

Homework #4: ECE 461 / 661

Code Composer Workbench, Mass Spring Systems, Rotational Systems

Code Composer Workbench / PLC Fiddle

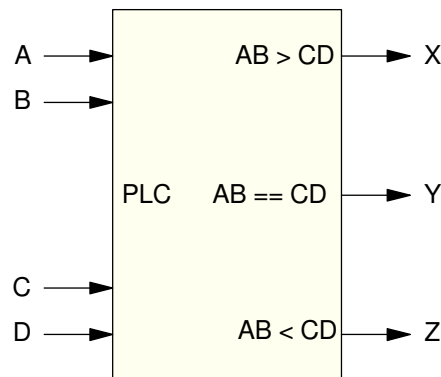
note: For the PLC homework assignments, you can use either

- Allen Bradley's Code Composer Workbench along with a Micro810 PLC (lab kits available), or
- PLC Fiddle (plcfiddle.com)

It's kind of fun seeing and hearing the lights and relays opening and closing. PLC Fiddle is a lot more convenient, however, being an online site where there's nothing to download and install. Your pick.

1) Assume there are two 2-digit numbers, AB and CD, taking on the values of 0 to 3 (00 to 11). Determine the logic for three outputs:

- X is true if $AB > CD$ *OUT-00 (red light) if using a Micro-810*
- Y is true if $AB == CD$ *OUT-01 (yellow light) if using a Micro-810*
- Z is true if $AB < CD$ *OUT-02 (green light) if using a Micro-810*



	X	CD			
		00	01	11	10
AB	00	0	0	0	0
	01	1	0	0	0
	11	1	1	0	1
	10	1	1	0	0

$$X = A\bar{C} + B\bar{C}\bar{D} + AB\bar{D}$$

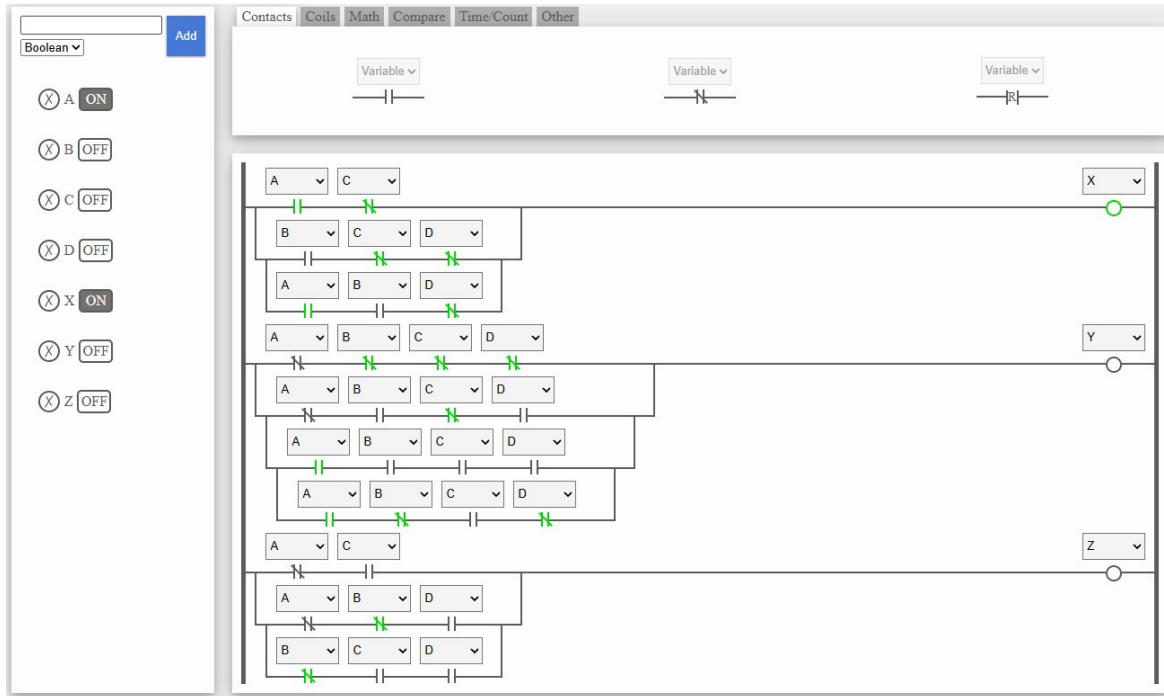
		CD			
		00	01	11	10
AB	00	1	0	0	0
	01	0	1	0	0
	11	0	0	1	0
	10	0	0	0	1

$$Y = A'B'C'D' + A'BC'D + ABCD + AB'CD'$$

		CD			
		00	01	11	10
AB	00	0	1	1	1
	01	0	0	1	1
	11	0	0	0	0
	10	0	0	1	0

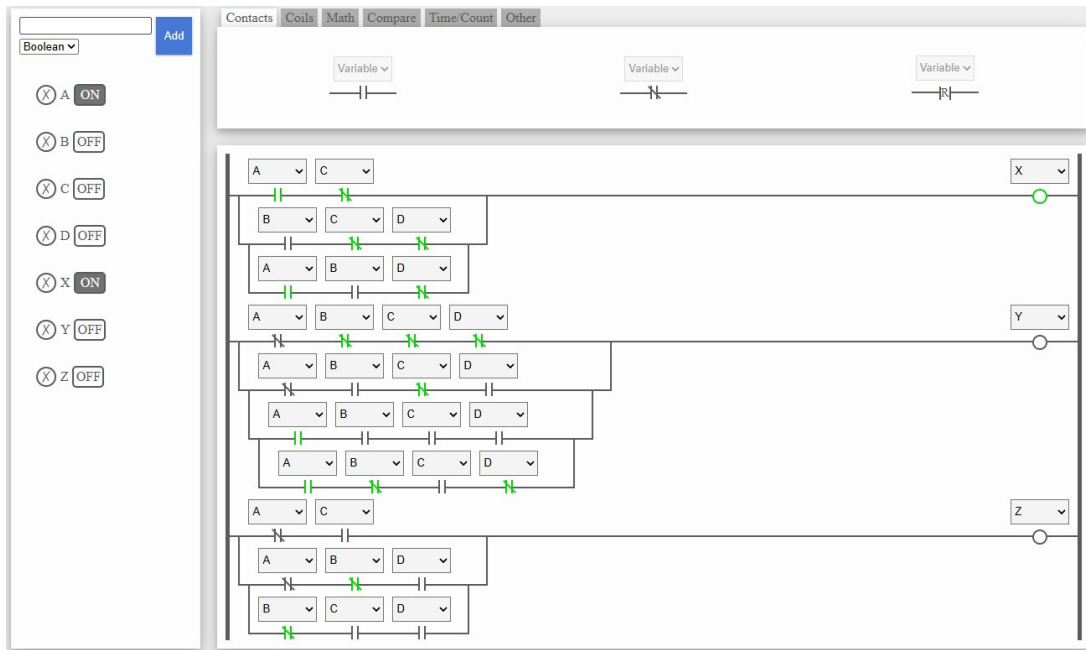
$$Z = A'C + A'B'D + B'CD$$

- 2) Implement these three outputs using ladder logic using Code Composer Workbench or PLC Fiddle
- It's kind of fun to see and hear the relays opening and closing with CCW and a Micro-810 PLC
 - PLC Fiddle is a lot more convenient. You don't have to install anything.
 - Your pick (you can use another program if you like)

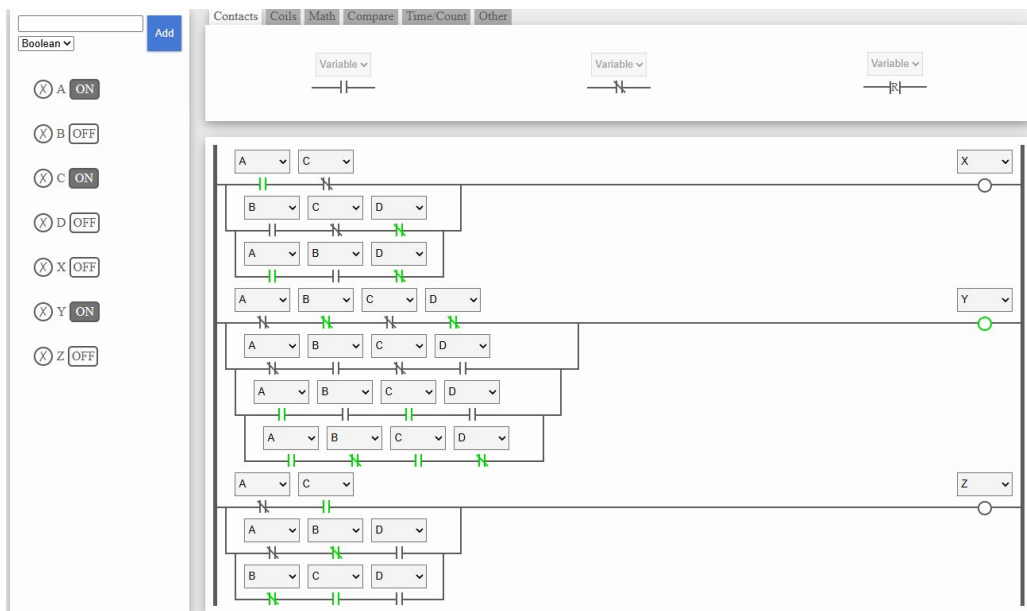


3) Demonstrate your working ladder logic program

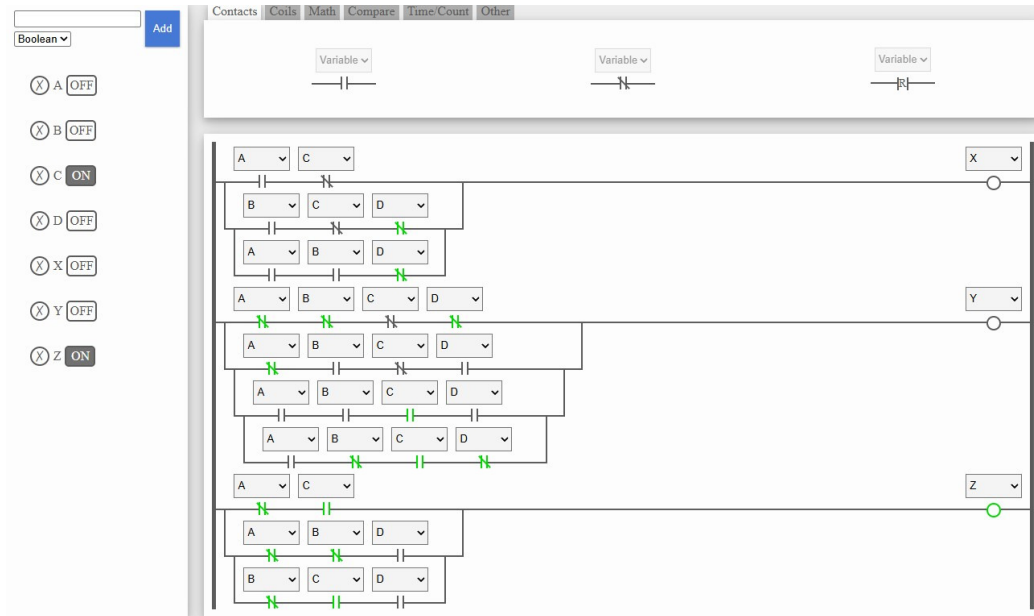
In-person, over zoom, attach a short video, take some screen shots



$AB > CD$



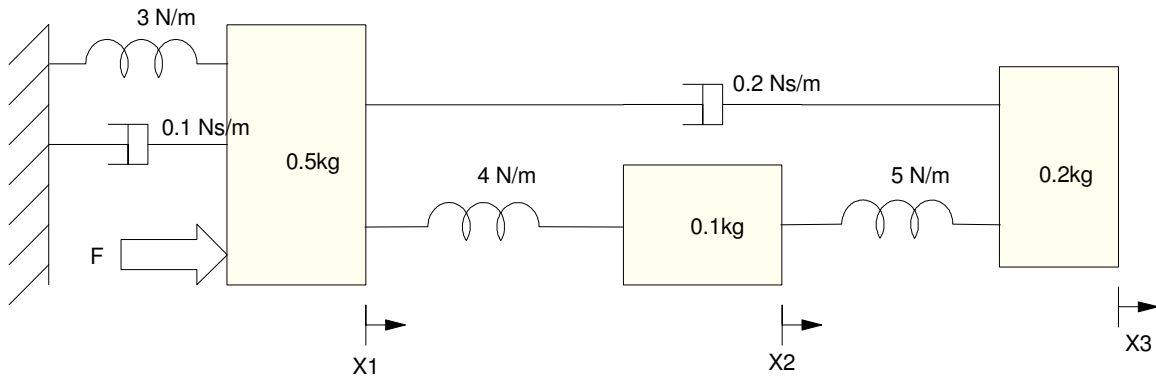
$AB == CD$



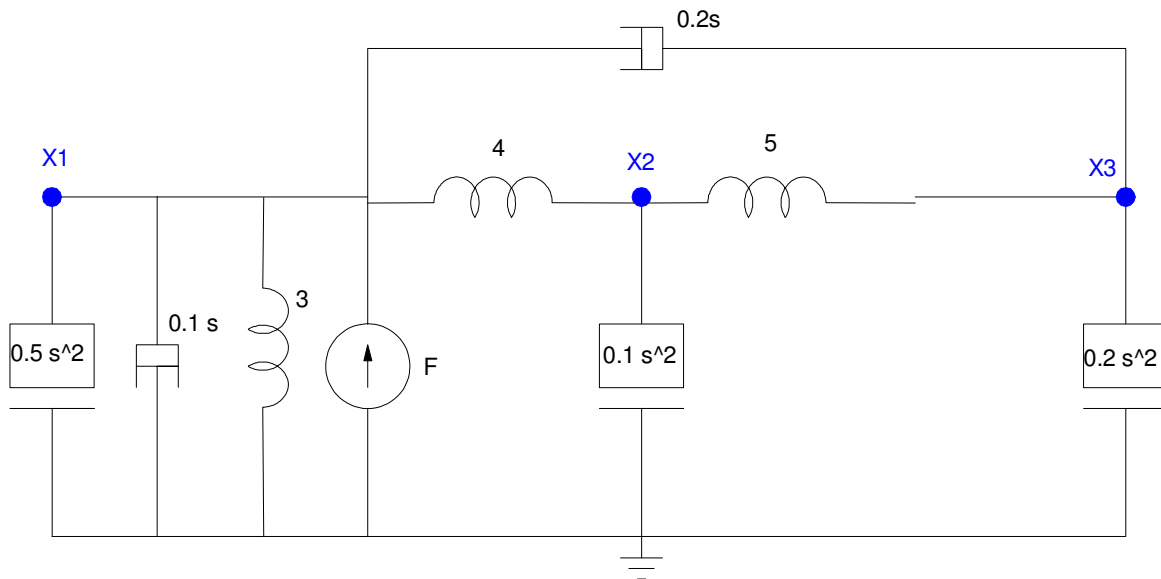
$AB < CD$

Mass-Spring Systems

Consider the following mass-spring system:



4) Draw the circuit equivalent for this system



5) Write the differential equations which describe this system

- i.e. Write the voltage node equations.

$$(0.5s^2 + 0.1s + 3 + 4 + 0.2s)x_1 - (4)x_2 - (0.2s)x_3 = F$$

$$(0.1s^2 + 4 + 5)x_2 - (4)x_1 - (5)x_3 = 0$$

$$(0.2s^2 + 5 + 0.2s)x_3 - (0.2s)x_1 - (5)x_2 = 0$$

6) Find the transfer function from F to X3

Solve for the highest derivative

$$s^2x_1 = -(0.6s + 14)x_1 + (8)x_2 + (0.4s)x_3 + 2F$$

$$s^2x_2 = (40)x_1 - (90)x_2 + (50)x_3$$

$$s^2x_3 = (s)x_1 + (25)x_2 - (s + 25)x_3$$

Place in matrix form

$$s \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ sx_1 \\ sx_2 \\ sx_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ \dots & \dots & \dots & & \dots & \dots & \dots \\ -14 & 8 & 0 & \vdots & -0.6 & 0 & 0.4 \\ 40 & -90 & 50 & \vdots & 0 & 0 & 0 \\ 0 & 25 & -25 & \vdots & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ sx_1 \\ sx_2 \\ sx_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ 2 \\ 0 \\ 0 \end{bmatrix} F$$

Throw in to Matlab with Y = X3

```
>> a11 = zeros(3,3);
>> a12 = eye(3,3);
>> a21 = [-14,8,0;40,-90,50;0,25,-25];
>> a22 = [-0.6,0,0.4;0,0,0;1,0,-1];
>> A = [a11,a12 ; a21, a22]
```

```

      0      0      0      1.0000      0      0
      0      0      0      0      1.0000      0
      0      0      0      0      0      1.0000
-14.0000    8.0000      0    -0.6000      0    0.4000
 40.0000   -90.0000   50.0000      0      0      0
      0    25.0000  -25.0000      1.0000      0   -1.0000
```

```
>> B = [0;0;0;2;0;0]
```

```

      0
      0
      0
      2
      0
      0
```

```
>> C = [0,0,1,0,0,0]
```

```
C =      0      0      1      0      0      0
```

```
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)
```

```

      2 (s+7.113) (s^2 - 7.113s + 140.6)
-----
(s^2 + 0.1377s + 3.179) (s^2 + 1.382s + 17.43) (s^2 + 0.08047s + 108.3)
```

```
>>
```

7) Assume $F = 0$ and $X_3(0) = 1$ meter. What initial condition on X_1 and X_2 result in the energy in the system decaying

- As fast as possible?
- As slow as possible?

This is an eigenvalue / eigenvector problem:

```
>> [M,V] = eig(A)

    0.0008 + 0.0079i    0.0008 - 0.0079i    0.0306 + 0.1243i    0.0306 - 0.1243i    0.0010 - 0.2146i    0.0010 + 0.2146i
   -0.0004 - 0.0914i   -0.0004 + 0.0914i    0.0001 - 0.0572i    0.0001 + 0.0572i   -0.0061 - 0.2893i   -0.0061 + 0.2893i
   -0.0020 + 0.0271i   -0.0020 - 0.0271i   -0.0308 - 0.1835i   -0.0308 + 0.1835i   -0.0128 - 0.3307i   -0.0128 + 0.3307i
   -0.0821 + 0.0077i   -0.0821 - 0.0077i   -0.5327 + 0.0400i   -0.5327 - 0.0400i    0.3823 + 0.0165i    0.3823 - 0.0165i
    0.9509             0.9509             0.2353 + 0.0400i    0.2353 - 0.0400i    0.5158 + 0.0091i    0.5158 - 0.0091i
   -0.2818 - 0.0221i   -0.2818 + 0.0221i    0.7768             0.7768             0.5901             0.5901

   -0.0402 -10.4057i   -0.0402 +10.4057i   -0.6909 - 4.1172i   -0.6909 + 4.1172i   -0.0688 - 1.7817i   -0.0688 + 1.7817i
```

The slow mode is the first eigenvector (real / imag = sine / cosine pretty much)

```
>> X0 = real(M(:,1));
>> X0 = X0 / X0(3)
```

```
x1    -0.3828
x2     0.1749
x3     1.0000
sx1    40.6651
sx2  -470.7802
sx3   139.5477
```

```
>> X0 = imag(M(:,1));
>> X0 = X0 / X0(3)
```

```
x1     0.2912
x2    -3.3726
x3     1.0000
sx1     0.2852
sx2      0
sx3    -0.8159
```

The fast mode is the third and fourth eigenvector

```
>> X0 = real(M(:,3));
>> X0 = X0 / X0(3)
```

```
x1    -0.9931
x2    -0.0039
x3     1.0000
s1    17.3006
sx2   -7.6417
sx3  -25.2255
```

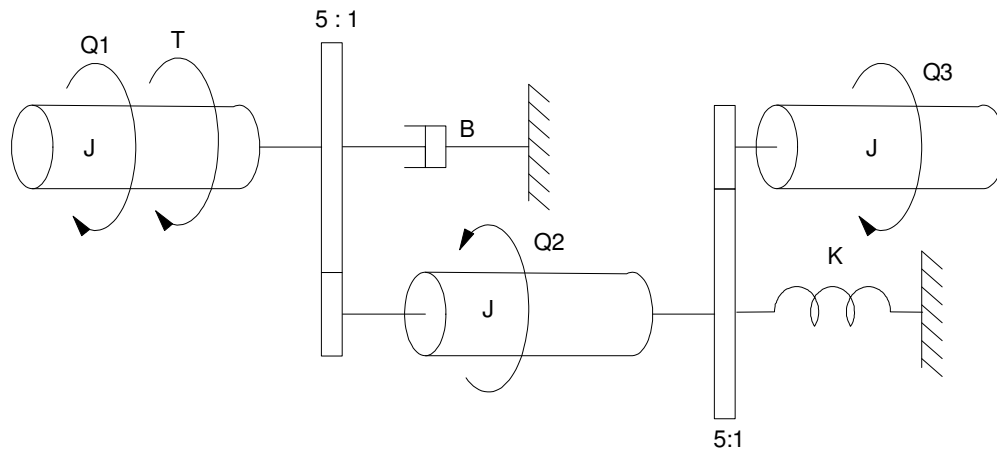
```
>> X0 = imag(M(:,3));
>> X0 = X0 / X0(3)
```

```
x1    -0.6772
x2     0.3116
x3     1.0000
sx1   -0.2182
sx2   -0.2180
sx3      0
```

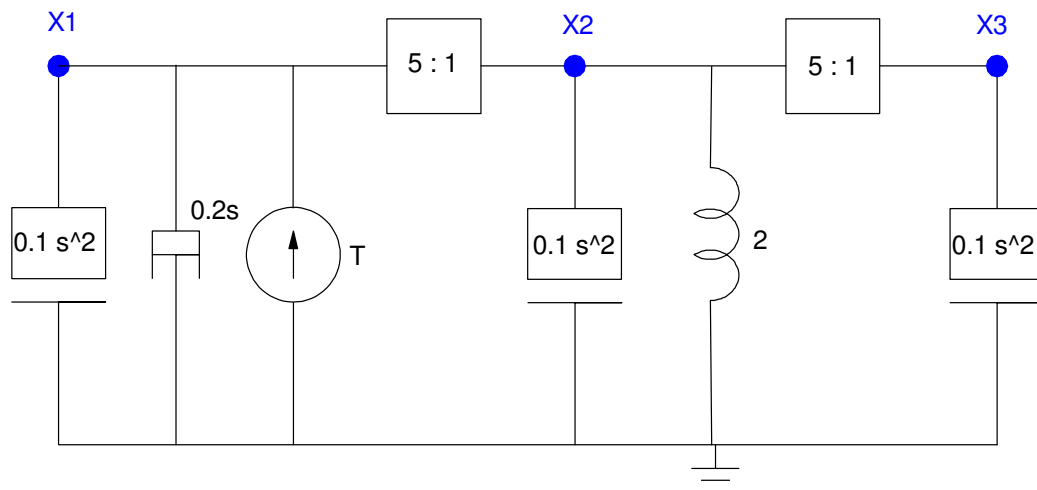
Rotational Systems

Assume for the following rotational system

- $J = 0.1 \text{ kg m}^2$
- $K = 2 \text{ Nm / radian}$
- $B = 0.1 \text{ Nms / radian}$

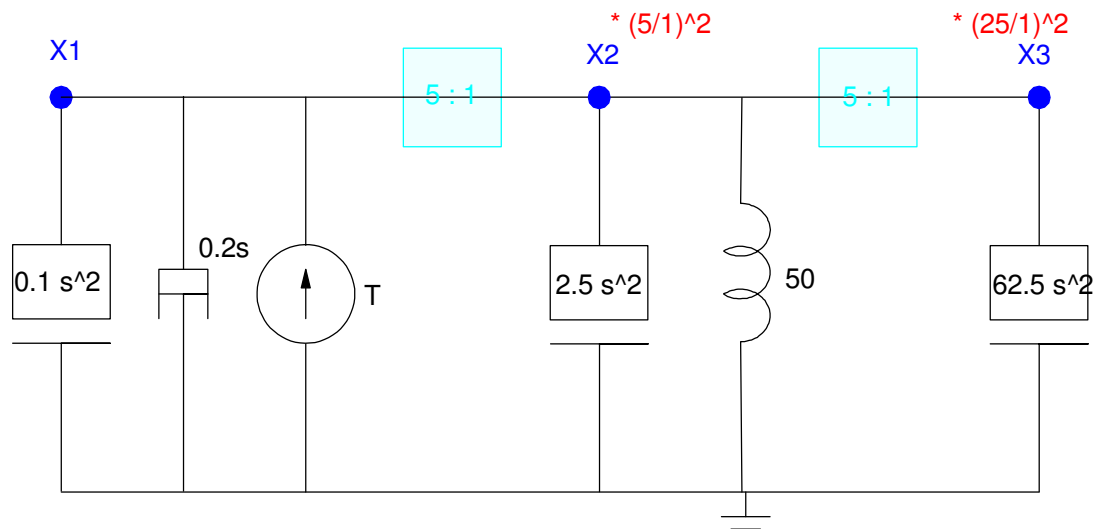


8) Draw the circuit equivalent for this system



9) Translate all of the impedances over to angle Q1 and write the differential equations which describe this system.

Remove the gears



Write the node equations

$$(0.1s^2 + 0.2s + 2.5s^2 + 50 + 62.5s^2)x_1 = T$$

10) Find the transfer function from T to Q1

$$x_1 = \left(\frac{1}{65.1s^2 + 0.2s + 50} \right) F$$