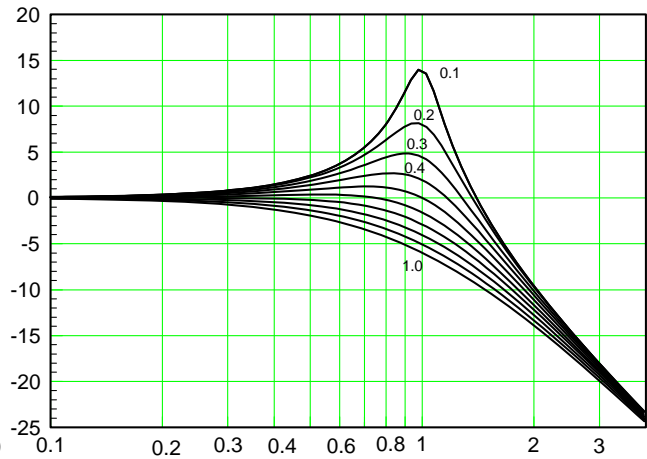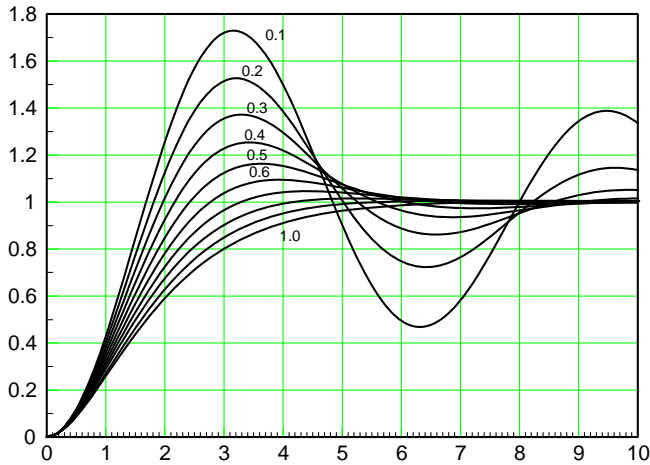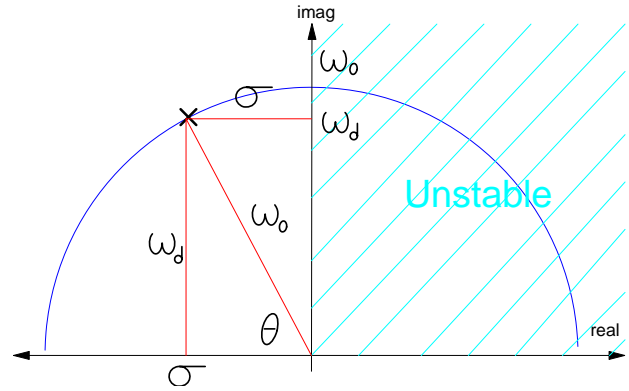# 2nd-Order Approximations

$$G(s) = \left(\frac{k \cdot \omega_o^2}{s^2 + 2\zeta\omega_o s + \omega_o^2} \right) = \left(\frac{k \cdot \left(\sigma^2 + \omega_d^2\right)}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)} \right)$$

$$s = -\sigma \pm j\omega_d = \omega_o \angle \pm \theta$$

$$\zeta = \cos\theta \qquad \text{damping ratio}$$

$$\%OS = \exp\left(-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)\right) \qquad \text{\% Overshoot}$$

$$\omega_m = \omega_o\sqrt{1-2\zeta^2} \qquad \text{Max gain frequency}$$

$$T_p = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}} \qquad \text{time to peak}$$

$$T_s = T_{2\%} = \frac{4}{\sigma} \qquad \text{2\% Settling Time}$$

$$M_m = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad \text{Max gain}$$

$$\frac{1}{2\zeta} \qquad \text{Gain at corner freq}$$

ζ	Tp	%OS	ω_m	Mm	Mm (dB)
0.1	3.15	72.9%	0.99	5.03	14.02
0.2	3.21	52.7%	0.96	2.55	8.14
0.3	3.29	37.2%	0.91	1.75	4.85
0.4	3.43	25.4%	0.82	1.36	2.7
0.5	3.63	16.3%	0.71	1.15	1.25
0.6	3.93	9.5%	0.53	1.04	0.35
0.7	4.4	4.6%	0.14	1	0
0.8	5.24	1.5%	0	1	0
0.9	7.21	0.2%	0	1	0
1.0	-	0	0	1	0