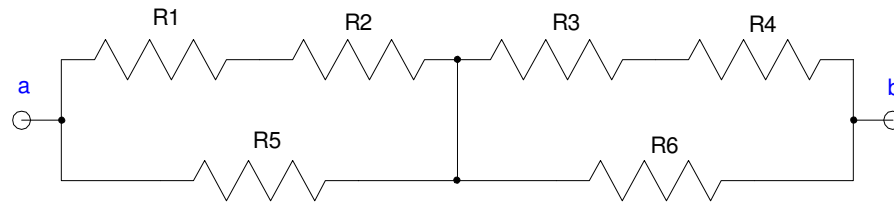


ECE 341 - Homework #9

Weibull Distribution, Central Limit Theorem

Weibull Distribution

1) Determine and plot the cdf for the voltage, Y , in homework set #7 problem #3



Homework #7, problem #3. Find the pdf for the resistance R_{ab} . All resistors are 100 Ohms, 10% tolerance

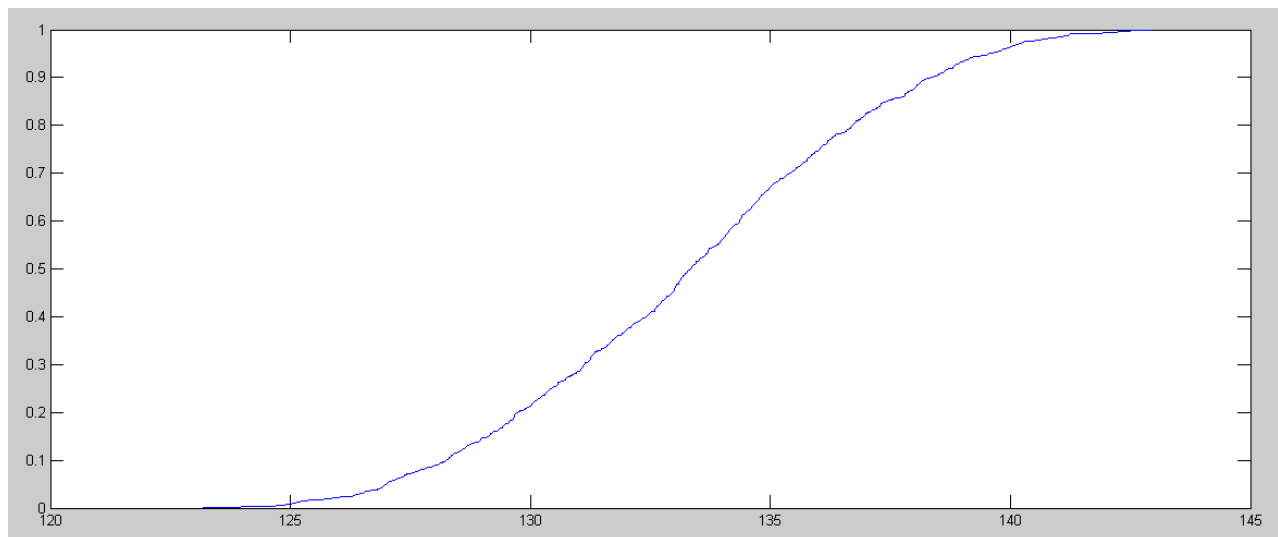
```
Rab = zeros(1000,1);

for i=1:1000
    R1 = 100 * (0.9 + 0.2*rand);
    R2 = 100 * (0.9 + 0.2*rand);
    R3 = 100 * (0.9 + 0.2*rand);
    R4 = 100 * (0.9 + 0.2*rand);
    R5 = 100 * (0.9 + 0.2*rand);
    R6 = 100 * (0.9 + 0.2*rand);

    R34 = R3 + R4;
    Ra = 1 / (1/R34 + 1/R6);
    R12 = R1 + R2;
    Rb = 1 / (1/R12 + 1/R5);

    Rab(i) = Ra + Rb;
end

Rab = sort(Rab);
n = [1:1000]';
plot(Rab,n/1000)
```



2) Determine and plot the pdf for this voltage using a Weibull approximation for the cdf

Once you have the data from problem #1, save to a file

```
>> p = [1:1000]' / 1000;  
>> save HW9 Rab p
```

Create a function which

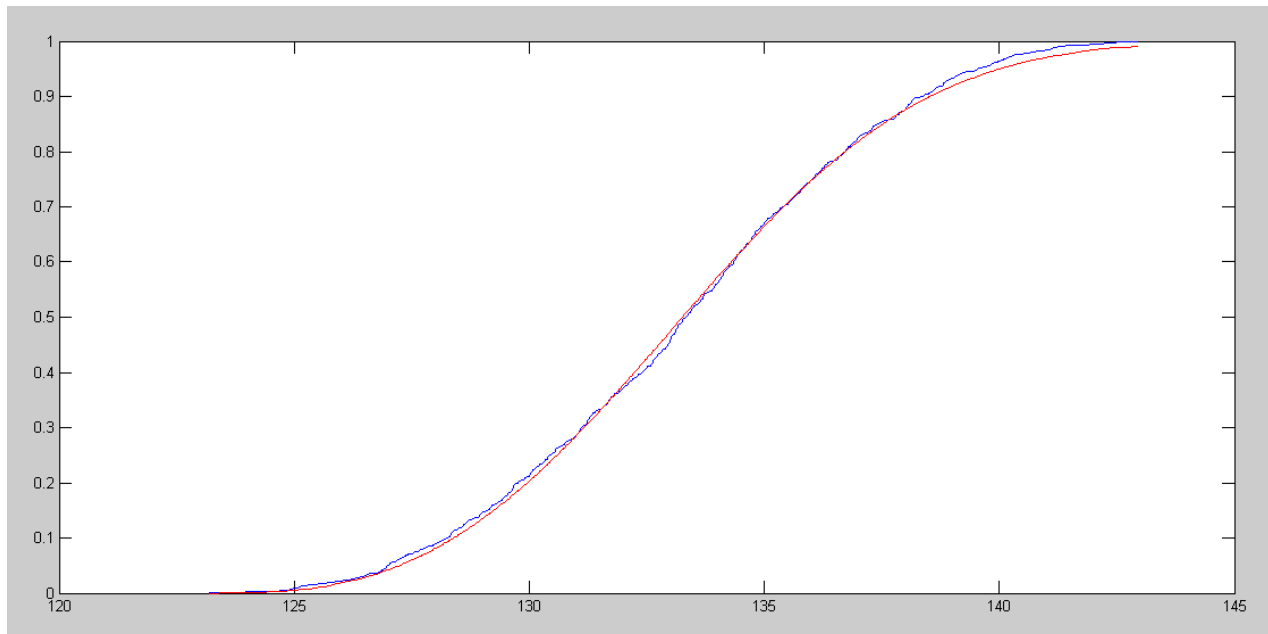
- Is passed Weibull parameters k and lambda
- Returns the sum-squared difference between the Weibull cdf and the data's cdf

```
function y = Weibull(z)  
% Weibull distribution curve fit  
k = z(1);  
L = z(2);  
  
load HW9  
  
x = Rab;  
x0 = min(x);  
  
% p(Vce) = target  
W = 1 - exp( -( (x-x0)/L) .^ k ));  
  
e = p - W;  
  
plot(Rab, p, 'b', Rab, W, 'r');  
pause(0.01);  
y = sum(e.^2);  
  
end
```

Optimize using fminsearch()

```
>> [Z,e] = fminsearch('Weibull',[2,8])
```

```
      k      lambda  
Z =    2.8529    11.4501
```



This tells you that the CDF is

$$Z = \begin{matrix} k & \text{lambda} \\ 2.8529 & 11.4501 \end{matrix}$$

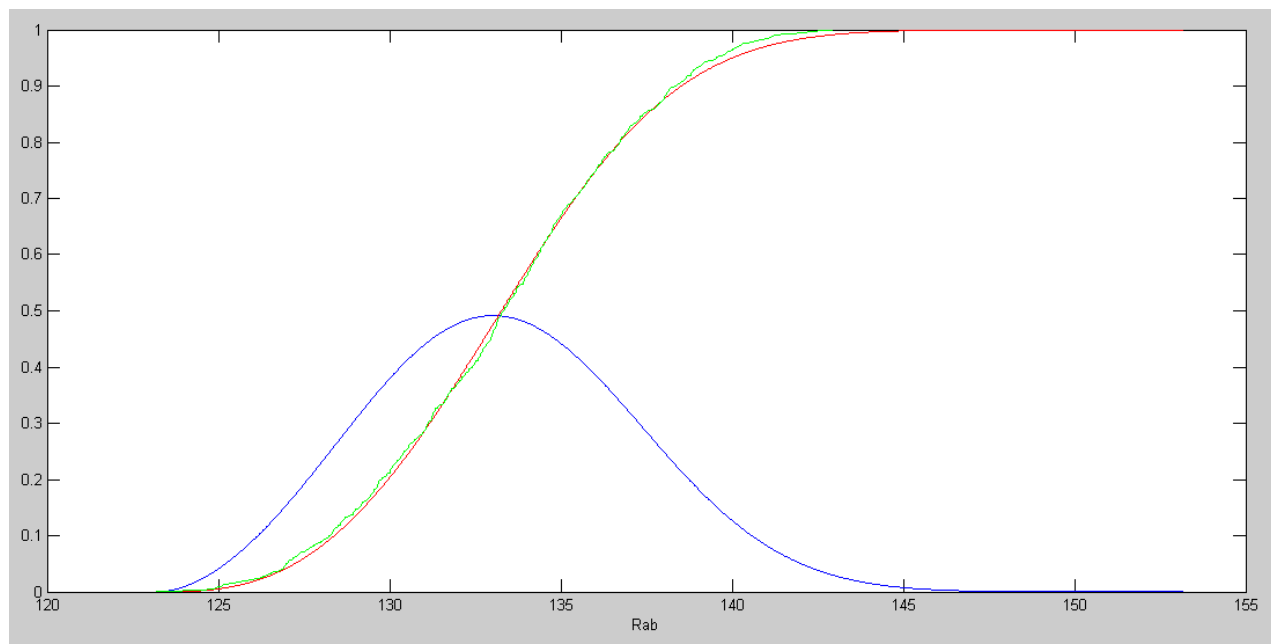
$$F(x) \approx \left(1 - \exp \left(- \left(\frac{x}{\lambda} \right)^k \right) \right) + x_0$$

the PDF is

$$f(x) = \frac{k}{\lambda} \left(\frac{x}{\lambda} \right)^{k-1} \exp \left(- \left(\frac{x}{\lambda} \right)^k \right) u(k) + x_0$$

$$f(x) = \frac{2.8529}{11.4501} \left(\frac{x}{2.8529} \right)^{1.8529} \exp \left(- \left(\frac{x}{11.4501} \right)^{2.8529} \right) u(k) + 123.19$$

```
>> k = Z(1);
>> L = Z(2);
>> x0 = min(Rab);
>> x = [0:0.01:30]';
>> pdf = (k/L) * (x/L) .^(k-1) .* exp(-(x/L).^k);
>> cdf = 1 - exp(-(x/L).^k);
>> plot(x+x0,pdf*5,'b',x+x0,cdf,'r', Rab, p, 'g');
>> xlabel('Rab');
```



cdf (red and green) and pdf*5 (blue)

Central Limit Theorem

The mean and standard deviation for a 4, 6, and 8-sided die are

Die	d4	d6	d8	d10	d12
mean	2.5	3.5	4.5	5.5	6.5
std dev	1.1180	1.7078	2.2191	2.8723	3.4521
variance	1.2500	2.9166	5.2487	8.25	11.9167

Let player A roll ten dice: $A = 3d6 + 3d8 + 4d12$

3) What is the

- mean of A?
- variance of A?

Mean:

$$\mu_a = 3 \cdot 3.5 + 3 \cdot 4.5 + 4 \cdot 6.5$$

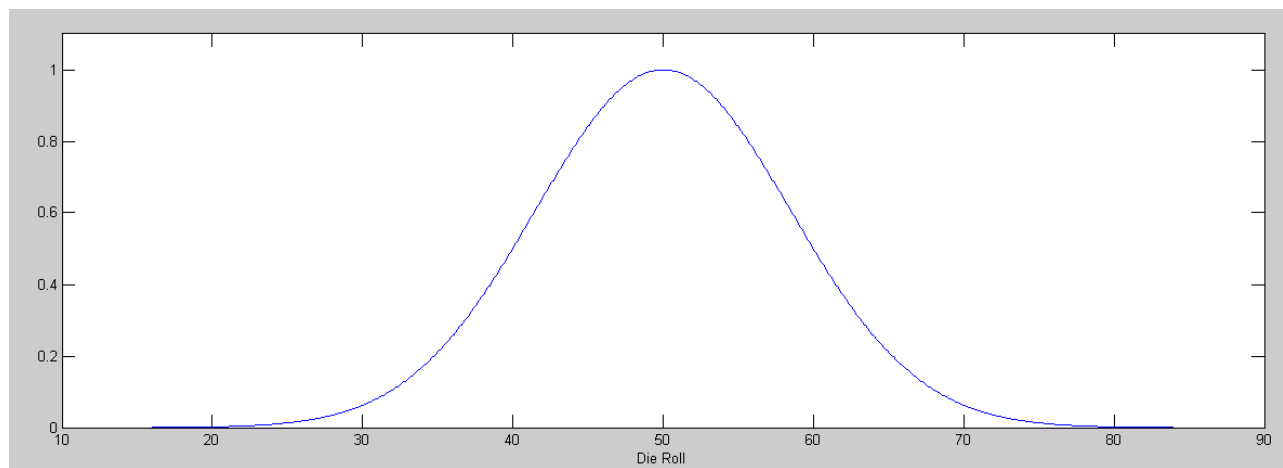
$$\mu_a = 50.00$$

Variance

$$\sigma_a^2 = 3 \cdot 2.9166 + 3 \cdot 5.2487 + 4 \cdot 11.9167$$

$$\sigma_a^2 = 72.1627$$

$$\sigma_a = 8.4949$$



4) Using a normal approximation, determine the probability that

$$p(59.5 < A < 60.5) \quad (A = 60)$$

Find the z-scores

```
>> x = 50;  
>> s = 8.4949;  
>> zL = (59.5 - x) / s  
  
zL = 1.1183  
  
>> pL = ( erf(-zL/sqrt(2)) + 1) / 2  
  
pL = 0.1317  
  
>> zR = (60.5 - x) / s  
  
zR = 1.2360  
  
>> pR = ( erf(-zR/sqrt(2)) + 1) / 2  
  
pR = 0.1082  
  
>> p = pL - pR  
  
p = 0.0235
```

There is a 2.35% chance of rolling a 60

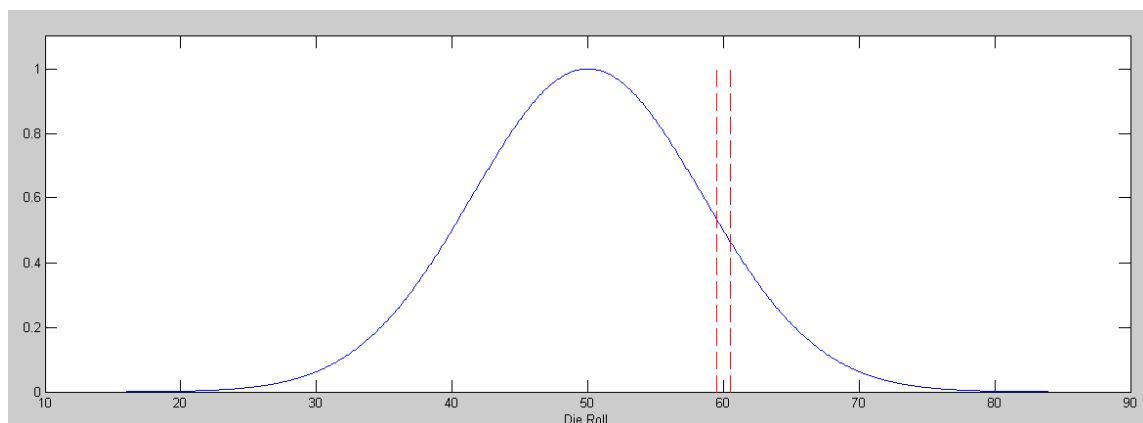
exact odds 2.40% from HW4

$$p(A > 60.5) \quad (A > 60)$$

```
>> zR = (60.5 - x) / s  
  
zR = 1.2360  
  
>> pR = ( erf(-zR/sqrt(2)) + 1) / 2  
  
pR = 0.1082
```

There is a 10.82% chance of rolling more than 60

exact odds 11.05% from homework #4



Let B roll ten 10-sided dice (10d10)

5) What is the

- mean of B?
- variance of B?

$$\mu_b = 10 \cdot 5.5$$

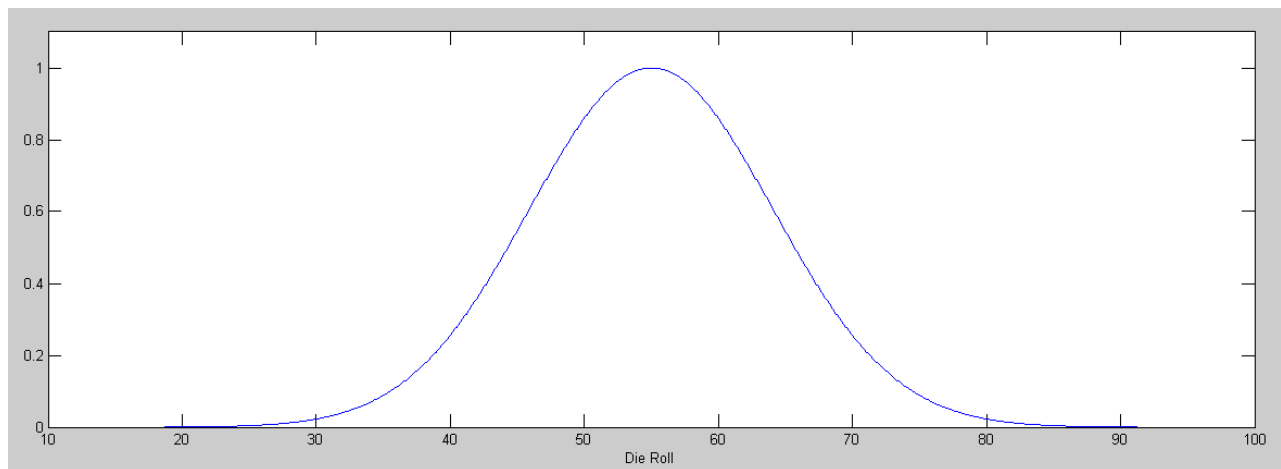
$$\mu_b = 55$$

$$\sigma_b^2 = 10 \cdot 8.25$$

$$\sigma_b^2 = 82.5$$

$$\sigma_b = 9.083$$

This tells you the pdf for B looks roughly like the following (it's actually discrete)



6) Using a normal approximation, determine the probability that

- $59.5 < B < 60.5$ ($B = 60$)
- $B > 60.5$ ($B > 60$)

note: Results should be similar to homework #4 problem #6

$p(59.5 < B < 60.5)$

```
>> x = 55;  
>> s = sqrt(82.5);  
>>  
>> z1 = (59.5 - x) / s  
z1 = 0.4954  
  
>> p1 = ( erf(-z1/sqrt(2)) + 1) / 2  
p1 = 0.3101  
  
>> z2 = (60.5 - x) / s  
z2 = 0.6055  
  
>> p2 = ( erf(-z2/sqrt(2)) + 1) / 2  
p2 = 0.2724  
  
>> p = p1 - p2  
p = 0.0377
```

There is a 3.77% chance of rolling a 60

exact odds 3.75% using convolution (HW4 problem 6)

$p(B > 60)$

```
>> z2 = (60.5 - x) / s  
z2 = 0.6055  
  
>> p2 = ( erf(-z2/sqrt(2)) + 1) / 2  
p2 = 0.2724
```

There is a 27.24% chance of rolling more than 60

exact odds 27.51% using convolution (HW4 problem 6)

