ECE 461/661: Handout #30

s to z conversions

Determine the discrete-time equivalent of G(s).   Assume T = 0.1 seconds

$$G(s) = \left(\frac{20}{(s+2)(s+5)}\right)$$

Determine the continuous-time equivalent of G(z). Assume T = 0.1 second

$$G(z) = \left(\frac{0.2z}{(z-0.8)(z-0.6)}\right)$$

Solution

z-Transform

Determine the discrete-time equivalent of G(s). Assume T = 0.1 seconds

$$G(s) = \left(\frac{20}{(s+2)(s+5)}\right)$$

The conversion from the s to z plane is

$$z = e^{sT}$$

s = -2

$$z = e^{(-2)(0.1)} = e^{-0.2} = 0.8187$$

s = -5

$$z = e^{(-5)(0.1)} = e^{-0.5} = 0.6065$$

so

$$G(z) = \left(\frac{k}{(z-0.8187)(z-0.6065)}\right)$$

To find k, match the DC gain

$$\left(\frac{20}{(s+2)(s+5)}\right)_{s=0} = 2$$

$$\left(\frac{k}{(z-0.8187)(z-0.6065)}\right)_{z=1} = 2$$

$$k = 0.1426$$

giving

$$G(z) = \left(\frac{0.1426}{(z-0.8187)(z-0.6065)}\right)$$

Note: this has a little too much delay (check the phast at s = j1). A slightly more accurate model is

$$G(z) = \left(\frac{0.1426z}{(z-0.8187)(z-0.6065)}\right)$$

2) Determine the continuous-time equivalent of G(z). Assume T = 0.1 second

$$G(z) = \left(\frac{0.2z}{(z-0.8)(z-0.6)} \right)$$

The conversion from the s to z plane is

$$z = e^{sT}$$

$$s = \frac{1}{T} \ln(z)$$

z = 0.8

$$s = \left(\frac{1}{0.1}\right) \ln(0.8) = -2.23$$

z = 0.6

$$s = \left(\frac{1}{0.1}\right) \ln(0.6) = -5.11$$

z = 0

$$s = \left(\frac{1}{0.1}\right) \ln(0) = -\infty \quad \textit{ignore}$$

So

$$G(s) \approx \left(\frac{k}{(s+2.23)(s+5.11)} \right)$$

To find k, match the DC gain

$$\left(\frac{0.2z}{(z-0.8)(z-0.6)} \right)_{z=1} = 2.50$$

$$\left(\frac{k}{(s+2.23)(s+5.11)} \right)_{s=0} = 2.50$$

$$k = 28.49$$

giving

$$G(s) \approx \left(\frac{28.49}{(s+2.23)(s+5.11)} \right)$$