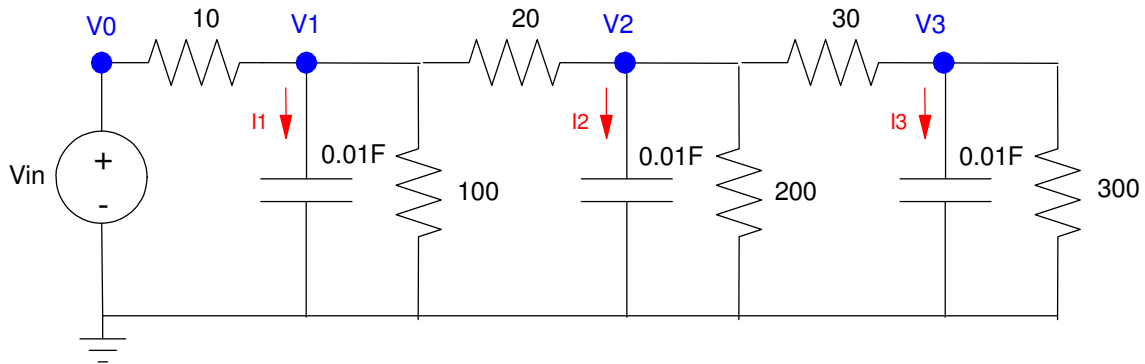


Homework #6: ECE 461/661

Mass-Spring Systems, Rotational Systems

Electrical Circuits

1) Using state-space methods, find the transfer function from V_{in} to V_3



$$I_1 = CsV_1 = \left(\frac{V_0 - V_1}{10} \right) + \left(\frac{V_2 - V_1}{20} \right) - \left(\frac{V_1}{100} \right)$$

$$I_2 = CsV_2 = \left(\frac{V_1 - V_2}{20} \right) + \left(\frac{V_3 - V_2}{30} \right) - \left(\frac{V_2}{200} \right)$$

$$I_3 = CsV_3 = \left(\frac{V_2 - V_3}{30} \right) - \left(\frac{V_3}{300} \right)$$

Simplifying and solving for sV_x

$$sV_1 = 10V_0 - 16V_1 + 5V_2$$

$$sV_2 = 5V_1 - 8.833V_2 + 3.333V_3$$

$$sV_3 = 3.333V_2 - 3.667V_3$$

In state-space

$$\begin{bmatrix} sV_1 \\ sV_2 \\ sV_3 \end{bmatrix} = \begin{bmatrix} -16 & 5 & 0 \\ 5 & -8.833 & 3.333 \\ 0 & 3.333 & -3.667 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} V_0$$

$$y = V_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Use Matlab to find the transfer function

```
>> A = [-16,5,0 ; 5,-8.833,3.333 ; 0,3.333,-3.667]
```

A =

```
   -16.0000    5.0000         0
         5.0000   -8.8330    3.3330
              0    3.3330   -3.6670
```

```
>> B = [10;0;0]
```

B =

```
   10
     0
     0
```

```
>> C = [0,0,1]
```

C =

```
     0     0     1
```

```
>> G = ss(A,B,C,0);
```

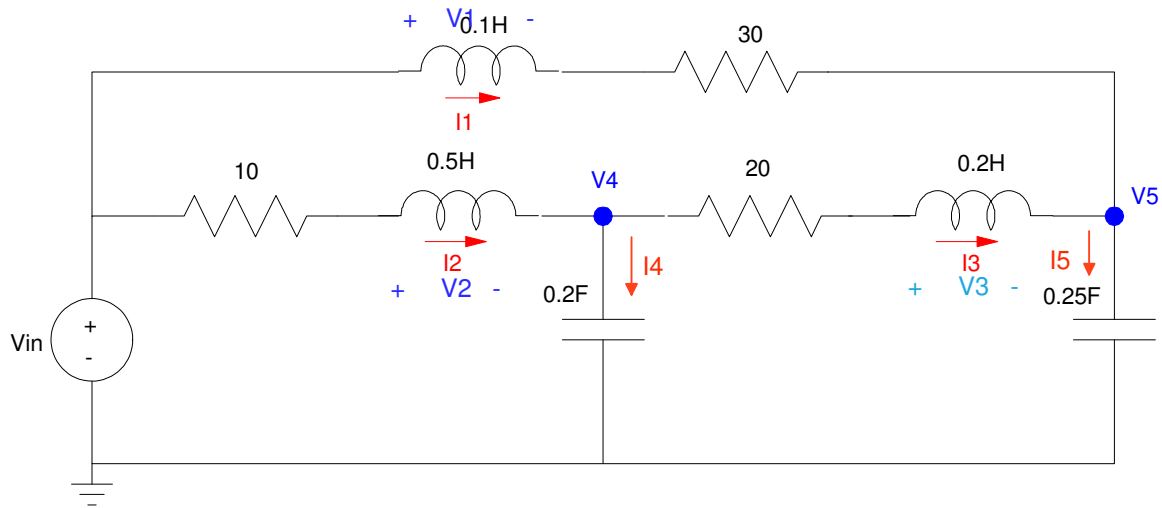
```
>> zpk(G)
```

166.65

(s+18.73) (s+8.138) (s+1.633)

2) Express the dynamics for the following RLC circuit in state-space form.

- Find the transfer function from V_{in} to V_5



The energy in the system is defined by the current in the inductors and the voltage across the capacitors. Let the states be $\{ I_1, I_2, I_3, V_4, V_5 \}$

$$V_1 = 0.1sI_1 = V_{in} - 30I_1 - V_5$$

$$V_2 = 0.5sI_2 = V_{in} - 10I_2 - V_4$$

$$V_3 = 0.2sI_3 = V_4 - 20I_3 - V_5$$

$$I_4 = 0.2sV_4 = I_2 - I_3$$

$$I_5 = 0.25sV_5 = I_1 + I_3$$

Simplifying

$$sI_1 = 10V_{in} - 300I_1 - 10V_5$$

$$sI_2 = 2V_{in} - 20I_2 - 2V_4$$

$$sI_3 = 5V_4 - 100I_3 - 5V_5$$

$$sV_4 = 5I_2 - 5I_3$$

$$sV_5 = 5I_1 - 5I_3$$

Place in matrix (state-space) form

$$\begin{bmatrix} sI_1 \\ sI_2 \\ sI_3 \\ sV_4 \\ sV_5 \end{bmatrix} = \begin{bmatrix} -300 & 0 & 0 & 0 & -10 \\ 0 & -20 & 0 & -2 & 0 \\ 0 & 0 & -100 & 5 & -5 \\ 0 & 5 & -5 & 0 & 0 \\ 5 & 0 & -5 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \end{bmatrix} + \begin{bmatrix} 10 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

Use Matlab to find the transfer function to V5

```
>> A = [-300,0,0,0,-10 ; 0,-20,0,-2,0 ; 0,0,-100,5,-5;0,5,-5,0,0;4,0,4,0,0]
```

```
    -300         0         0         0    -10  
         0     -20         0     -2         0  
         0         0    -100         5     -5  
         0         5        -5         0         0  
         4         0         4         0         0
```

```
>> B = [10;2;0;0;0]
```

```
    10  
     2  
     0  
     0  
     0
```

```
>> C = [0,0,0,0,1];
```

```
>> D = 0;
```

```
>> G = ss(A,B,C,D);
```

```
>> zpk(G)
```

```
          40 (s+99.88) (s+18.5) (s+1.624)  
-----  
(s+299.9) (s+99.55) (s+19.48) (s+0.8701) (s+0.2372)
```

3) Assume $V_{in} = 0$. Specify the initial conditions so that the total energy at $t = 0$ is 1.0 Joules and

- The transients decay as slow as possible
- The transients decay as fast as possible

This is an eigenvalue / eigenvector problem

```
>> [M,V] = eig(A)
```

M (eigenvector)

0.9999	0.0020	-0.0300	0.0117	0.0001
-0.0000	0.0013	-0.0439	-0.0971	-0.9675
-0.0003	0.9979	-0.0233	0.0646	0.0159
-0.0000	0.0501	0.4338	0.9291	0.2524
-0.0133	-0.0402	0.8991	-0.3506	-0.0033

V (eigenvalues)

-299.8667	0	0	0	0
0	-99.5479	0	0	0
0	0	-0.2372	0	0
0	0	0	-0.8701	0
0	0	0	0	-19.4782

The red eigenvector decays the fastest. Its energy is

$$E = \sum \left(\frac{1}{2} L I^2 + \frac{1}{2} C V^2 \right) =$$

```
>> X0 = M(:,1);
```

```
>> Energy = 0.5 * [0.1, 0.5, 0.2, 0.2, 0.25] * X0.^2
```

```
Energy = 0.0500
```

```
>> X0 / Energy
```

```
I1 19.9929A
I2 -0.0000A
I3 -0.0067A
V4 -0.0001V
V5 -0.2666V
```

The 3rd eigenvector (blue) is the slow mode. For its initial energy to be 1.00 Joule

```
>> X0 = M(:,3);
```

```
>> Energy = 0.5 * [0.1, 0.5, 0.2, 0.2, 0.25] * X0.^2
```

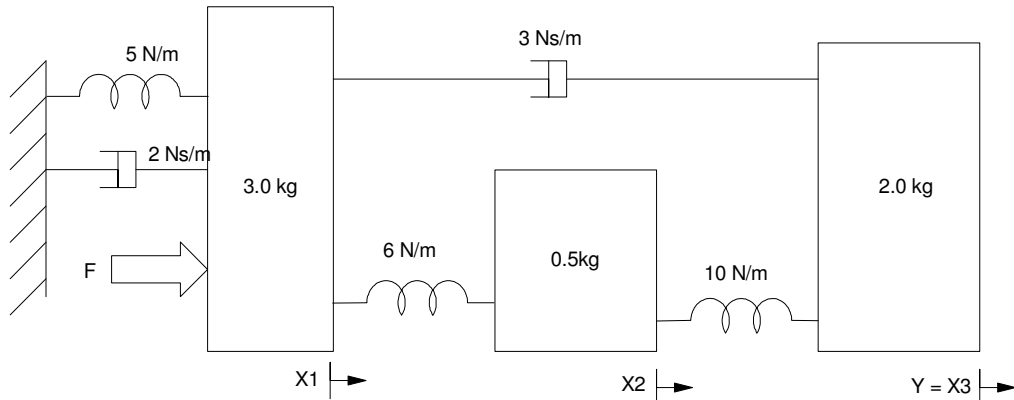
```
Energy = 0.1205
```

```
>> X0 / Energy
```

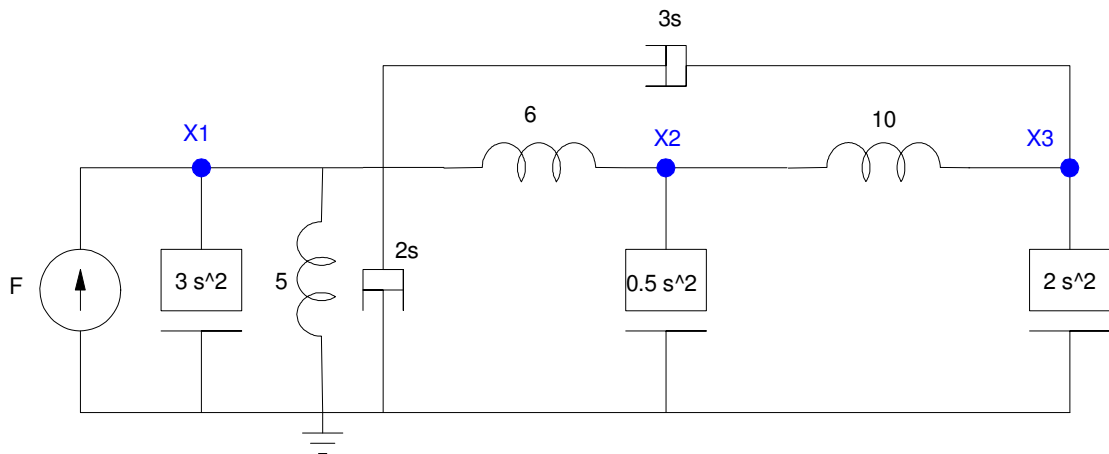
```
I1 -0.2490A
I2 -0.3645A
I3 -0.1936A
V4 3.6014V
V5 7.4645V
```

Mass Spring systems

4) For the following mass-spring system:



Draw the circuit equivalent for the following mass-spring systems.



Express the dynamics in state-space form

$$(3s^2 + 2s + 5 + 6 + 3s)X_1 - (6)X_2 - (3s)X_3 = F$$

$$(0.5s^2 + 6 + 10)X_2 - (6)X_1 - (10)X_3 = 0$$

$$(2s^2 + 3s + 10)X_3 - (3s)X_1 - (10)X_2 = 0$$

Solve for the highest derivative

$$s^2 X_1 = \left(-\frac{5}{3}s - \frac{11}{3} \right) X_1 + 2X_2 + sX_3 + \frac{1}{3}F$$

$$s^2 X_2 = -32X_2 + 12X_1 + 20X_3$$

$$s^2 X_3 = \left(-\frac{3}{2}s - 5 \right) X_3 + \frac{3}{2}sX_1 + 5X_2$$

Place in matrix form

$$s \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \dots \\ sX_1 \\ sX_2 \\ sX_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ -\frac{11}{3} & 2 & 0 & \vdots & -\frac{5}{3} & 0 & 1 \\ 12 & -32 & 20 & \vdots & 0 & 0 & 0 \\ 0 & 5 & -5 & \vdots & \frac{3}{2} & 0 & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \dots \\ sX_1 \\ sX_2 \\ sX_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ \frac{1}{3} \\ 0 \\ 0 \end{bmatrix} F$$

Find the transfer function from F to X2

```
>> a11 = zeros(3,3);
>> a12 = eye(3,3);
>> a21 = [-11/3,2,0 ; 12,-32,20 ; 0,5,-5];
>> a22 = [-5.3,0,1 ; 0,0,0 ; 3/2,0,-3/2];
>> A = [a11,a12 ; a21,a22]
```

A =

0	0	0	1.0000	0	0
0	0	0	0	1.0000	0
0	0	0	0	0	1.0000
-3.6667	2.0000	0	-5.3000	0	1.0000
12.0000	-32.0000	20.0000	0	0	0
0	5.0000	-5.0000	1.5000	0	-1.5000

```
>> B = [0;0;0;1/3;0;0];
>> C = [0,1,0,0,0,0];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)
```

Zero/pole/gain:

4 (s^2 + 4s + 5)

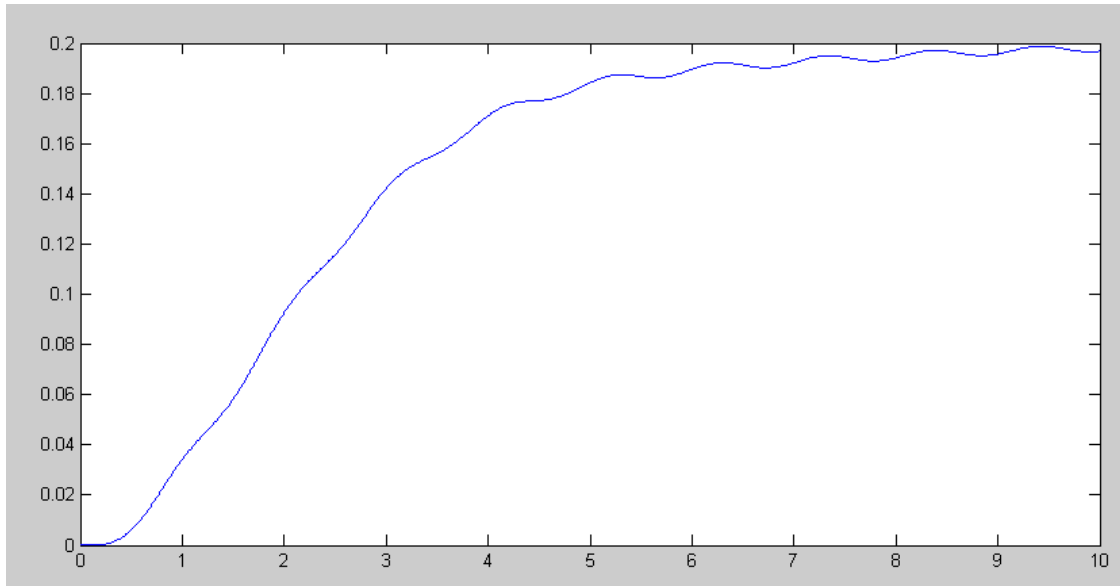
(s+4.91) (s+0.4769) (s^2 + 1.283s + 1.193) (s^2 + 0.1298s + 35.8)

Plot the step response from F to X2

```
>> t = [0:0.01:10]';  
>> y = step(G,t);  
>> plot(t,y)  
>> eig(A)
```

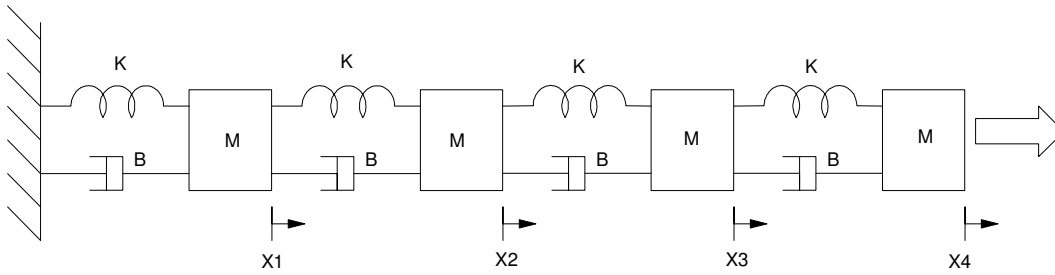
```
-0.0649 + 5.9832i  
-0.0649 - 5.9832i  
-4.9099  
-0.4769  
-0.6417 + 0.8838i  
-0.6417 - 0.8838i
```

```
>>
```



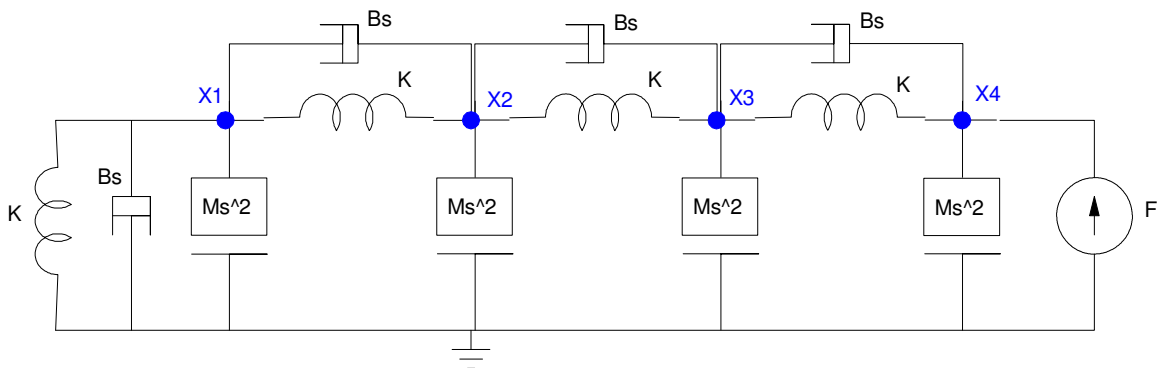
Problem 1

5) For the following mass-spring system...



Problem 6: $M = 2.0\text{kg}$, $B = 0.3\text{ Ns/m}$, $K = 10\text{ N/m}$

Draw the circuit equivalent



Express the dynamics in state-space form plug in ($K/M = 5$), ($B/M = 0.15$)

$$(Ms^2 + 2Bs + 2K)X_1 - (Bs + K)X_2 = 0$$

$$(Ms^2 + 2Bs + 2K)X_2 - (Bs + K)X_1 - (Bs + K)X_3 = 0$$

$$(Ms^2 + 2Bs + 2K)X_3 - (Bs + K)X_2 - (Bs + K)X_4 = 0$$

$$(Ms^2 + Bs + K)X_4 - (Bs + K)X_3 = F$$

Place in matrix form

$$\begin{bmatrix} sX_1 \\ sX_2 \\ sX_3 \\ sX_4 \\ \dots \\ s^2X_1 \\ s^2X_2 \\ s^2X_3 \\ s^2X_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \vdots & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \vdots & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{-2K}{M} & \frac{K}{M} & 0 & 0 & \vdots & \frac{-2B}{M} & \frac{B}{M} & 0 & 0 \\ \frac{K}{M} & \frac{-2K}{M} & \frac{K}{M} & 0 & \vdots & \frac{B}{M} & \frac{-2B}{M} & \frac{B}{M} & 0 \\ 0 & \frac{K}{M} & \frac{-2K}{M} & \frac{K}{M} & \vdots & 0 & \frac{B}{M} & \frac{-2B}{M} & \frac{B}{M} \\ 0 & 0 & \frac{K}{M} & \frac{-K}{M} & \vdots & 0 & 0 & \frac{B}{M} & \frac{-B}{M} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ \dots \\ sX_1 \\ sX_2 \\ sX_3 \\ sX_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dots \\ 0 \\ 0 \\ 0 \\ \frac{1}{M} \end{bmatrix} F$$

plug into Matlab noting that

- $K/M = 5$
- $B/M = 0.15$
-

Find the transfer function from F to X4

```
>> a11 = zeros(4,4);
>> a12 = eye(4,4);
>> a21 = [-10,5,0,0 ; 5,-10,5,0 ; 0,5,-10,5 ; 0,0,5,-5];
>> a22 = [-0.3,0.15,0,0 ; 0.15,-0.3,0.15,0 ; 0,0.15,-0.3,0.15 ; 0,0,0.15,-0.15];
>> A = [a11,a12 ; a21,a22]
```

A =

```
      0      0      0      0      1.0000      0      0      0
      0      0      0      0      0      1.0000      0      0
      0      0      0      0      0      0      1.0000      0
      0      0      0      0      0      0      0      1.0000
 -10.0000   5.0000      0      0     -0.3000   0.1500      0      0
   5.0000  -10.0000   5.0000      0     0.1500  -0.3000   0.1500      0
      0   5.0000  -10.0000   5.0000      0   0.1500  -0.3000   0.1500
      0      0   5.0000  -5.0000      0      0   0.1500  -0.1500
```

```
>> B = [0;0;0;0;0;0;0;0;1/2]
```

B =

```
      0
      0
      0
      0
      0
      0
      0
      0
 0.5000
```

```
>> C = [0,0,0,1,0,0,0,0,0];
```

```
>> D = 0;
```

```
>> G = ss(A,B,C,D);
```

```
??? Error using ==> ss.ss>ss.ss at 345
```

The values of the "a" and "c" properties must be matrices with the same number of columns.

```
>> C = [0,0,0,1,0,0,0,0];;
```

```
>> D = 0;
```

```
>> G = ss(A,B,C,D);
```

```
>> zpk(G)
```

```
      0.5 (s^2 + 0.08787s + 2.929) (s^2 + 0.3s + 10) (s^2 + 0.5121s + 17.07)
-----
(s^2 + 0.01809s + 0.6031) (s^2 + 0.15s + 5) (s^2 + 0.3521s + 11.74) (s^2 + 0.5298s + 17.66)
```

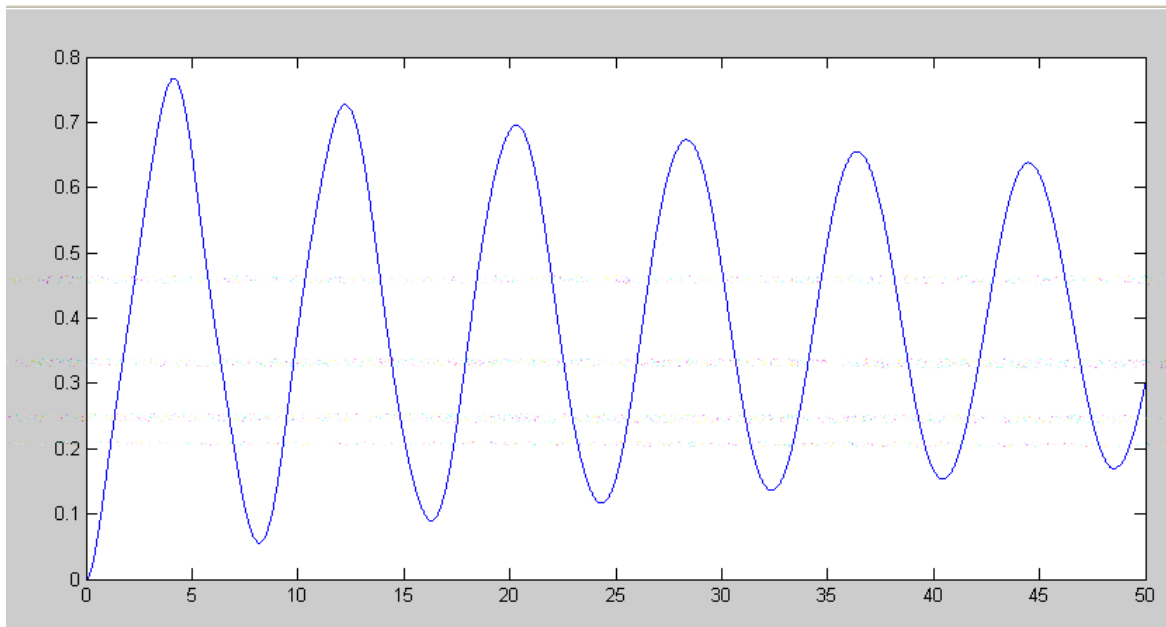
```
>> eig(A)
```

```
-0.2649 + 4.1941i
-0.2649 - 4.1941i
-0.1760 + 3.4213i
-0.1760 - 3.4213i
-0.0750 + 2.2348i
-0.0750 - 2.2348i
-0.0090 + 0.7765i
-0.0090 - 0.7765i
```

```
>>
```

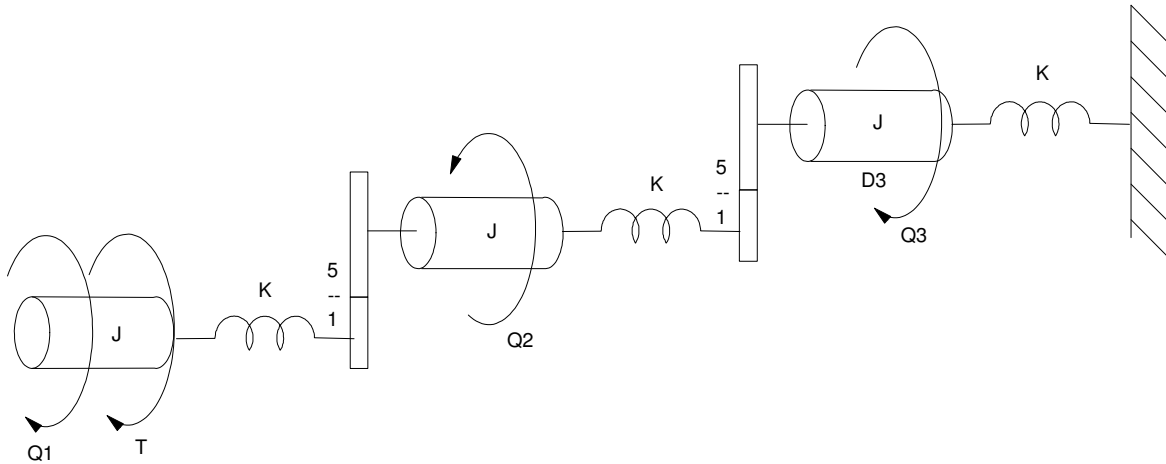
Plot the step response from F to X4

```
>> t = [0:0.01:50]';  
>> y = step(G,t);  
>> plot(t,y)
```



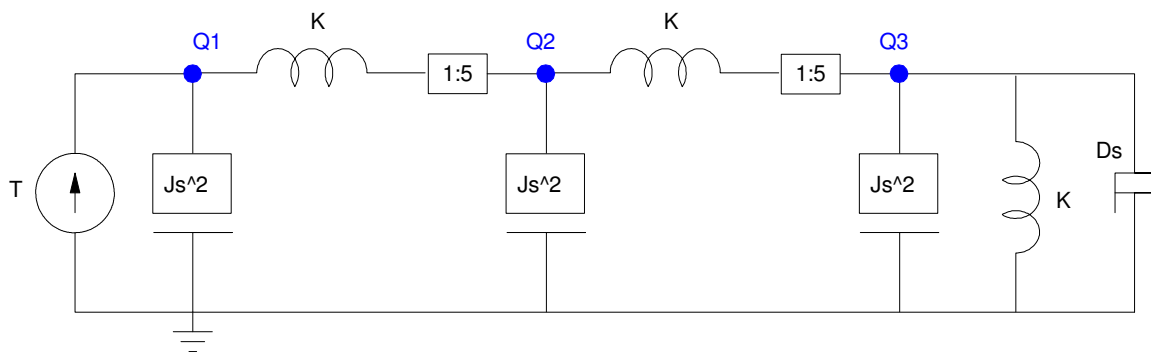
Rotational Systems

6) For the following rotational system...

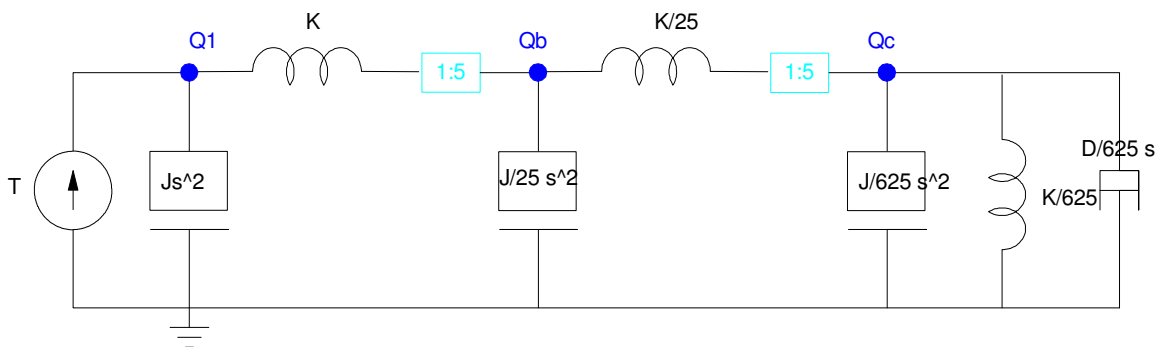


Problem 3: $J = 2.5 \text{ Kg m}^2 / \text{s}^2$. $K = 20 \text{ Nm/rad}$, $D_3 = 1.5 \text{ Nms/rad}$

Draw the circuit equivalent



Remove the gears



Write the node equations

$$(Js^2 + K)\theta_1 - (K)\theta_2 = T$$

$$\left(\frac{J}{25}s^2 + K + \frac{K}{25}\right)\theta_b - (K)\theta_1 - \left(\frac{K}{25}\right)\theta_c = 0$$

$$\left(\frac{J}{625}s^2 + \frac{D}{625}s + \frac{K}{625} + \frac{K}{25}\right)\theta_c - \left(\frac{K}{25}\right)\theta_b = 0$$

Solving for the highest derivative

Problem 3: $J = 2.5 \text{ Kg m}^2 / \text{s}^2$, $K = 20 \text{ Nm/rad}$, $D = 1.5 \text{ Nms/rad}$

$$s^2\theta_1 = \left(-\frac{K}{J}\right)\theta_1 + \left(\frac{K}{J}\right)\theta_b + \left(\frac{1}{J}\right)T$$

$$s^2\theta_b = \left(\frac{25K}{J}\right)\theta_1 - \left(\frac{26K}{J}\right)\theta_b + \left(\frac{K}{J}\right)\theta_c$$

$$s^2\theta_c = \left(\frac{25K}{J}\right)\theta_b - \left(\frac{D}{J}s + \frac{26K}{J}\right)\theta_c$$

Plugging in numbers and placing in matrix form ($K/J = 8$, $D/J = 0.6$)

$$\begin{bmatrix} s\theta_1 \\ s\theta_b \\ s\theta_c \\ \dots \\ s^2\theta_1 \\ s^2\theta_b \\ s^2\theta_c \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ -8 & 8 & 0 & \vdots & 0 & 0 & 0 \\ 200 & -208 & 8 & \vdots & 0 & 0 & 0 \\ 0 & 200 & -208 & \vdots & 0 & 0 & -0.6 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_b \\ \theta_c \\ \dots \\ s\theta_1 \\ s\theta_b \\ s\theta_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ 0.4 \\ 0 \\ 0 \end{bmatrix} T$$

Find the transfer function from T to Q1

```
>> a11 = zeros(3,3);
>> a12 = eye(3,3);
>> a21 = [-8,8,0 ; 200,-208,8 ; 0,200,-208];
>> a22 = [0,0,0 ; 0,0,0 ; 0,0,-0.6];
>> A = [a11,a12 ; a21,a22]
```

```

      0      0      0      1.0000      0      0
      0      0      0      0      1.0000      0
      0      0      0      0      0      1.0000
 -8.0000  8.0000      0      0      0      0
200.0000 -208.0000  8.0000      0      0      0
      0  200.0000 -208.0000      0      0 -0.6000
```

```
>> B = [0;0;0;0.4;0;0];
>> C = [1,0,0,0,0,0];
>> D = 0;
```

$$0.4 (s^2 + 0.3003s + 168.2) (s^2 + 0.2997s + 247.7)$$

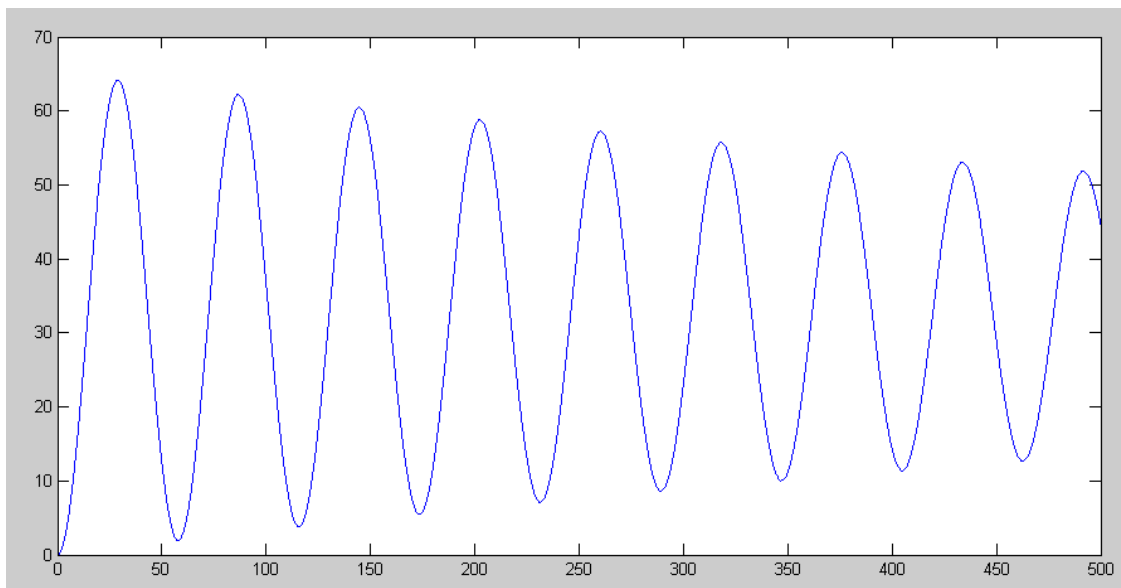
$$(s^2 + 0.0008499s + 0.0118) (s^2 + 0.3282s + 172.8) (s^2 + 0.271s + 251.1)$$

```
>> eig(A)

-0.1355 +15.8467i
-0.1355 -15.8467i
-0.1641 +13.1429i
-0.1641 -13.1429i
-0.0004 + 0.1086i
-0.0004 - 0.1086i
>>
```

Plot the step response from T to Q1

```
>> t = [0:0.1:500]';
>> y = step(G,t);
>> plot(t,y)
```



Motors

7) Find the transfer function for the following DC servo motor

<http://www.baldor.com/catalog/CDP3306>

Allen Bradley CDP3306: 1/4 HP Servo Motor

- \$716ea
- $K_t = 1.117 \text{ ft-lb @ } 1.68\text{A}$
- $R_a = 8.54 \text{ Ohms}$
- $L_a = 60.59\text{mH}$
- $J = 4.680 \text{ lb-ft}^2$
- $1.43\text{A @ } 1690\text{rpm @ } 0.945 \text{ ft-lb load}$
- $0.1\text{A @ } 180\text{V @ } 1830\text{rpm @ } 0 \text{ ft-lb load}$
- Weight 23.0kg

$$K_t = \left(\frac{1.117 \text{ ft-lb}}{1.68\text{A}} \right) \left(\frac{1\text{m}}{3.281\text{ft}} \right) \left(\frac{4.445\text{N}}{\text{lb}} \right) = 0.9008 \frac{\text{Nm}}{\text{A}}$$

$$J = 4.608(\text{lb} \cdot \text{ft}^2) \left(\frac{0.4536\text{kg}}{\text{lb}} \right) \left(\frac{1\text{m}}{3.281\text{ft}} \right)^2 = 0.1942 \cdot \text{kg} \cdot \text{m}^2$$

D: Power In = Power Out

$0.1\text{A @ } 180\text{V @ } 1830\text{rpm @ } 0 \text{ ft-lb load}$

$$\omega = 1830\text{rpm} \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 191.6 \frac{\text{rad}}{\text{sec}}$$

$$180\text{V} \cdot 0.1\text{A} = 18\text{W} = T \cdot \omega = (D \cdot \omega)\omega$$

$$D = 0.000490 \frac{\text{Nm}}{\text{rad/sec}}$$

Plugging in numbers

$$\omega = \left(\frac{K_t}{(Js+D)(Ls+R)+K_t^2} \right)$$

In Matlab

```
>> Kt = 0.9008;  
>> J = 0.1942;  
>> D = 0.000490;  
>> R = 8.54;  
>> L = 0.06059;  
>> den = [J*L, J*R + D*L, D*R + Kt^2];  
>> G = tf(Kt, [J*L, J*R + D*L, D*R + Kt^2]);  
>> zpk(G)
```

76.5558

(s+140.5) (s+0.4935)

```
>>
```

Note: I think the data sheets are incorrect with respect to the rotor inertia.

- The motor weighs 23lb (10.43kg).
- The motor diameter is 4.50 inches (11.43cm).

Assume

- The rotor is 1/2 of the mass of the motor (probably a high estimate).
- The rotor is 1/2 the diameter of the motor (5.71cm diameter).
- The rotor is a solid cylinder (for modeling)

The rotational inertia of a solid cylinder (Wikipedia) is

$$J = \frac{1}{2}mr^2$$

$$J = \frac{1}{2}(5.216kg)(0.02858m)^2$$

$$J = 0.002130 \cdot kg \cdot m^2$$

which is 89x smaller than the data sheets. I think the data sheets are *wrong*

Using this inertia

```
>> Kt = 0.9008;  
>> J = 0.00213;  
>> D = 0.00049;  
>> R = 8.54;  
>> L = 0.06059;  
>> den = [J*L, J*R + D*L, D*R + Kt^2];  
>> G = tf(Kt, [J*L, J*R + D*L, D*R + Kt^2]);  
>> zpk(G)
```

6979.8778

(s^2 + 141.2s + 6320)

```
>> eig(G)
```

```
-70.5887 +36.5668i  
-70.5887 -36.5668i
```

This actually makes more sense. The poles are complex, meaning some overshoot. Motors are designed to drive something. Add inertia to the rotor and the poles will become real. (problem #5)

8) Assume this motor is used to power an electric bicycle at 20mph

- Motor speed @ 20mph = 1750 rpm
- Gear (wheel) used to convert 1750 rpm to 20mph
- Bicycle weight = 100kg

What is the gear reduction (wheel diameter) to convert 1750rpm to 20mph?

$$x = r\theta$$

$$\dot{x} = r\dot{\theta}$$

$$20mph \left(\frac{1609m}{mile} \right) \left(\frac{1hr}{3600sec} \right) = 8.939 \frac{m}{s}$$

$$1750rpm \left(\frac{2\pi rad}{rev} \right) \left(\frac{1min}{60s} \right) = 183.3 \frac{rad}{sec}$$

$$8.939 \frac{m}{s} = r \cdot 183.3 \frac{rad}{sec}$$

$$r = 0.04878m \quad (48.78mm = 2 \text{ inch diameter})$$

What is the inertia relative to the DC servo motor (bring the 100kg mass back to the motor through a gear)

The net inertia is

$$J = J_{motor} + r^2 M_{cart}$$

$$J = (0.00213 \cdot kg \cdot m^2) + (0.04878m)^2 \cdot 100kg$$

$$J = 0.2401 \cdot kg \cdot m^2$$

What is the transfer function (dynamics) for the bicycle / servo motor combination?

```
>> J = 0.2401;  
>> G = tf(Kt, [J*L, J*R + D*L, D*R + Kt^2]);  
>> zpk(G)
```

```
      61.9206  
-----  
(s+140.6) (s+0.3988)
```

The extra inertia made the complex pole real like you'd expect. If used to drive an electric bike, this motor would get you up to speed in 10 seconds (2% settling time).