ECE 461/661: Handout #29

z-Transform

1) Determine the difference equation that relates X and Y

$$Y = \left(\frac{0.2z}{(z-0.8)(z-0.6)} \right) X$$

2) Assume $x(k) = u(k)$ (unit step). Find $y(k)$

$$Y = \left(\frac{0.2z}{(z-0.8)(z-0.6)} \right) X$$

Solution

1) Determine the difference equation that relates X and Y

$$Y = \left(\frac{0.2z}{(z-0.8)(z-0.6)} \right) X$$

Multiply out and cross multiply

$$(z-0.8)(z-0.6)Y = (02z)X$$

$$(z^2 - 1.4z + 0.48)Y = (0.2z)X$$

zY means 'the next value of y'

$$y(k+2) - 1.4y(k+1) + 0.48y(k) = 0.2x(k+1)$$

or with a change of variable (shift time by 2)

$$y(k) - 1.4y(k-1) + 0.48y(k-2) = 0.2x(k-1)$$

Either answer is correct

2) Assume x(k) = u(k) (unit step). Find y(k)

$$Y = \left(\frac{0.2z}{(z-0.8)(z-0.6)} \right) X$$

Use the z-transform for a step

$$Y = \left(\frac{0.2z}{(z-0.8)(z-0.6)} \right) \left(\frac{z}{z-1} \right)$$

Pull out a z in the numerator (we'll need this later)

$$Y = \left(\frac{0.2z}{(z-1)(z-0.8)(z-0.6)} \right) z$$

Do partial fraction expansion

$$Y = \left(\left(\frac{2.5}{z-1} \right) + \left(\frac{-4}{z-0.8} \right) + \left(\frac{1.5}{z-0.6} \right) \right) z$$

$$Y = \left(\frac{2.5z}{z-1} \right) + \left(\frac{-4z}{z-0.8} \right) + \left(\frac{1.5z}{z-0.6} \right)$$

Take the inverse-z transform

$$y(k) = \left(2.5 - 4(0.8)^k + 1.5(0.6)^k \right) u(k)$$