CoronaVirus & Markov Chains

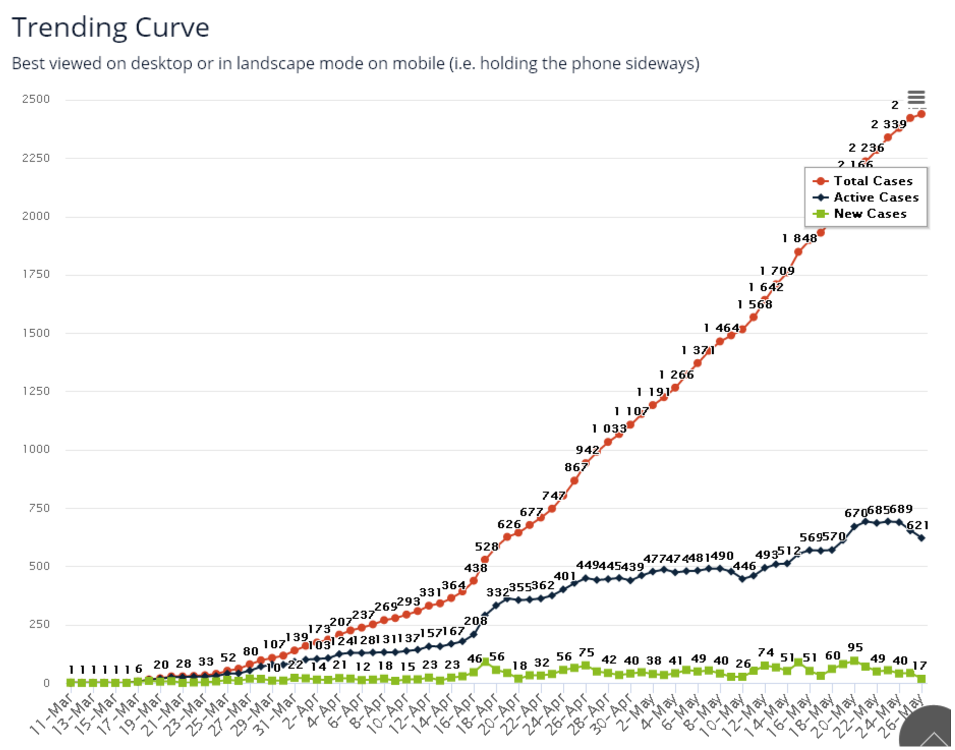
ECE 341: Random Processes

Lecture #22

note: All lecture notes, homework sets, and solutions are posted on www.BisonAcademy.com

CoronaVirus & Markov Chains

- X(k+1) = A X(k)
- Applies to more than just electronic circuits...



CoronaVirus infections in North Dakota as of May 28, 2020
www.health.nd.gov/diseases-conditions/coronavirus/north-dakota-coronavirus-cases

Disclaimer:

The simulations presented are very simplistic and are in no way intended to challenge the models made by the Center for Disease Control or to influence policy decisions made at the state or local levels. These models, simplistic as they are, are reasonably consistent with other (more sophisticated) models.

The point behind this lecture is that Markov chains are useful: they allow you to analyze systems ranging from tennis matches to electronic circuits to disease studies. They're kind of fun to play with too....

Coronavirus Outbreak (2020)

Using what we know from this course, we can simulate the outbreak and analyze different situations, such as

- What impact does social distancing have on the disease spread?
- What impact does wearing masks have on the spread?
- What impact would a vaccine have - which reduces the recovery time to 1/2?

Since we don't have actual numbers for many of the parameters used in this lecture, we'll take a scientific wild-ass guess (SWAG) at them.

System Modeling

Assume there are 4 groups of people:

- H: Healthy and uninfected: People who have not caught the Coronavirus and are susceptible to it.
- I: Infected: People who have the Coronavirus and can spread the disease
- C: Cured: People who had the Coronavirus but have recovered.
- D: Dead: Self explanatory.

Assume

- People who are cured cannot be reinfected (?) and or spread the disease (?)

$$\begin{bmatrix} H(k+1) \\ I(k+1) \\ C(k+1) \\ D(k+1) \end{bmatrix} = \begin{bmatrix} ? & & 0 & 0 \\ & ? & 0 & 0 \\ & & 1 & 0 \\ & & 0 & 1 \end{bmatrix} \begin{bmatrix} H(k) \\ I(k) \\ C(k) \\ D(k) \end{bmatrix}$$

Define:

- N = Contact Rate: # of people you come in close contact with each day.
- p = The transmission rate: p(catching the disease) if healthy contacts infected
- Cure Rate = 5%: The chance that an infected person will recover on a given day, and
- Death Rate = 0.07% The chance that in infected person will die on any given day.

Cure Rate:

- Doesn't quite fit into a Markov chain
- Cured 20 days on average (?) after catching the Corona virus.
- Assume that the cure rate is a geometric distribution with a mean of 20 days
- Cure Rate = 0.05 (each day has a 5% chance you'll be cured the next day)

$$p(cure) = \left(\frac{1}{20}\right)\left(\frac{19}{20}\right)^{k-1} u(k-1)$$

Death Rate:

- Assume 1.4%[1] of the cure rate or 0.007% chance per day infected

1 https://www.worldometers.info/coronavirus/coronavirus-death-rate/

Then, the number of new cases each day will be

- The number of infected people at day k, I(k), times
- The number of people each person comes in contact each day (N), times
- The chance of infecting someone (p), times
- The change that the person contacted was uninfected
- Minus the number of people who are cured (5% of the infected people),
- Minus the number of people who die (1.4% of 5% of the infected people)

or

$$\delta I(k) = I(k)\left(N \cdot p \cdot \left(\frac{H(k)}{H(k)+I(k)+C(k)}\right) - 0.05 - (0.014)(0.05)\right)$$

$$= (a - 0.05 - 0.0007)I(k)$$

$$a = N \cdot p \cdot \left(\frac{H(k)}{H(k)+I(k)+C(k)}\right)$$

The number of new cured people is

$$\delta C(k) = 0.05 \cdot I(k)$$

The number of new dead people is

$$\delta D(k) = 0.014 \cdot 0.05 \cdot I(k)$$

Cured and Dead are absorbing states (once there, you cannot leave).

This results in the Markov chain model being

- Note that all of the columns equal 1.000 (required for this to be a valid Markov chain).

$$\begin{bmatrix} H(k+1) \\ I(k+1) \\ C(k+1) \\ D(k+1) \end{bmatrix} = \begin{bmatrix} 1 & -a & 0 & 0 \\ 0 & 1+a-0.05-0.0007 & 0 & 0 \\ 0 & 0.05 & 1 & 0 \\ 0 & 0.0007 & 0 & 1 \end{bmatrix} \begin{bmatrix} H(k) \\ I(k) \\ C(k) \\ D(k) \end{bmatrix}$$

Simulation #1: March 1, 2020

Assume most of the population is healthy. This results in

$$a = N \cdot p \cdot \left(\frac{H(k)}{H(k)+I(k)+C(k)}\right) \approx Np$$

From a mathematical standpoint, we don't really care how you arrive at Np: only the net result matters.

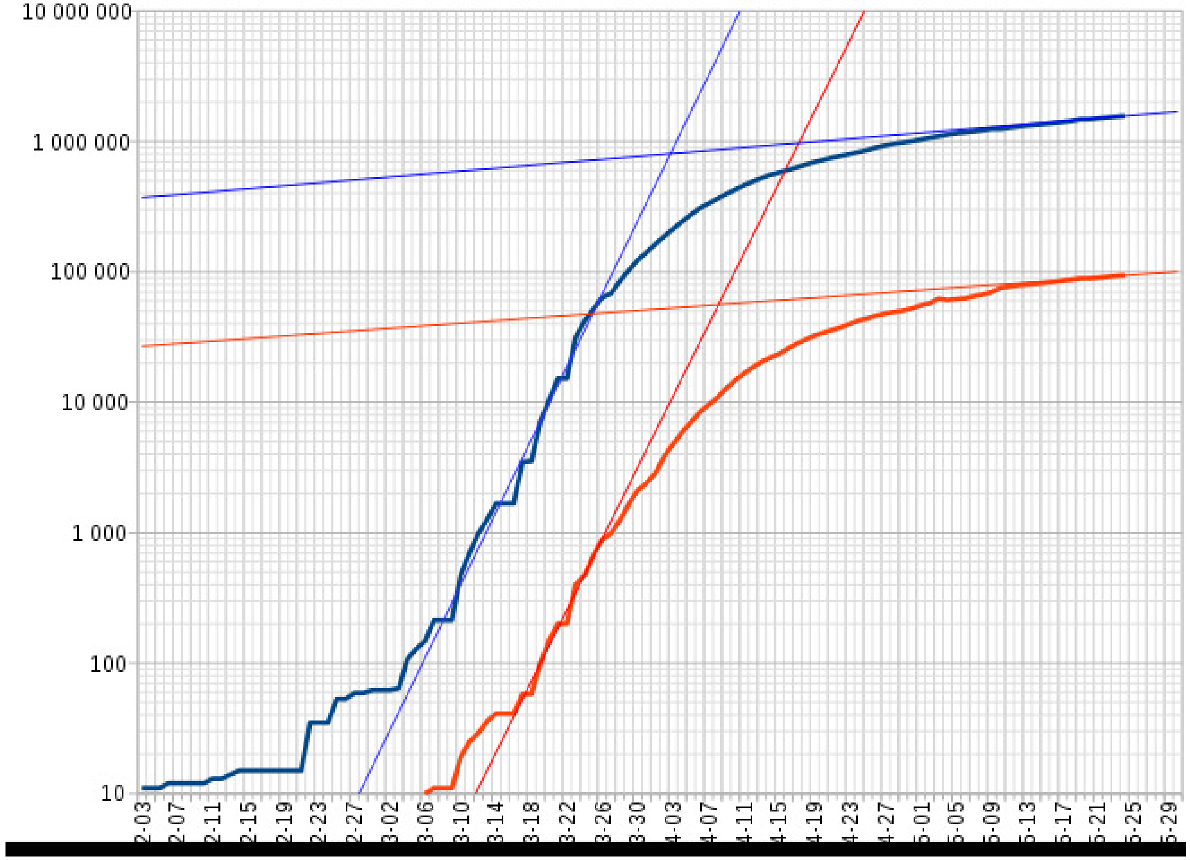
'a' determines the growth rate of the disease

$$I(k+1) = (1 + a - 0.05 - 0.0007)I(k)$$

or

$$I(k) = (1 + a - 0.05 - 0.007)^k \, I(0)$$

To determine this growth rate, look at a graph of the disease spread:



Disease spread of CoronaVirus in the United States
Blue = cases, orange = deaths.   Source: Wikipedia

In early March, the disease was growing at a rate of

- 10x increase every 9 days
- 2x increase every 3 days

This gives

$$(1+a-0.05-0.007)^9 = 10$$

$$1+a-0.05-0.007 = 1.29155$$

or

$$a = 0.34815$$

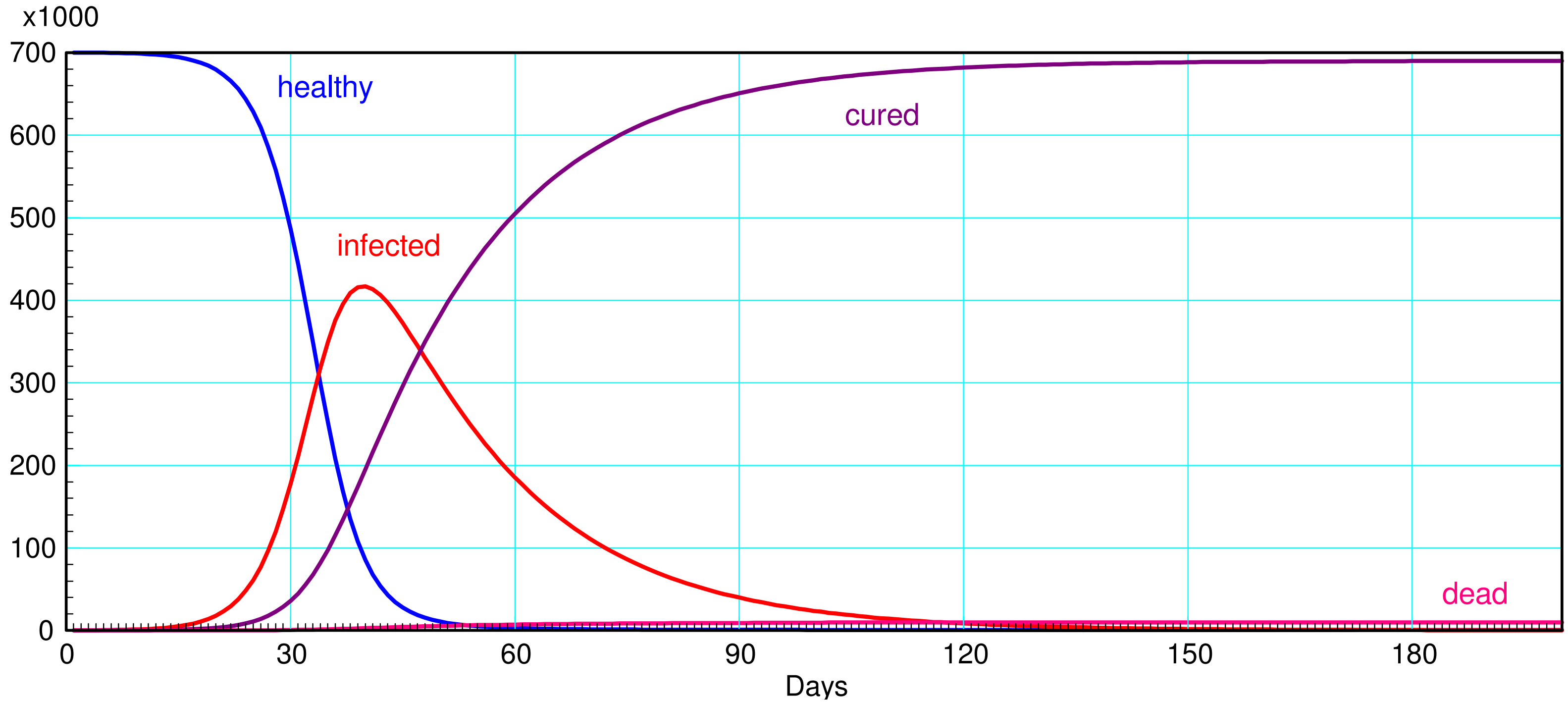
$$Np = 0.34815$$

Assume

- N = 10 *the average person comes in contact with 10 people each day*
- p = 0.0348 *the chance of catching Coronavirus is 3.48%*

Using these numbers, you can simulate what will happen in the state of North Dakota assuming an initial condition of

- H(0) = 700,000 *the population of North Dakota*
- I(0) = 100 *start with 100 people infected as of March 1, 2020*



Markov chain simulation with Np = 0.34815
Prediction of what would have happened if no preventive measures were taken

This model predicts that if nothing had changed as of March 1st (and the entire state was interacting at the same rate and there were no people entering or leaving, etc.),

- The peak in the number of people infected would have been in late April,
- 416,920 people would have been infected, and
- 9658 people will eventually die (about 1.4% of 700,000)

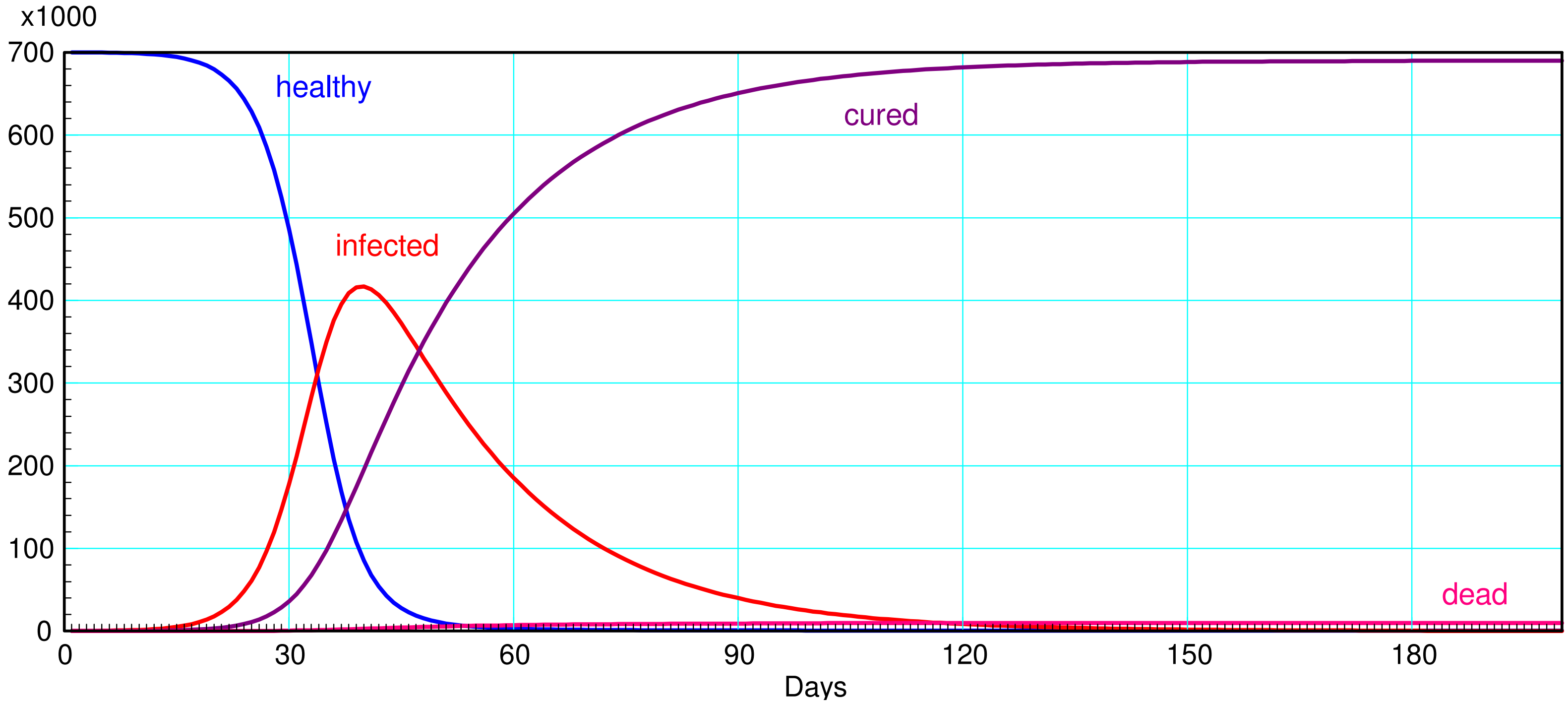
While this model is simplistic in the extreme, it does match up fairly well with more sophisticated models. It also points out some of the problems with not taking action:

- If 416,920 people were infected at the same time, our hospitals would be over whelmed.
- If hospitals could not see any new patients, the death rate would have gone up substantially
- etc.

The main thing to take from this is that

- Markov chains can be applied to more than just electric circuits, and
- They also allow you to ask various questions in simulation.

The latter helps policy makers understand the impacts of different policies.



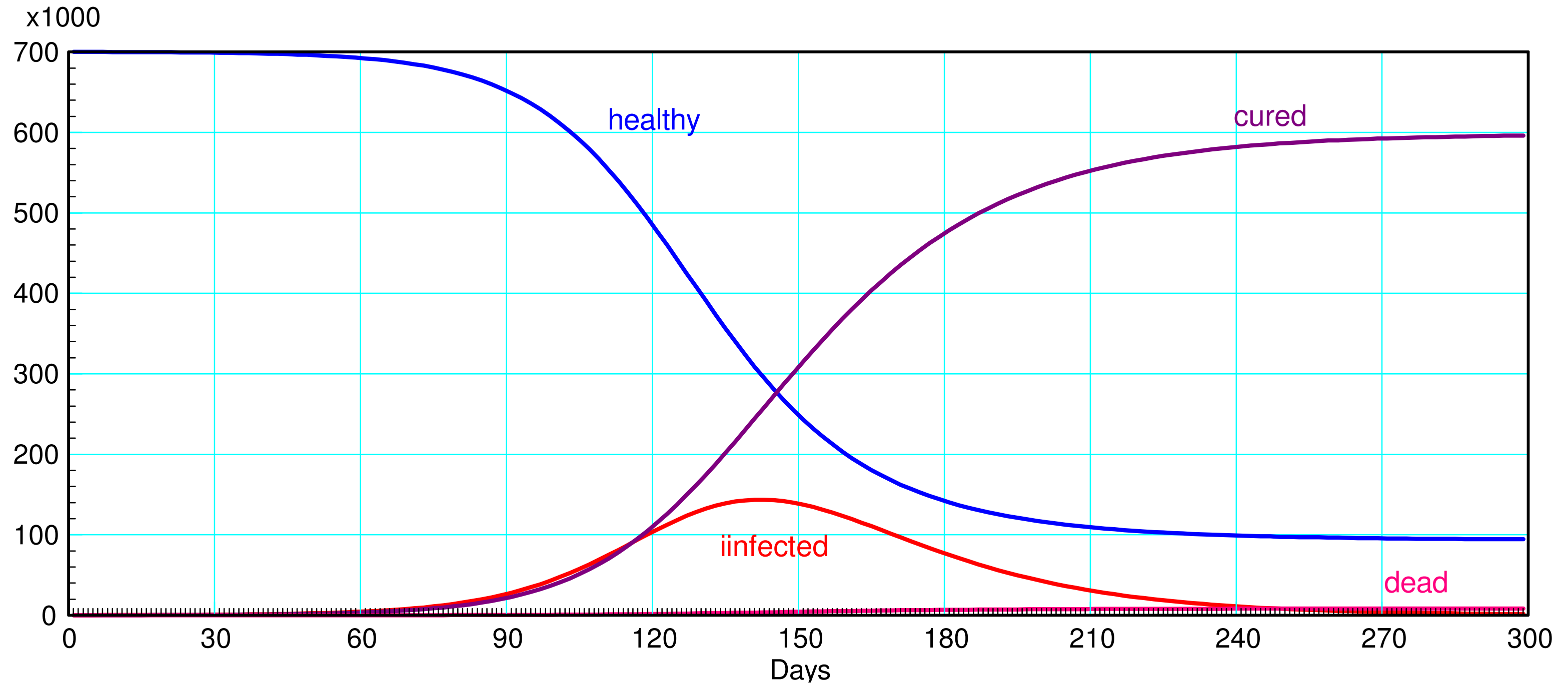
Matlab Code:

```
% X(1) = Uninfected
% X(2) = Infected
% X(3) = Cured
% X(4) = Dead;
X = [700000;100;0;0];

CureRate = 0.05;
DeathRate = 0.014 * CureRate;
Infectivity = .034815;
N = 10;              % interactions per person
% note:  a = N * Infectivity

Y = [];

for i=1:200

    NewInfections = X(1) * X(2) * Infectivity * N / sum(X(1:3)) ;
    Cures = CureRate * X(2);
    Deaths = DeathRate * X(2);

    X(1) = X(1) - NewInfections
    X(2) = X(2) + NewInfections - Cures - Deaths;
    X(3) = X(3) + Cures;
    X(4) = X(4) + Deaths;

    Y = [Y ; X'];
end
```

Simulation #2: Increase Social Distancing

With these models, you can start to ask questions

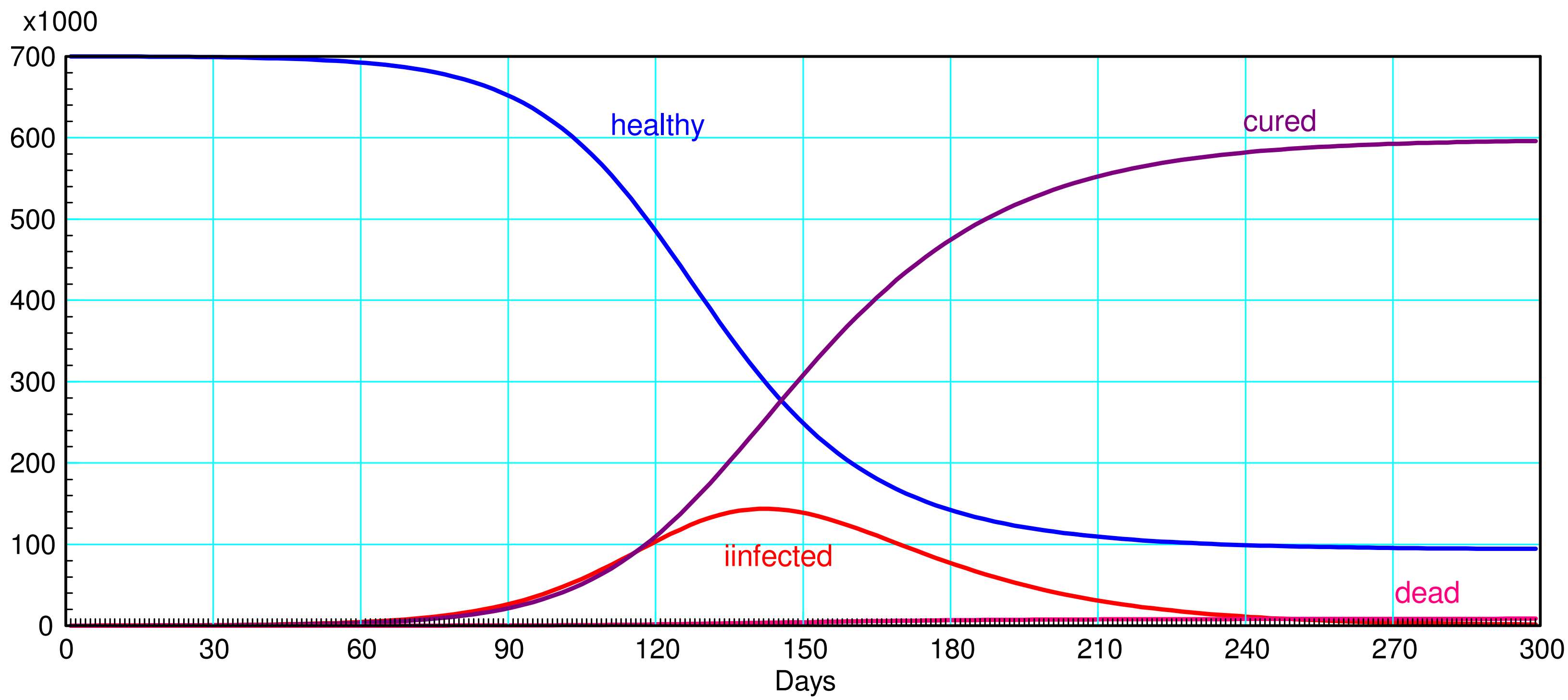
- What impact would it have if N were reduced 3x? (self isolation, closing schools, etc), or
- What impact would it have if p were reduced 3x? (people start wearing face masks)



Markov chain simulation with Np = 0.34815 / 3
Prediction of how the Coronavirus would spread if the Np were reduced by 3x

Note that by dropping either term by 3x

- Max infected drops from 416,920 down to 143,710
- The peak is pushed out to 140 days (4.5 months),
- The disease eventually dissipates with 83,000 people never having been infected, and
- The final death toll is 7427 - slightly less than simulation #1



Simulation #3: Increase in social distancing and wear masks

- Assume N is reduced 3x? (self isolation, closing schools, etc), and
- p is reduced 3x? (people start wearing face masks)

This is more akin to what actually happened in mid March with schools going to remote learning, bars, health clubs being closed, and people being asked to wear face masks and social distance.

By reducing Np 9x, the growth rate drops to less than one - meaning that the disease should dissipate exponentially.

$$y(k+1) = (1 + Np - 0.05 - 0.0007)\ y(k)$$

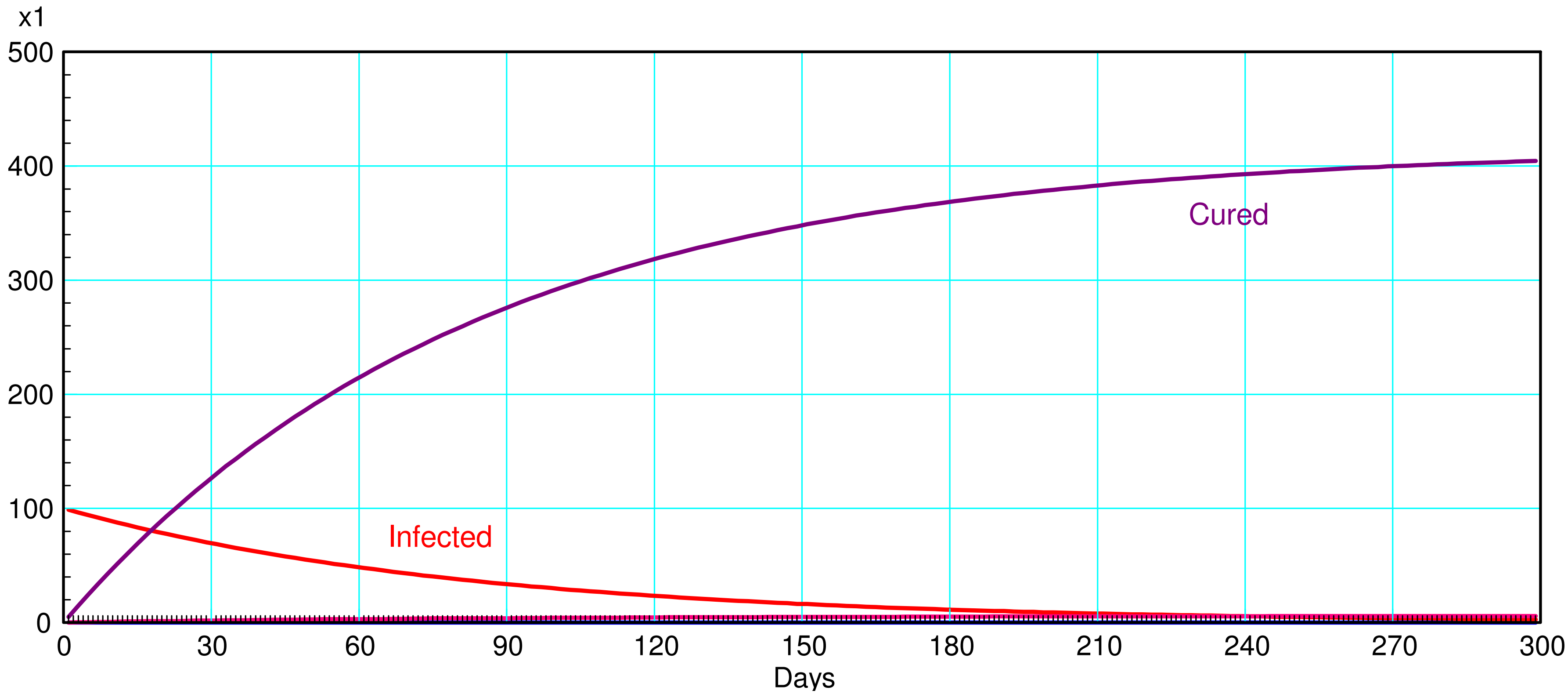
$$y(k+1) = (1 - 0.012)\ y(k)$$

resulting in the disease never taking hold. The result is

- The total number of people infected is 412
- The total number of deaths = 5

Result:

- Death total = 140
- Only 402 people are ever infected
- The pandemic never happens



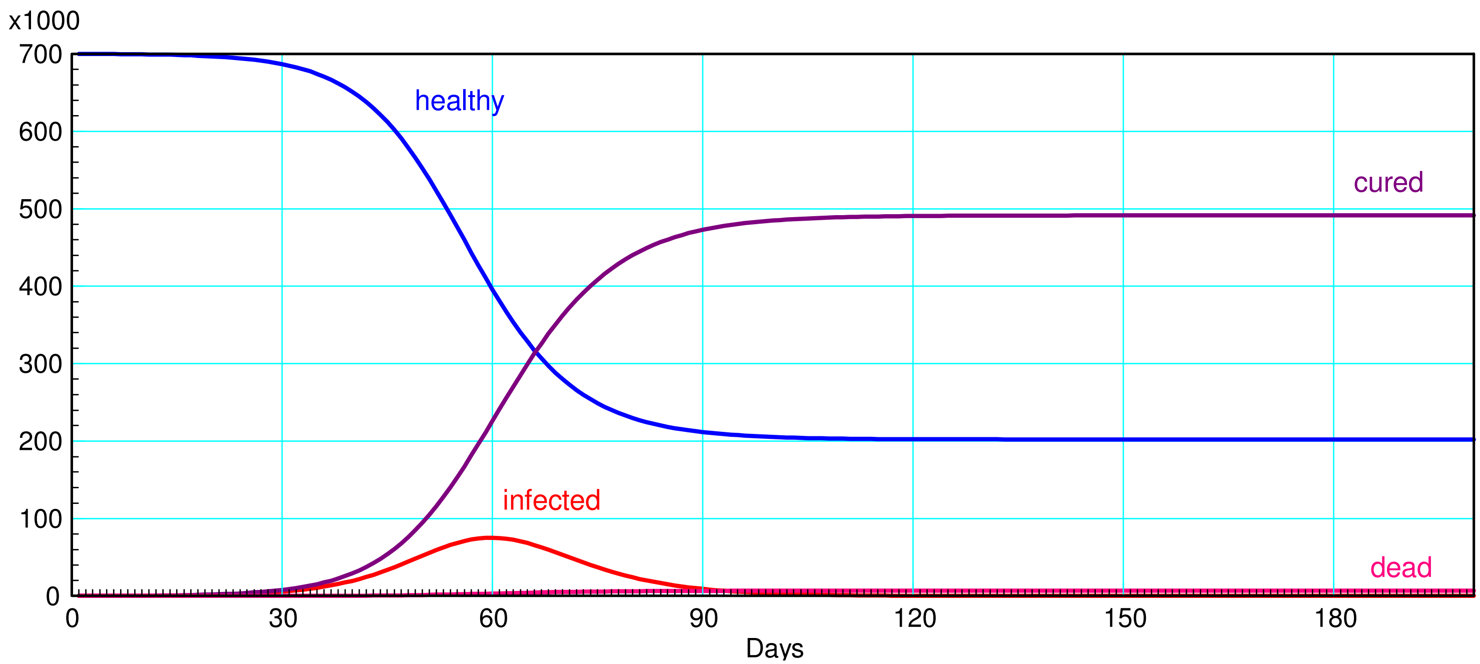
Simulation #4: Vaccine

What happens if a vaccine is found?

- Model as reduce the the infection time from 20 days to 5 days

Repeat without social distancing (N = 10) or wearing masks (p = 0.0348)

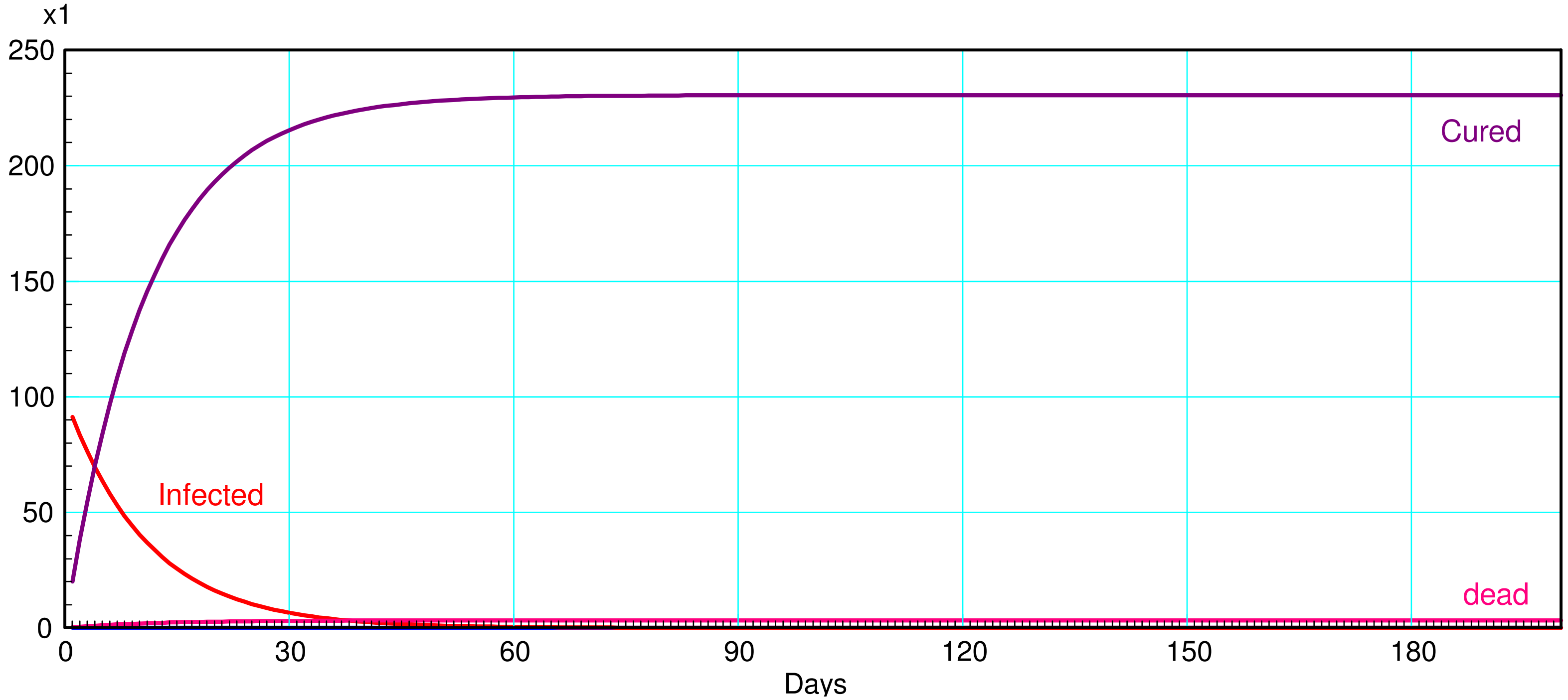
- 83,000 infected, 6922 deaths, peak is 60 days out



Simulation #5: Vaccine & Keep Wearing Masks

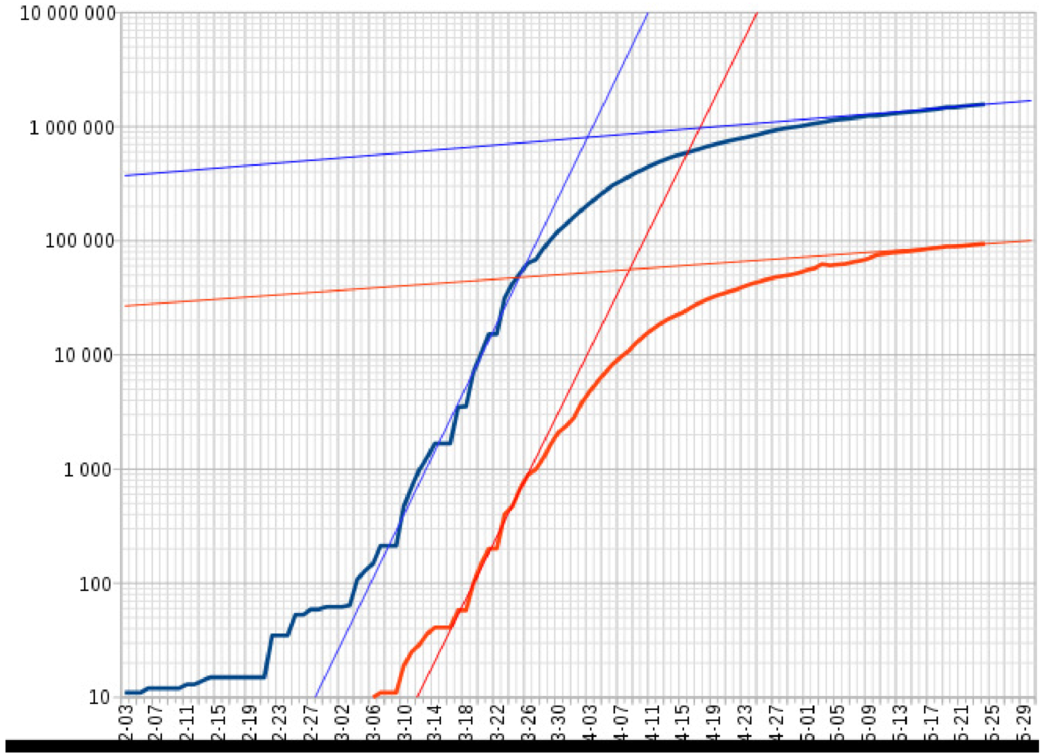
- The infectivity rate drops to less than 1
- The total number of people infected becomes just 240, and
- Only 80 people die from the disease.

Masks may become a permanent fixture - even with a vaccine



Simulation #6: May 1, 2020

- Schools are closed (sort of) with classes being held via remote learning
- Bars, restaurants, health clubs are closed,
- Nonessential workers are asked to self isolate,
- Travel is strongly discouraged - except to buy essentials such as groceries, and
- People are asked to wear face masks in public.



Pre May 1, 2020

- Disease doubles every 3 days

Post May 1, 2020

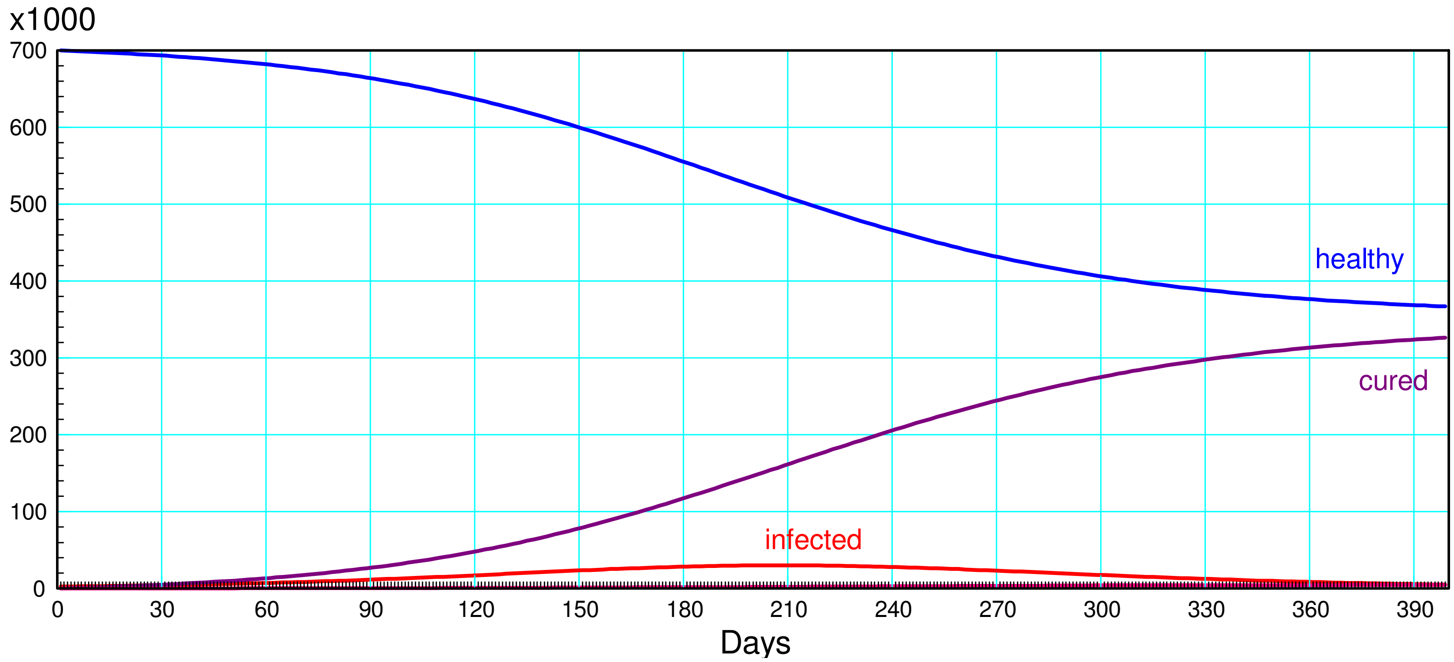
- Doubles every 56 days

This implies that the growth rate (a) dropped to

$$(1+a-0.05-0.0007)^{56}=2 \quad \Rightarrow \quad a=0.06298$$

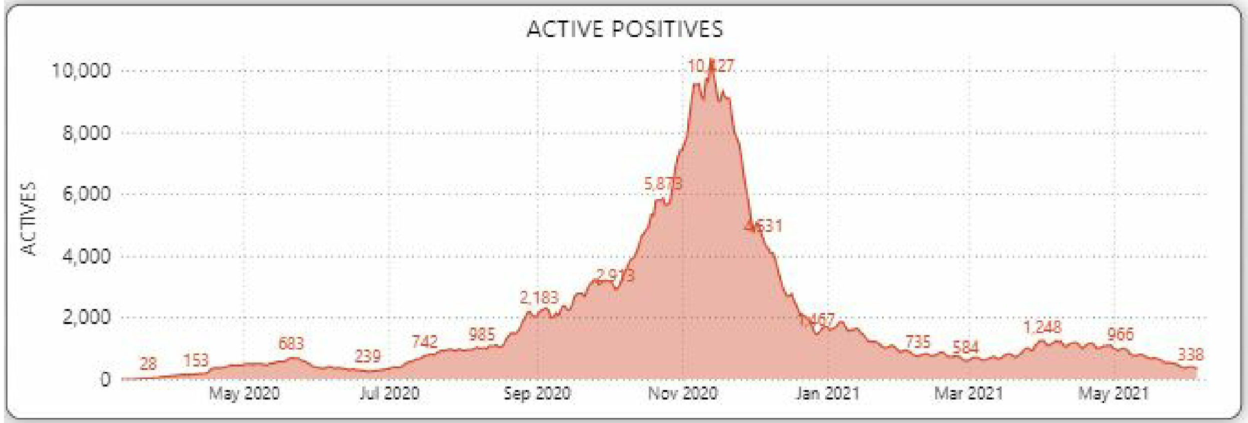
Note with this simulation

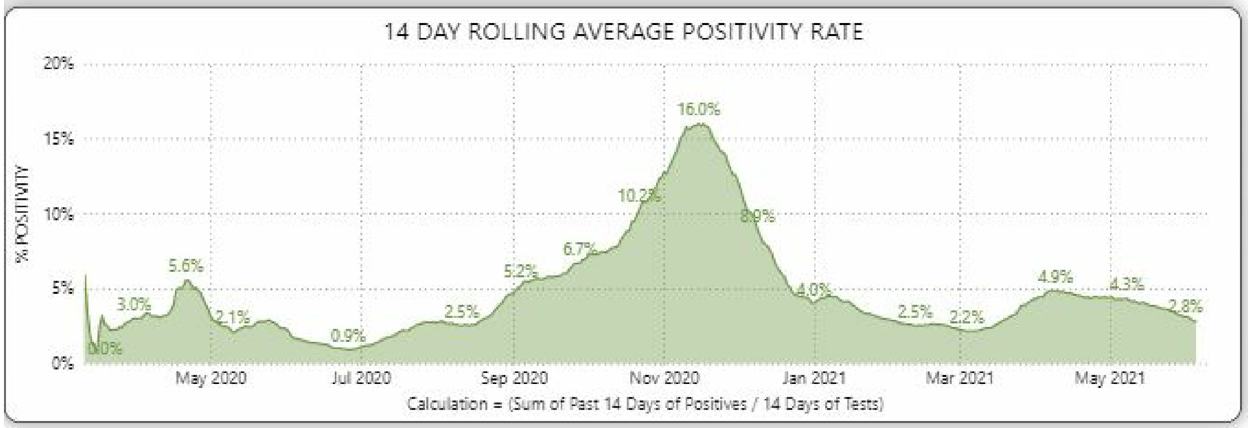
- The maximum number of people infected is 16,540 (December 2020)
- The total number of deaths in North Dakota is 3067, and
- The disease is around for over a year.



June, 2021 Status

ACTIVE POSITIVES
338





CUMULATIVE

POSITIVITY RATE
24.95%

TESTS

TOTAL PROCESSED
2,030,728

SUSCEPTIBLE ENCOUNTERS
1,801,657

UNIQUE INDIVIDUALS
441,624

POSITIVES

TOTAL (PCR AND ANTIGEN TESTS)
110,199

CONFIRMED (PCR TESTS)
101,160

PROBABLE (ANTIGEN TESTS)
9,039

RECOVERED and DEATHS

RECOVERED
108,345

DEATHS
1,516

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