

ECE 341 - Homework #8

Gamma, Poisson, & Normal Distributions

Gamma Distributions

Let A be an exponential distribution with a mean of 30 seconds

The time until the next customer arrives

Let B be the time until three customers arrive (B has a gamma distribution)

1) Determine the pdf for B using LaPlace transforms.

- From your results, determine the pdf at B=20

The Moment generating functions (i.e. LaPlace transforms) are

$$A(s) = \left(\frac{1/30}{s+1/30} \right)$$

$$B(s) = \left(\frac{1/30}{s+1/30} \right)^3$$

From a table of LaPlace tranforms

$$t^2 e^{-bt} \leftrightarrow \left(\frac{2}{(s+b)^3} \right)$$

so

$$B(s) = \left(\frac{1}{30} \right)^3 \left(\frac{1}{2} \right) \left(\frac{2}{\left(s + \frac{1}{30} \right)^3} \right)$$

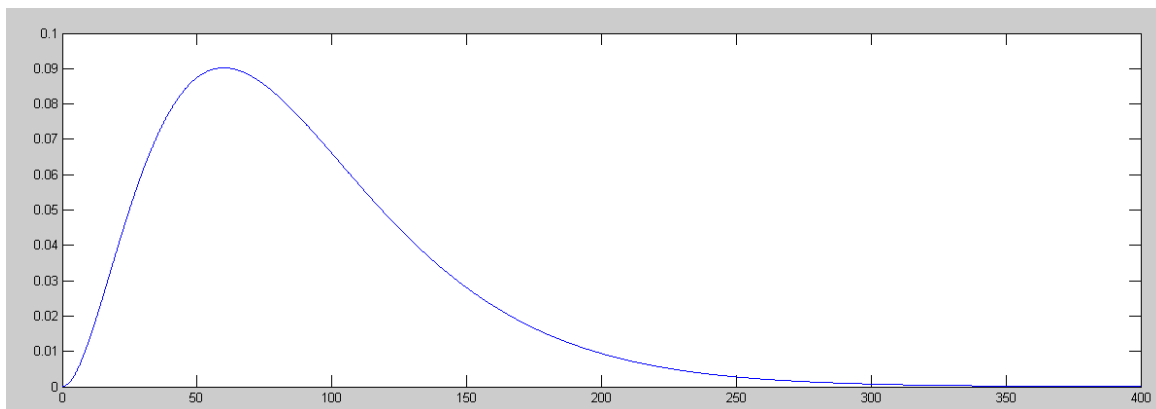
$$b(t) = \left(\frac{1}{54,000} \right) t^2 \cdot \exp\left(\frac{-t}{30}\right) u(t)$$

In Matlab

```
>> t = 20  
t = 20
```

```
>> b = 1/54000 * t^2 * exp(-t/30)
```

```
b = 0.0038
```



2) Determine the pdf of B using convolution

- From your results, determine the pdf at $B = 20$

```
>> dt = 0.01;  
>> t = [0:dt:400]';  
>> A = 1/30 * exp(-t/30);
```

Check that the areas is 1.000 (went far enough)

```
>> sum(A) * dt
```

```
ans =    1.0002
```

```
>> A2 = conv(A,A) * dt;  
>> A3 = conv(A2,A) * dt;  
>> A3(2000)
```

```
ans =    0.0038
```

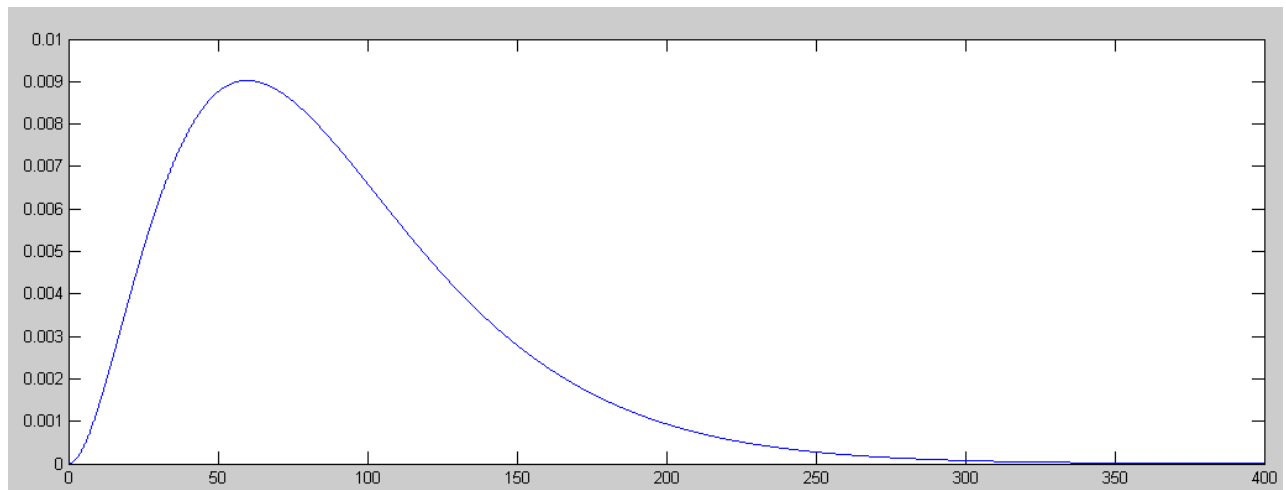
Same answer as problem #1

Just for fun, plot the pdf of B

```
>> size(t)
```

```
ans =    40001         1
```

```
>> plot(t,A3(1:40001));
```



Poisson Distributions

3) The odds of getting a straight-flush or royal-flush in poker are 40 in 2,598,960. What is the Poisson approximation for the number of straght/royal flushes you'll be dealt in 100,000 hands of poker?

$$f(k) \approx \frac{1}{k!} \cdot \lambda^k \cdot e^{-\lambda}$$

where

$$\lambda = np$$

For this problem

$$\lambda = (100,000) \left(\frac{40}{2,598,960} \right)$$

$$\lambda = 1.5391$$

4) From the Poisson approximation, what is the probability of being dealt

- 1 straight/royal flush in 100,000 hands
- 2 straight/royal flushes in 100,000 hands
- 3 straight/royal flushes in 100,000 hands

```
>> lambda = 1e5 * 40 / 2598960
```

```
lambda =      1.5391
```

```
>> k = 1;
```

```
>> p = 1 / factorial(k) * lambda^k * exp(-lambda)
```

```
p =      0.3303           33% chance of getting one straight-flush in 100,000 hands
```

```
>> k = 2;
```

```
>> p = 1 / factorial(k) * lambda^k * exp(-lambda)
```

```
p =      0.2541           25% chance of getting two straight-flushes
```

```
>> k = 3;
```

```
>> p = 1 / factorial(k) * lambda^k * exp(-lambda)
```

```
p =      0.1304           13% chance of getting three straight-flushes
```

```
>>
```

Normal Distribution

The high and low for the month in Fargo, ND is:

Monthly High (Degrees F: Fargo, ND)												
	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sep	Oct	Nov	Dec
mean	38.5363	41.0038	56.0625	78.1	87.8625	92.0138	94.625	94.6262	89.575	79.5	59.425	41.7875
st dev	6.4057	7.1528	10.7118	7.7909	4.5472	4.5281	4.0043	4.5967	5.6294	6.7842	7.4728	6.5327

5) What is the probability that it will break 100F in June?

Find the z-score

```
>> z = (100 - 92.0138) / 4.5281
```

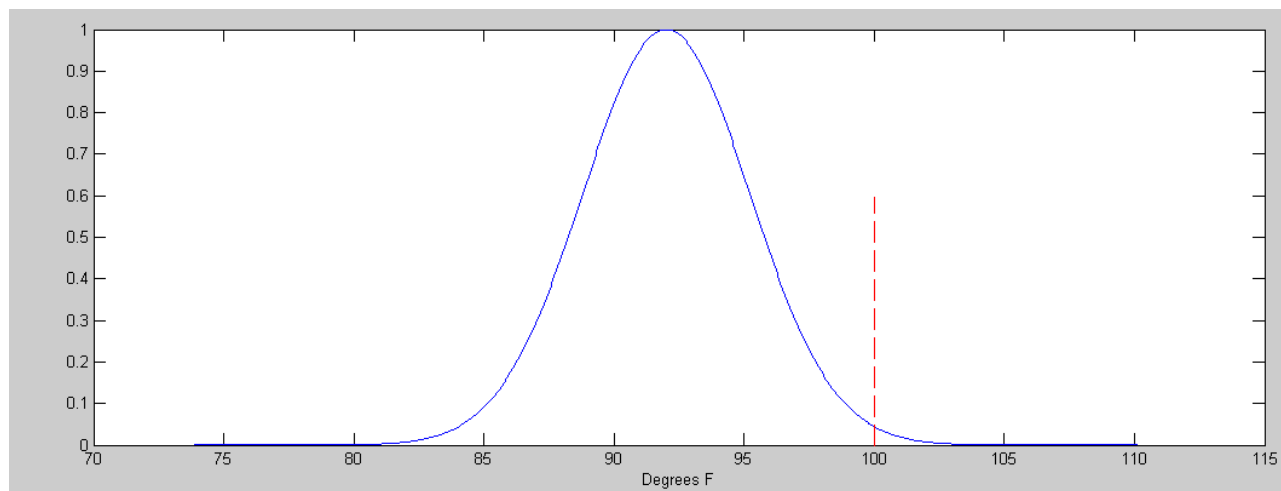
```
z = 1.7637
```

Use a standard normal table, StatTrek, or Matlab's erf() to find the area of the tail:

```
>> p = ( erf(-z/sqrt(2)) + 1) / 2
```

```
p = 0.0389
```

From the data, there is a 3.89% chance it will break 100F this June



6) What is the probability that it never gets above freezing in January? (high < 32.0F)

Find the z-score

```
>> z = (38.5363 - 32) / 6.4057
```

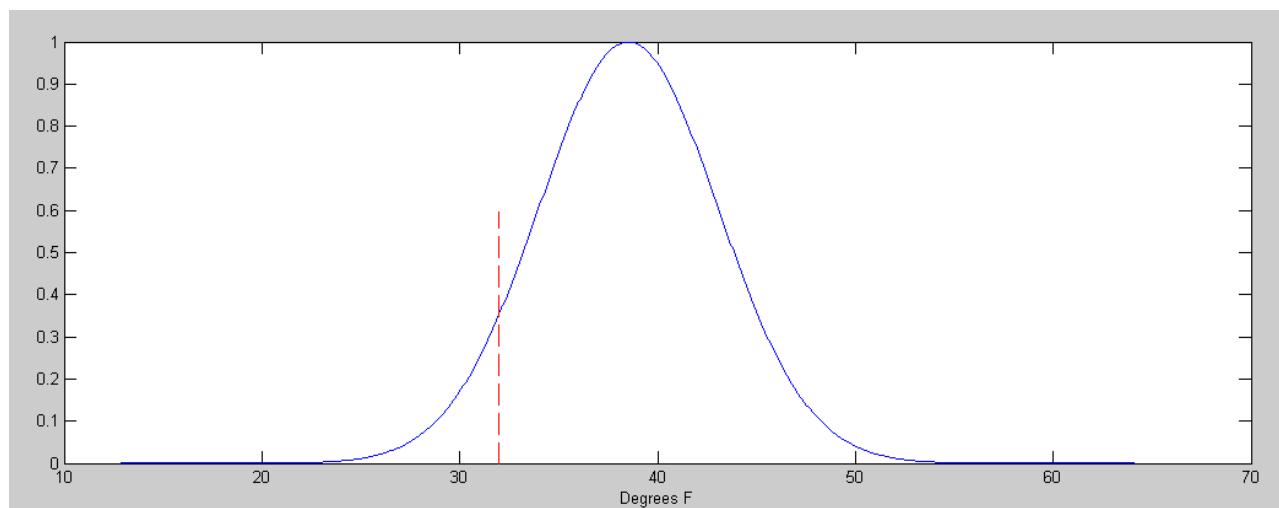
```
z = 1.0204
```

Convert to a probability

```
>> p = ( erf(-z/sqrt(2)) + 1) / 2
```

```
p = 0.1538
```

There is a 15% chance of never getting above freezing in January



Monthly Low (Degrees F: Fargo, ND)												
	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sep	Oct	Nov	Dec
mean	-23.8725	-20.6238	-8.1475	15.1775	27.3413	40.425	46.4875	43.3387	30.6763	19.15	-1.0875	-17.025
st dev	8.2179	7.8559	10.0237	7.0423	4.3864	4.1576	4.0938	4.1522	4.8861	5.5212	9.0417	9.1069

7) What is the probability that it will freeze in May? (low < 32.0F)

Find the z-score

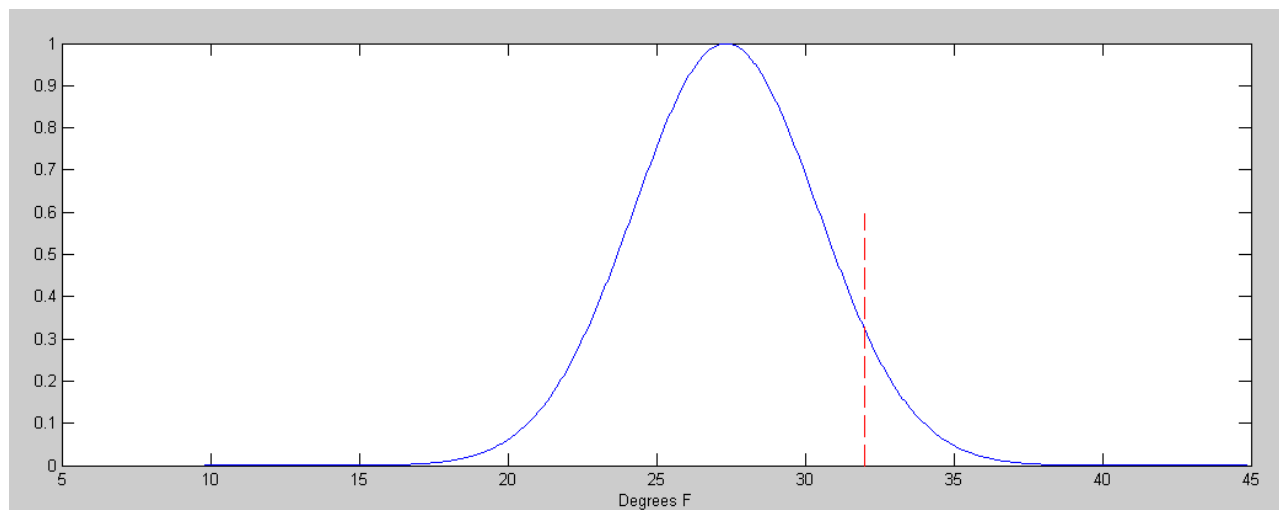
```
>> z = (32 - 27.3413) / 4.3864
```

```
z = 1.0621
```

Convert to a probability (the odds are more than 50%: the mean is less than freezing)

```
>> p = ( erf(z/sqrt(2)) + 1) / 2
```

```
p = 0.8559
```



8) What is the probability that it never get below 0F in April? (low > 0.0F)

```
>> z = (15.1775 - 0) / 7.0423
```

```
z = 2.1552
```

```
>> p = ( erf(z/sqrt(2)) + 1) / 2
```

```
p = 0.9844
```

There is a 98% chance it will not get below 0F in April

