

ECE 341 - Homework #7

Uniform and Exponential Distributions

Uniform Distributions

Let

- **a** be a sample from A, a uniform distribution over the range of (2, 3)
- **b** be a sample from B, a uniform distribution over the range of (0, 4)

1) Determine the pdf for **a + b** using moment generating functions (i.e. LaPlace transforms)

$$A(s) = \left(\frac{1}{s}\right)(e^{-2s} - e^{-3s})$$

$$B(s) = \left(\frac{1}{4s}\right)(1 - e^{-4s})$$

a ** b:

$$C(s) = A(s) \cdot B(s)$$

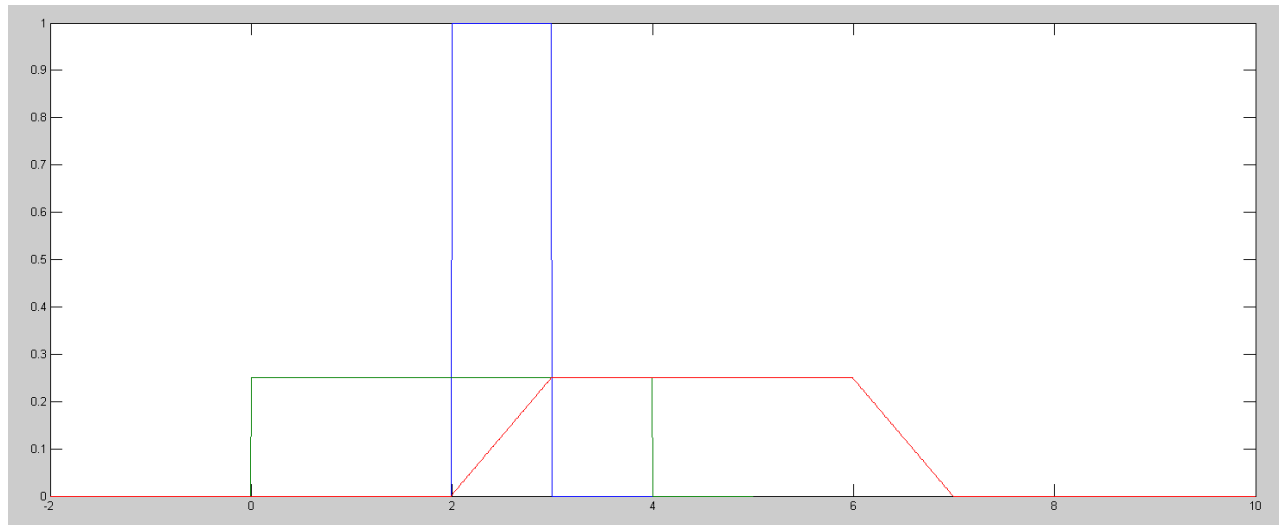
$$\begin{aligned} C(s) &= \left(\left(\frac{1}{s}\right)(e^{-2s} - e^{-3s})\right)\left(\left(\frac{1}{4s}\right)(1 - e^{-4s})\right) \\ &= \left(\frac{1}{4s^2}\right)(e^{-2s} - e^{-3s} - e^{-6s} + e^{-7s}) \end{aligned}$$

Take the inverse LaPlace transform

$$c(t) = \left(\frac{1}{4}\right)((t-2)u(t-2) - (t-3)u(t-3) - (t-6)u(t-6) + (t-7)u(t-7))$$

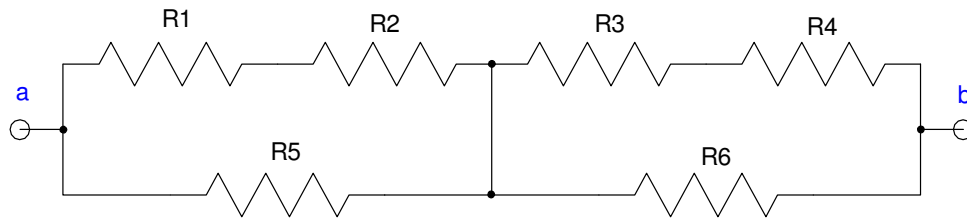
2) Determine the pdf for $\mathbf{a} + \mathbf{b}$ using convolution (by hand or Matlab)

```
>> dt = 0.01;  
>> t = [-1:dt:5]' + 1e-6;  
>> A = 1*(t>2).* (t<3);  
>> B = 1/4 * (t>0).* (t<4);  
>> AB = conv(A,B) * dt;  
>> t2 = [-2:dt:10]';  
>> plot(t,A,t,B,t2,AB)
```



A (blue), B (green), $A*B$ (red)

3) Determine R_{ab} as a function of $R_1..R_6$



```
R34 = R3 + R4;  
Ra = 1 / (1/R34 + 1/R6);  
R12 = R1 + R2;  
Rb = 1 / (1/R12 + 1/R5);  
  
Rab(i) = Ra + Rb;
```

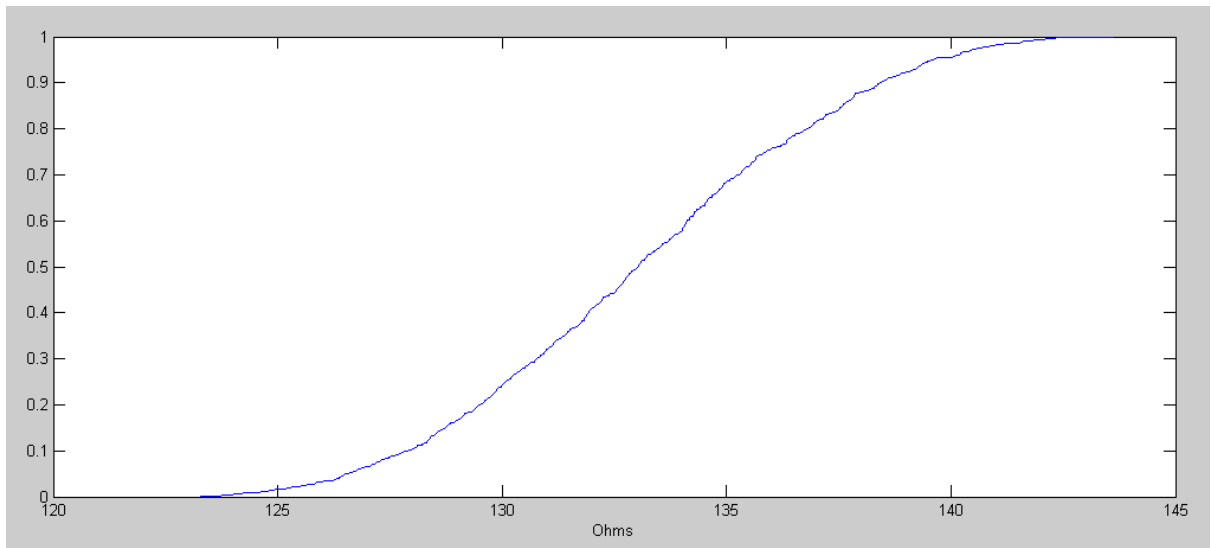
4) Assume each resistor has a nominal value of 100 Ohms and 10% tolerance

```
R1 = 100 * (0.9 + 0.2*rand);
```

Determine the net resistance, R_{ab} , for 1000 random values of R 's.

Matlab Code

```
Rab = zeros(1000,1);  
  
for i=1:1000  
    R1 = 100 * (0.9 + 0.2*rand);  
    R2 = 100 * (0.9 + 0.2*rand);  
    R3 = 100 * (0.9 + 0.2*rand);  
    R4 = 100 * (0.9 + 0.2*rand);  
    R5 = 100 * (0.9 + 0.2*rand);  
    R6 = 100 * (0.9 + 0.2*rand);  
  
    R34 = R3 + R4;  
    Ra = 1 / (1/R34 + 1/R6);  
    R12 = R1 + R2;  
    Rb = 1 / (1/R12 + 1/R5);  
  
    Rab(i) = Ra + Rb;  
end  
  
Rab = sort(Rab);  
n = [1:1000]';  
plot(Rab,n/1000)
```



cdf for the total resistance R_{ab}

Exponential Distribution

5) A telephone call has an exponential distribution with mean = 2 minutes. What is probability that a given phone call will last more than 10 minutes?

$$pdf(x) = a \cdot e^{-ax}$$

$$cdf(x) = 1 - e^{-ax}$$

$$a = \frac{1}{2} \text{ minute}$$

the cdf of 10 minutes is

$$cdf(10) = 1 - \exp\left(-\frac{10}{2}\right)$$

$$cdf(10) = 0.9933$$

This is the probability that a call will last *less* than 10 minutes. The probability for *more* than 10 minutes is 1-this

$$p(x < 10) = 0.9933$$

$$p(x > 10) = 0.0067$$

6) Yellowstone volcano is an exponential distribution with a mean of 725,000 years. What is the probability that it will remain dormant for the next 1000 years?

$$pdf(x) = a \cdot e^{-ax}$$

$$cdf(x) = 1 - e^{-ax}$$

$$a = \frac{1}{725,000}$$

So

$$cdf(1000) = 1 - \exp\left(\frac{-1000}{725,000}\right)$$

$$cdf(1000) = 0.0014$$

The odds are 725 : 1 against Yellowstone going off in the next 1000 years.

7) Assume

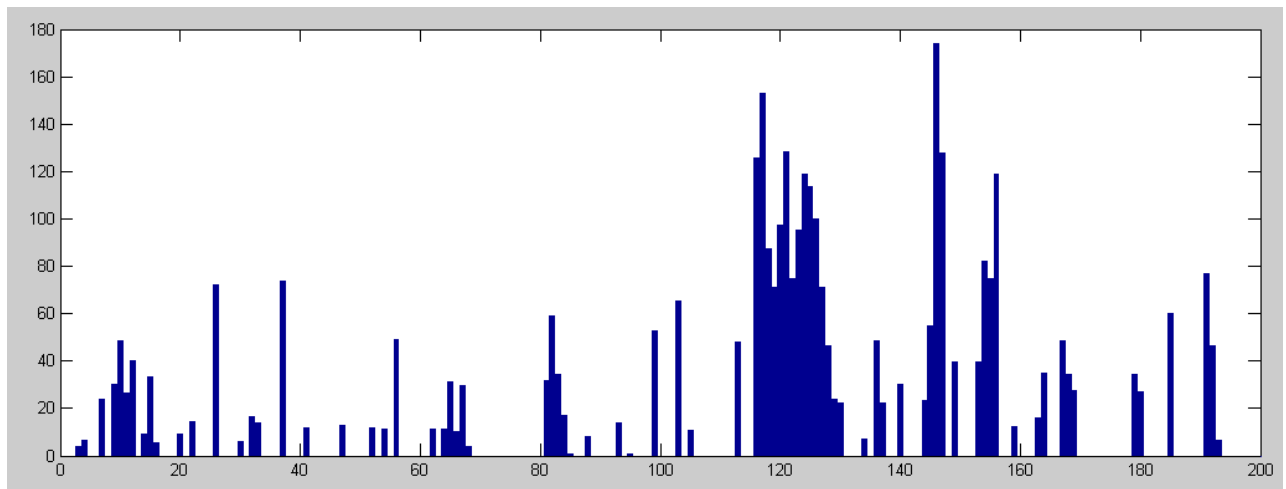
- Customers arrive at a restaurant with an exponential distribution with a mean of 80 seconds.
- Once a customer's order is taken, the time it takes to fill a customer's order is an exponential distribution with a mean of 30 seconds.

For 200 customers, determine

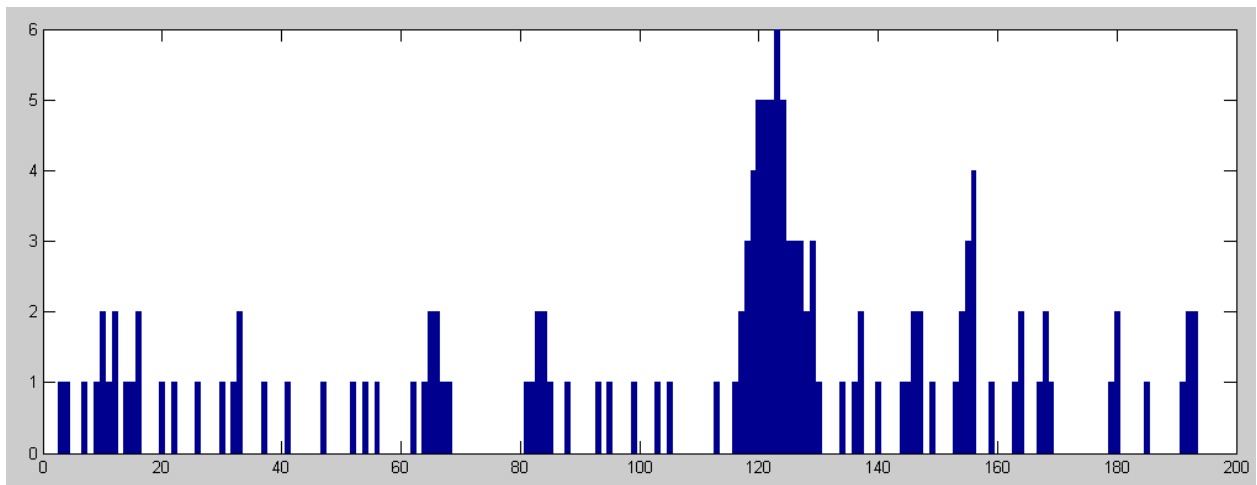
- The maximum queue size and
- The maximum wait time a customer sees.

Results (time in minutes)

---Problem #7 -----					
n	Tarr	Torder	Tdone	Queue	Wait (sec)
1.0000	0.7218	0.7218	1.9532	0	0
2.0000	1.0617	1.9532	2.1234	1.0000	53.4903
3.0000	6.2151	6.2151	6.9961	0	0
4.0000	6.7345	6.9961	7.3919	1.0000	15.6956
5.0000	7.6769	7.6769	8.1980	0	0
6.0000	8.3318	8.3318	8.7235	0	0
7.0000	9.9601	9.9601	10.0486	0	0
8.0000	11.3685	11.3685	11.3837	0	0
9.0000	12.5820	12.5820	12.9048	0	0
10.0000	13.4789	13.4789	13.7577	0	0
11.0000	13.6292	13.7577	14.2482	1.0000	7.7104
12.0000	13.6725	14.2482	14.7685	2.0000	34.5421
13.0000	15.9300	15.9300	16.2154	0	0
14.0000	16.4183	16.4183	16.7456	0	0
15.0000	16.8791	16.8791	16.9286	0	0
16.0000	17.0542	17.0542	17.7133	0	0
17.0000	17.1647	17.7133	18.5453	1.0000	32.9153
18.0000	19.0768	19.0768	19.3864	0	0
19.0000	21.7593	21.7593	22.5001	0	0
20.0000	23.5734	23.5734	24.1472	0	0
21.0000	26.2634	26.2634	27.1673	0	0
22.0000	28.0032	28.0032	28.5349	0	0
23.0000	30.1294	30.1294	31.4483	0	0
24.0000	30.3097	31.4483	32.3906	1.0000	68.3116
25.0000	30.6168	32.3906	32.4182	2.0000	106.4276
26.0000	32.0111	32.4182	33.8339	2.0000	24.4228
27.0000	32.1203	33.8339	33.9600	3.0000	102.8176
28.0000	33.9730	33.9730	34.2889	0	0
29.0000	37.4519	37.4519	37.5322	0	0
30.0000	37.8331	37.8331	38.7006	0	0



Customer Wait Times (seconds)



Queue Size

Matlab Code

```
Tarr = zeros(200,1);
Torder = zeros(200,1);
Tdone = zeros(200,1);
Queue = zeros(200,1);
Wait = zeros(200,1);

Tarr(1) = -80 * log(rand);
Torder(1) = Tarr(1);
Tdone(1) = Tarr(1) - 30 * log(rand);

n = 1
disp('---Problem #7 ----')
disp('      n      Tarr      Torder      Tdone      Queue      Wait (sec)')
disp([n,Tarr(n)/60, Torder(n)/60, Tdone(n)/60, Queue(n), Wait(n)])

for n=2:200
    a = -80 * log(rand);
    b = -30 * log(rand);
    Tarr(n) = Tarr(n-1) + a;
    Torder(n) = max(Tarr(n), Tdone(n-1));
    Tdone(n) = Torder(n) + b;
    Queue(n) = sum(Tarr(n) < Tdone(1:n-1));
    Wait(n) = Torder(n) - Tarr(n);
    disp([n,Tarr(n)/60, Torder(n)/60, Tdone(n)/60, Queue(n), Wait(n)])
    pause(0.1)
end
```