- Check the steady-state response



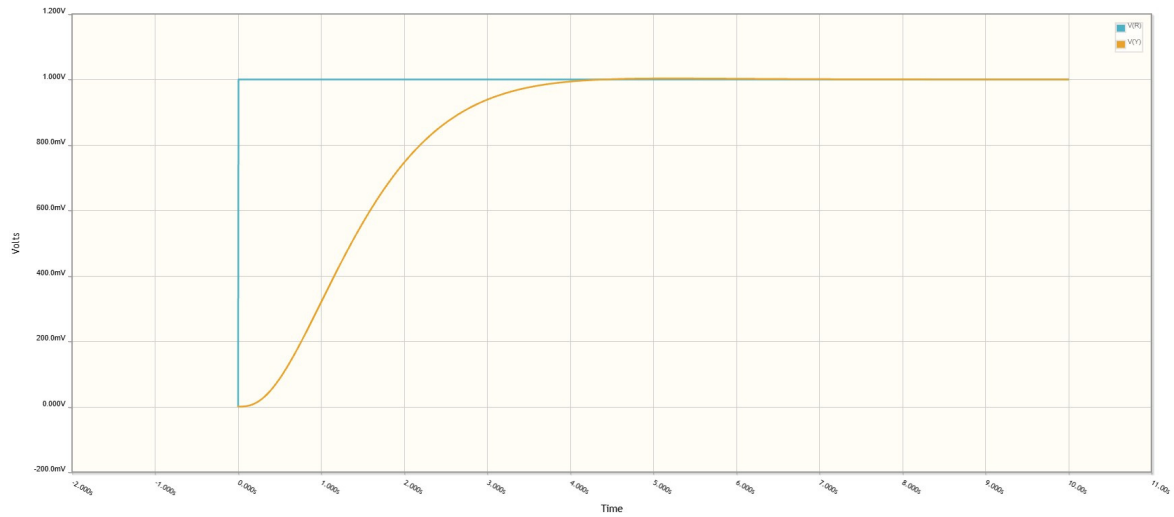
a) $G(s) = \left(\frac{10}{(s+3)(s+5)} \right)$

- $E(\text{step}) = 0.6$



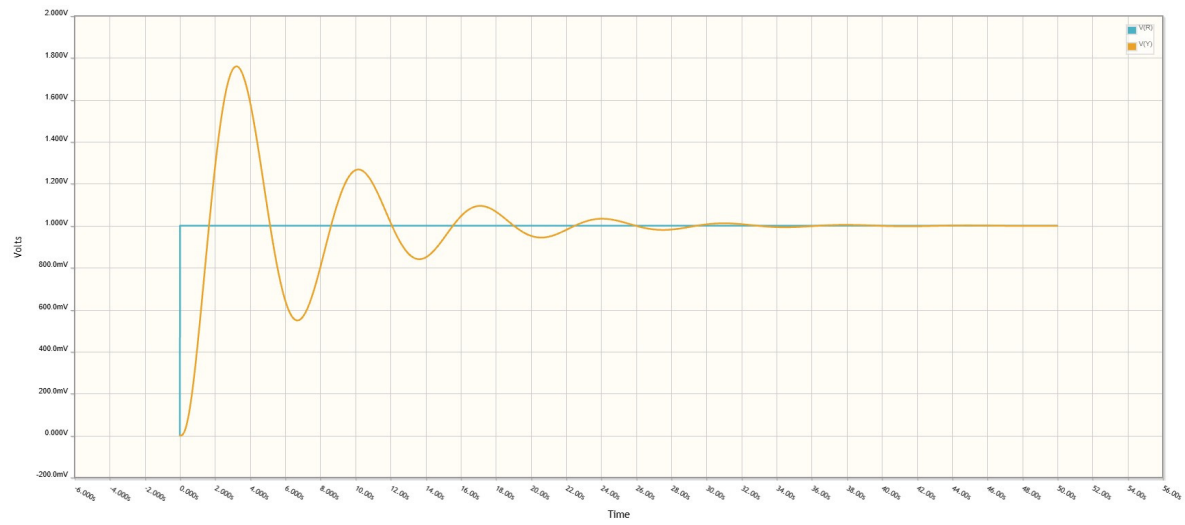
b) $G(s) = \left(\frac{10}{s(s+3)(s+5)} \right)$

- $E(\text{step}) = 0$



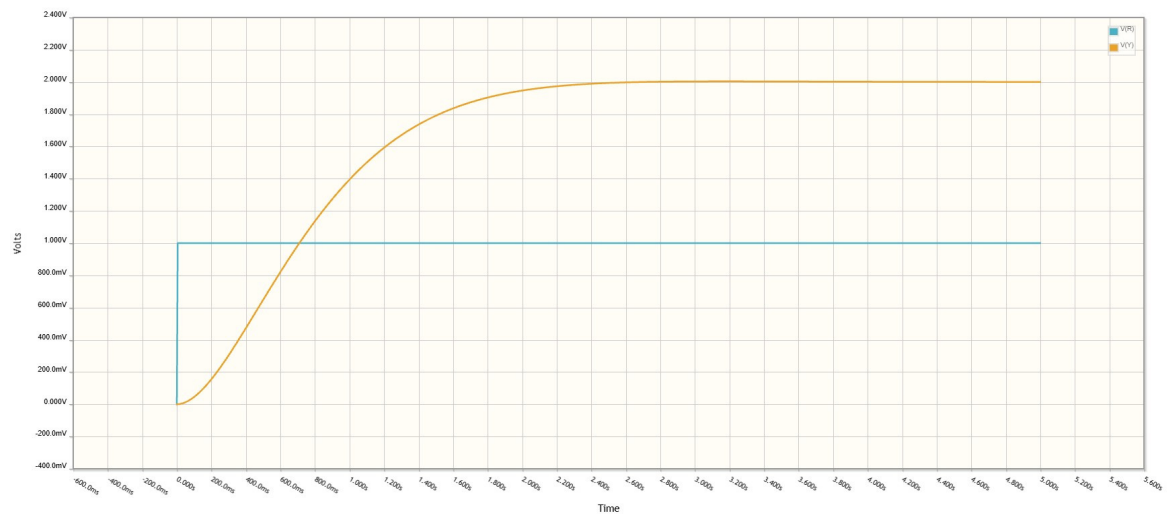
c) $G(s) = \left(\frac{10(s+1)}{s^2(s+3)(s+5)} \right)$

- E(step) = 0



d) $G(s) = \left(\frac{10}{(s-1)(s+5)} \right)$

- E(step) = -1.00



Routh Criteria

5) Set up a Routh table and determine the range of k that results in a stable system (i.e. a negative definite polynomial)

$$(s + 1)(s + 3)(s + 7)(s + 10) + 2k = 0$$

Multiply out the polynomial

```
>> X = poly([-1,-3,-7,-10])
X =      1      21     141     331     210
```

$$s^4 + 21s^3 + 141s^2 + 331s + 210 + 2k = 0$$

Place in a Routh table

1	141	210 + 2k
21	331	0
125.2381 (a)	210 + 2k	0
295.79 - 0.3354k (b)	0	0
210 + 2k	0	0

$$k < 881.9955$$

$$k > -105.00$$

Result: $-105 < k < 866.2338$

(a)

$$-\frac{\begin{vmatrix} 1 & 141 \\ 21 & 331 \end{vmatrix}}{21} = 125.2381$$

(b)

$$-\frac{\begin{vmatrix} 21 & 331 \\ 125.2381 & 210+2k \end{vmatrix}}{125.2381} = 295.7871 - 0.3354k$$

Checking with Matlab (not required):

Multiply out the numerator (Z) and denominator (P) polynomials

```
>> P = poly([-1,-3,-7,-10])  
P =      1      21     141     331     210  
  
>> Z = [0,0,0,0,2]  
Z =      0      0      0      0      2
```

Check if there is a root on the jw axis at the extremes for k:

k = -105.000

```
>> roots(P - 105*Z)  
  
      0  
-10.7435  
-5.1282 + 2.1238i  
-5.1282 - 2.1238i
```

k = +866.2338

```
>> roots(P + 866.2338*Z)  
  
-10.4824 + 3.8474i  
-10.4824 - 3.8474i  
-0.0176 + 3.9470i  
-0.0176 - 3.9470i
```

The latter is off by a little

6) Set up a Routh table and determine the range of k that results in a stable system (i.e. a negative definite polynomial)

$$(s-1)(s+2)(s+5)(s+7) + 2(s+1)k = 0$$

Multiply out the polynomials. In Matlab

```
>> X = poly([1,-2,-5,-7])
```

```
X =      1      13      45      11     -70
```

$$s^4 + 13s^3 + 45s^2 + 11s - 70 + 2ks + 2k = 0$$

$$(1)s^4 + (13)s^3 + (45)s^2 + (11 + 2k)s + (2k - 70) = 0$$

Put this into a Routh table

1	45	2k - 70
13	11 + 2k	0
44.15 - 0.1538k (a)	2k - 70	0
(b)	0	0
2k - 70	0	0

$$k < 287.06$$

$$-20.825 < k < 217.89$$

$$k > 35$$

Result: $35 < k < 217.89$.

(a)

$$-\frac{\begin{vmatrix} 1 & 45 \\ 13 & 11+2k \end{vmatrix}}{13} = 44.1538 - 0.1538k$$

(b)

$$-\frac{\begin{vmatrix} 13 & 11+2k \\ 44.1538-0.1538k & 2k-70 \end{vmatrix}}{44.15-0.1538k} = \frac{1395.7+60.6016k-0.3076k^2}{(44.15-0.1538k)}$$

Check that there is a root on the $j\omega$ axis for each extreme:

Multiply out the numerator and denominator polynomials:

```
>> P = poly([1,-2,-5,-7])
```

```
P =      1      13      45      11     -70
```

```
>> Z = [0,0,0,2,2]
```

```
Z =      0      0      0      2      2
```

Check the roots $P + k*Z$

$k = +35.000$

```
>> roots(P + 35*Z)
```

```
      0  
-9.0000  
-2.0000 + 2.2361i  
-2.0000 - 2.2361i
```

$k = +217.89$

```
>> roots(P + 217.89*Z)
```

```
-12.1228  
 0.0004 + 5.8623i  
 0.0004 - 5.8623i  
-0.8780
```

```
>>
```