

ECE 341 - Homework #2

Combinatorics & Conditional Probability

Combinatorics

In 7-card stud poker, each player is dealt 7 cards. The best 5 cards constitutes your hand. Determine using combinatorics the odds of being dealt in 7-card stud:

1) 4-of-a-kind: (xxxx aaa) or (xxxx aa b) or (xxxx abc)

From homework #1, the odds are about 180 in 100,000 hands

$M = 52$ choose 7

$$N = \binom{52}{7} = 133,784,560$$

$N = (13 \text{ choose } 1 \text{ for } x)(4 \text{ choose } 4)(48 \text{ remaining cards choose } 3 \text{ for } abc)$

$$M = \binom{13}{1} \binom{4}{4} \binom{48}{3} = 224,848$$

The odds are then

$$p = \left(\frac{M}{N} \right) = 0.001681$$

or 168 in 100,000 hands

- Matches Monte Carlo (HW1)
- 180 in 100,000 hands

2) Full-House (xxx yy a) or (xxx yy aa) or (xxx yy ab)

From homework #1, the odds are 2582 in 100,000 hands (approx)

$N = 13c7$ like before

$$N = \binom{52}{7} = 133,784,560$$

$M_1 = (13 \text{ choose } 2 \text{ for } xy)(4 \text{ x's choose } 3)(4 \text{ y's choose } 3)(11 \text{ choose } 1 \text{ for } a)(4 \text{ choose } 1)$

$$M_1 = \binom{13}{2} \binom{4}{3} \binom{4}{3} \binom{11}{1} \binom{4}{1} = 54,912$$

$M_2 = (13 \text{ values choose } 1 \text{ for } x)(4 \text{ choose } 3)(12 \text{ values choose } 2 \text{ for } ya)(4c2 \text{ for } y)(4c2 \text{ for } a)$

$$M_2 = \binom{13}{1} \binom{4}{3} \binom{12}{2} \binom{4}{2} \binom{4}{2} = 123,552$$

$M_3 = (13c1 \text{ for } x)(4c3)*(12c1 \text{ for } y)(4c2)*(11c2 \text{ for } ab)(4c1 \text{ for } a)(4c1 \text{ for } b)$

$$M_3 = \binom{13}{1} \binom{4}{3} \binom{12}{1} \binom{4}{2} \binom{11}{2} \binom{4}{1} \binom{4}{1} = 3,294,720 \text{ Adding it up}$$

$$M = M_1 + M_2 + M_3$$

$$M = 3,473,184$$

The odds are then

$$p = \left(\frac{M}{N} \right) = 0.025961$$

or 2596 in 100,000 hands

- Matches Monte Carlo
- 2582 in 100,000 hands

3) Flush (5, 6, or 7 cards of the same suit)

$$N = 52c3$$

$$N = \binom{52}{7} = 133,784,560$$

7 of a suit

$$M_1 = (4c1 \text{ for the suit})(13c7)$$

$$M_1 = \binom{4}{1} \binom{13}{7} = 6848$$

6 of a suit

$$M_2 = (4c1 \text{ for the suit})(13c6)(39 \text{ remaining cards choose } 1)$$

$$M_2 = \binom{4}{1} \binom{13}{6} \binom{39}{1} = 267,696$$

5 of a suit

$$M_3 = (4c1 \text{ for the suit})(13c5)(39 \text{ remaining cards choose } 2)$$

$$M_3 = \binom{4}{1} \binom{13}{5} \binom{39}{2} = 3,814,668$$

The number of ways to get a flush in 7-card stud are

$$M = M_1 + M_2 + M_3$$

$$M = 4,089,228$$

The odds are

$$p = \left(\frac{M}{N}\right) = 0.030566$$

or 3,056 hands in 100,000

- Matches Monte-Carlo (HW1)
- 2845 in 100,000 hands

Conditional Probability

4) In a game of dice, you roll three 6-sided dice (3d6). Determine the odds of getting in one roll:

- 3-of-a-kind (xxx)
- Pair (xx y)
- No pairs (xyz)

N = number of rolls

$$N = 6^3 = 216$$

3-of-a-kind

$$M = (6c1 \text{ for } x)(3 \text{ spots choose } 3)$$

$$M = \binom{6}{1} \binom{3}{3} = 6$$

$$p = \left(\frac{M}{N} \right) = 0.027778$$

Pair

$$M = (6c1 \text{ for } x)(3 \text{ spots choose } 2)(5c1 \text{ for } a)(1 \text{ spot choose } 1)$$

$$M = \binom{6}{1} \binom{3}{2} \binom{5}{1} \binom{1}{1} = 90$$

$$p = \left(\frac{M}{N} \right) = 0.416667$$

No Pairs

$$M = (6c3 \text{ for } xyz)(3 \text{ spots choose } 1 \text{ for } x)(2 \text{ spots choose } 1 \text{ for } y)(1 \text{ spot choose } 1 \text{ for } z)$$

$$M = \binom{6}{3} \binom{3}{1} \binom{2}{1} \binom{1}{1} = 120$$

$$p = \left(\frac{M}{N} \right) = 0.555556$$

Probabilities add up to 1.0000

5) Using conditional probabilities, determine the odds of getting 3-of-a-kind in two rolls:

- If you rolled 3-of-a-kind on the first roll, stop.
- If you rolled a pair (xx y), re-roll the y
- If you rolled no pairs (xyz), re-roll y and z

$$p(3ok) = p(3|roll 1)p(3) + p(3|roll 2)p(2) + p(3|roll 1)p(1)$$

a) rolled 3 of a kind

$$p(3|xxx) = 1$$

$$p(xxx) = 0.027778$$

b) rolled pair

$$p(3|xxy) = \frac{1}{6}$$

$$p(xxy) = 0.416667$$

c) rolled no pairs. Keep one die

$$p(3|xyz) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$$

$$p(xyz) = 0.55556$$

So...

$$p(xxx) = (1)(0.027778) + \left(\frac{1}{6}\right)(0.416667) + \left(\frac{1}{36}\right)(0.55556)$$

$$p = 0.112654$$

6) Determine the odds of getting a flush in 5-card draw poker:

- If you are dealt 5 cards of the same suit, stop (you have a flush)
- If you are dealt 4 cards of the same suit, discard the off card and draw one
- If you are dealt a high-card hand, keep the one card whose suit doesn't match any others and draw 4.

a) Dealt a flush

$$N = \binom{52}{5} = 4,598,960$$

$$M = \binom{4}{1} \binom{13}{5} = 5148$$

$$p = \left(\frac{M}{N}\right) = 0.001981$$

b) Dealt 4 cards of a suit

$$M = \binom{4}{1} \binom{13}{4} \binom{39}{1} = 111,540$$

$$p = \left(\frac{M}{N}\right) = 0.042917$$

The odds of the card you draw being the right suit

$$p(\text{flush} | 4) = \left(\frac{9}{47}\right)$$

c) Dealt high card and draw 4

- assume you keep the card that doesn't match the suit of any other card

$$p(\text{high card}) = 0.5011777 \text{ (lecture notes)}$$

$$N = \binom{47}{4}$$

$$M = \binom{12}{4} = 495$$

$$p(\text{flush} | \text{high}) = \left(\frac{M}{N}\right) = 0.002775$$

The odds of a flush are then

$$p(\text{flush}) = p(\text{flush} | \text{dealt 5}) * p(\text{dealt 5}) + p(\text{flush} | \text{dealt 4}) * p(\text{dealt 4}) + p(\text{flush} | \text{high card}) * p(\text{high card})$$

$$p = (1)(0.001981) + \left(\frac{9}{47}\right)(0.042917) + (0.002775)(0.5011777)$$

$$p = 0.011619$$