

ECE 341 - Homework #14

Chi-Squared Tests

Loaded Dice

1) The following Matlab code generates 90 random numbers from 1..6 (sixty 6-sided dice)

```
RESULT = zeros(1,6);  
for i=1:90  
    d6 = ceil(rand*6);  
    RESULT(d6) = RESULT(d6) + 1;  
end
```

Use a chi-squared test to determine if this is a fair die.

Collect data

RESULT = 13 14 13 13 20 17

Calculate the chi-squares score

Roll	p	np	N	chi-squared
1	1/6	15	13	0.2667
2	1/6	15	14	0.0667
3	1/6	15	13	0.2667
4	1/6	15	13	0.2667
5	1/6	15	20	1.6667
6	1/6	15	17	0.2667
Total				2.8

Convert the chi-squared score to a probability using a chi-squared table (or StatTrek)

$p = 0.26921$

There is a 26.9% chance that this die is loaded

no conclusion

- Enter value for degrees of freedom.
- Enter a value for one, and only one, of the other textboxes.
- Click **Calculate** to compute a value for the remaining textbox.

Degrees of freedom

Chi-square value (x)

Probability: $P(\chi^2 \leq 2.8)$

Probability: $P(\chi^2 \geq 2.8)$

Calculate

2) The following Matlab code generates 90 random die rolls with 15% loading (15% of the time you roll a 6)

```
RESULT = zeros(1,6);  
for i=1:90  
    if(rand < 0.15) d6 = 6;  
    else d6 = ceil(rand*6);  
    end  
    RESULT(d6) = RESULT(d6) + 1;  
end
```

Use a chi-squared test to determine if this is a fair die.

Collect data

```
RESULT =      16      14      14      14      12      20
```

Calculate the chi-squared score

Roll	p	np	N	chi-squared
1	1/6	15	16	0.0667
2	1/6	15	14	0.0667
3	1/6	15	14	0.0667
4	1/6	15	14	0.0667
5	1/6	15	12	0.6
6	1/6	15	20	1.6667
Total				2.5333

Convert to a probability using a chi-squared table (or StatTrek)

$p = 0.22853$

There is a 22.8% chance this is a loaded die

note: That happens: the data is random.
Sometimes, the loaded die looks more fair than the fair die.

- Enter value for degrees of freedom.
- Enter a value for one, and only one, of the other textboxes.
- Click **Calculate** to compute a value for the remaining textbox.

Degrees of freedom

5

Chi-square value (x)

2.5333

Probability: $P(\chi^2 \leq 2.5333)$

0.22853

Probability: $P(\chi^2 \geq 2.5333)$

0.77147

Calculate

Mendel's Experiment

3) Mendel showed that there are dominant and recessive genes. When two hybrid plants are crossed (Gg x Gg), 75% of the offspring should display the dominant trait (GG or Gg) while 25% show the recessive trait (gg). Repeat this experiment in Matlab.

Suppose you flip a coin which has a 75% chance of success ($p = 0.75$). What is the probability of getting *exactly* 3 heads in 4 tosses?

note: This is a binomial distribution.

$$x(m) = \binom{n}{m} p^m (1-p)^{n-m}$$

$$x(3) = \binom{4}{3} (0.75)^3 (0.25)^1$$

$$x(3) = 0.4219$$

There is a 42.19% chance of getting 3 dominant and 1 recessive offspring with four trials

4) Suppose you run this experiment n times. Each experiment of four tosses, you get *exactly* 3 heads and 1 tails. Use a chi-squared test to determine how large n has to be until you can be 99% certain the experiment is rigged.

Outcome	p	np	N	chi-squared
3 : 1	0.4219	0.4219n	n	0.7921n
other	0.5781	0.5781n	0	0.5781n
Total				1.3702n

From StatTrek, to be 99% certain this is not a fair die, the chi-squared value has to be 6.63477

$$1.3702n = 6.63477$$

$$n = 4.842$$

After 5 experiments where you get *exactly* 3 dominant and 1 recessive offspring, you can be 99% certain the experiment is rigged.

Am I psychic?

5) Shuffle a deck of 52 playing cards. Without looking at the top card, predict the suit (clubs, diamonds, hearts, and spades). Repeat for all 52 cards, keeping track of how many you got right and how many you got wrong.

- From the results, use a chi-squared test to determine if you are just guessing (25% chance of getting the suit correct.)

Result:

Correct: 15 times

Incorrect: 37 times

Compute the chi-squared score:

Result	p	n*p	N	$\chi^2 = \left(\frac{(np-N)^2}{np} \right)$
Correct	1/4	13	15	0.3077
Incorrect	3/4	39	37	0.1026
			Total	0.4103

This corresponds to a probability of 0.47818

There is a 47.818% chance I'm not just guessing (no conclusion)

- Enter value for degrees of freedom.
- Enter a value for one, and only one, of the other textboxes.
- Click **Calculate** to compute a value for the remaining textbox.

Degrees of freedom

1

Chi-square value (x)

0.4103

Probability: $P(X^2 \leq 0.4103)$

0.47818

Probability: $P(X^2 \geq 0.4103)$

0.52182

Calculate