

ECE 341 - Homework #5

Geometric & Pascal Distributions

Geometric Distributions

In 5-card stud poker, the probability of getting 2-pair or better is $p = 0.076255$

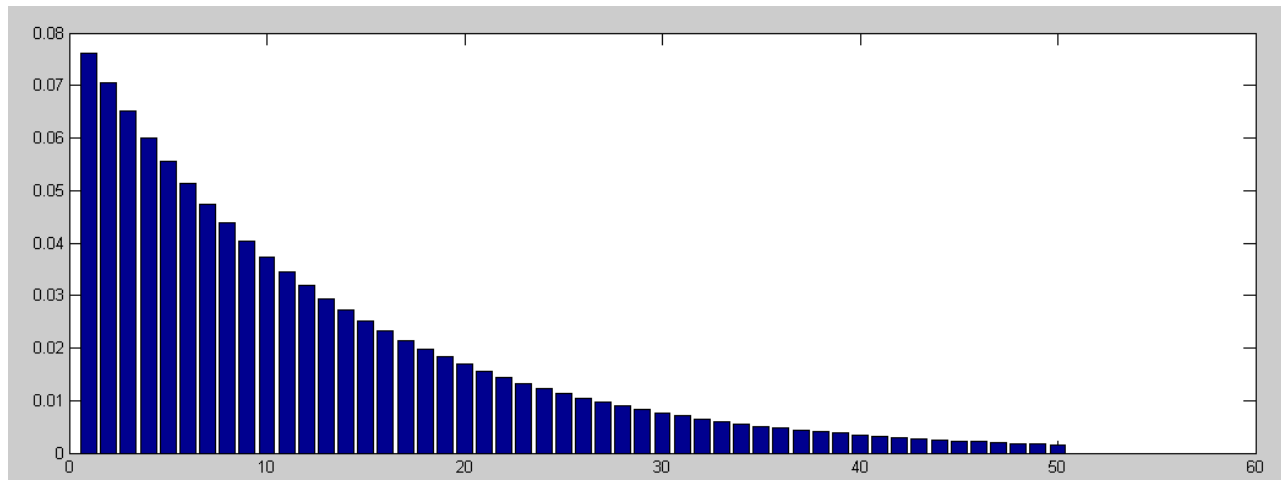
1) Determine the pdf for the number of hands you play if you keep playing until you get 2-pair or better.

This is a geometric distribution

$$f(n) = p \cdot (1 - p)^{n-1}$$

```
>> n = [1:50]';  
>> p = 0.076255;  
>> f = p * (1-p) .^(n-1);  
>> bar(n,f);  
>> [n,f]
```

hands	pdf
1.0000	0.0763
2.0000	0.0704
3.0000	0.0651
4.0000	0.0601
5.0000	0.0555
6.0000	0.0513
7.0000	0.0474
8.0000	0.0438
9.0000	0.0404
10.0000	0.0373



2) Determine the cdf for the number of hands you play if you keep playing until you get 2-pair or better.

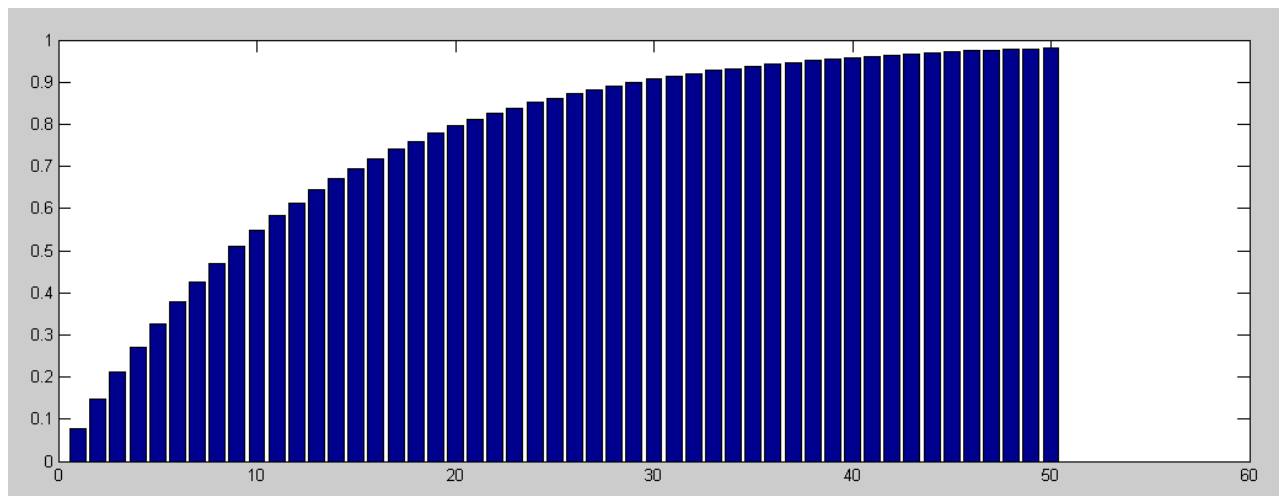
The cdf is the integral of the pdf. From the lecture notes

$$cdf(n) = 1 - (q)^n$$

In matlab

```
>> q = 1-p;  
>> cdf = 1 - q.^n;  
>> bar(n,cdf)
```

n	pdf	cdf
1.0000	0.0763	0.0763
2.0000	0.0704	0.1467
3.0000	0.0651	0.2118
4.0000	0.0601	0.2719
5.0000	0.0555	0.3274
6.0000	0.0513	0.3787
7.0000	0.0474	0.4261
8.0000	0.0438	0.4698
9.0000	0.0404	0.5103
10.0000	0.0373	0.5476
11.0000	0.0345	0.5821



3) What is the probability that you play 20 hands of poker and never get 2-pair or better?

```
>> cdf(20)
```

```
ans =    0.7953
```

There is a 79.53% chance that you'll get 2-pair or better after 20 hands of poker

- **There is a 20.47% chance of never getting 2-pair or better after 20 hands of poker**

Pascal Distribution

4) Suppose I play poker until

- I am dealt 2-pair or better ($p = 0.076255$). I then keep playing until
- I am dealt 3-of-a-kind or better ($p = 0.0287146$).

What is the pdf for the number of hands I'll end up playing?

- What is the probability that I will play exactly 50 hands ($k = 60$)?

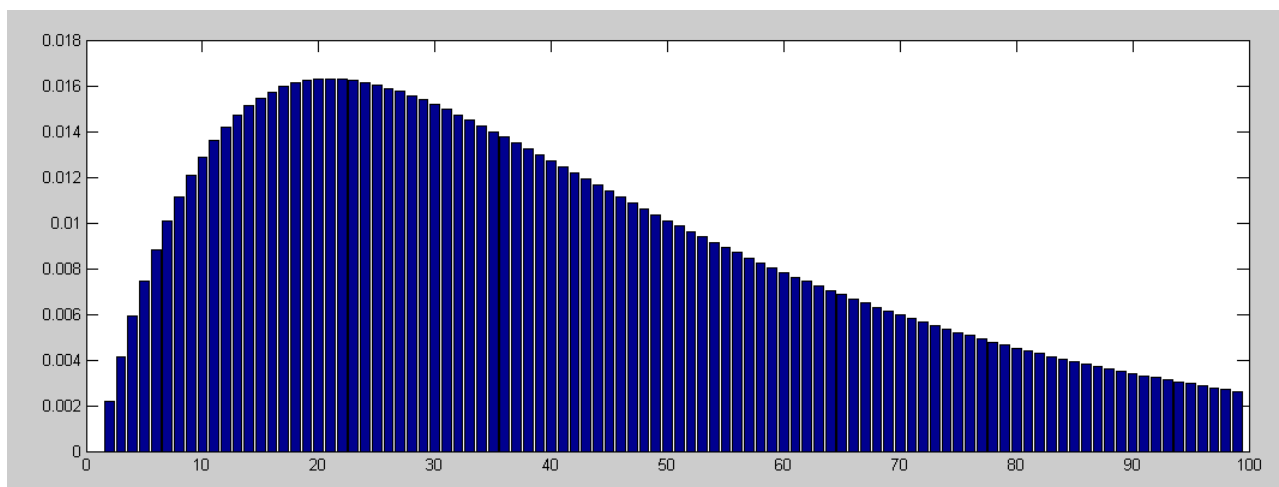
Let

- A be the number of hands dealt until I get 2-pair or better ($p_A = 0.076255$)
- B be the number of hands dealt until I get 3-of-a-kind or better ($p_B = 0.0287146$)

Solve using convolution.

```
pA = 0.076255;
pB = 0.0287146;
n = [0:99]';
A = pA * (1-pA).^(n-1) .* (n>0);
B = pB * (1-pB).^(n-1) .* (n>0);
AB = conv(A,B);
AB = AB(1:100);
[n,A,B,AB]
```

n	A	B	A**B
0	0	0	0
1.0000	0.0763	0.0287	0
2.0000	0.0704	0.0279	0.0022
3.0000	0.0651	0.0271	0.0041
4.0000	0.0601	0.0263	0.0059
5.0000	0.0555	0.0256	0.0075
6.0000	0.0513	0.0248	0.0088
7.0000	0.0474	0.0241	0.0101
8.0000	0.0438	0.0234	0.0111
9.0000	0.0404	0.0227	0.0121
10.0000	0.0373	0.0221	0.0129
:			
50.0000	0.0016	0.0069	0.0101
:			



5) Repeat problem #6 using z-transforms.

The z-transform for an geometric distribution is

$$f(n) = \left(\frac{p}{z-q} \right)$$

For A and B

$$AB = \left(\frac{0.076255}{z-0.923745} \right) \left(\frac{0.0287146}{z-0.9712854} \right)$$

Do a partial fraction expansion

$$AB = \left(\frac{-0.04605833}{z-0.923745} \right) + \left(\frac{0.04605833}{z-0.9712854} \right)$$

Multiply both sides by z

$$zAB = \left(\frac{-0.04605833z}{z-0.923745} \right) + \left(\frac{0.04605833z}{z-0.9712854} \right)$$

Take the inverse z-transform

$$zAB = \left(-0.04605833(0.923745)^k + 0.04605833(0.9712854)^k \right) u(k)$$

Divide by z

$$AB(k) = \left(-0.04605833(0.923745)^{k-1} + 0.04605833(0.9712854)^{k-1} \right) u(k-1)$$

Checking in Matlab

```
k = [0:99]';
ABk = 0.04605833 * ( (0.9712854).^ (k-1) - (0.923745).^ (k-1) ) .* (k>1);
[n, AB, ABk]
```

n	A**B	z-transform
0	0	0
1	0	0
2	0.002189631823000	0.002189631431532
3	0.004149418869392	0.004149418127549
4	0.005898693365920	0.005898692311336
5	0.007455261518653	0.007455260185782
6	0.008835521368312	0.008835519788675
7	0.010054571615787	0.010054569818205
8	0.011126312107644	0.011126310118454
9	0.012063536618790	0.012063534462040
10	0.012878018520828	0.012878016218463
:		
50	0.010103682015259	0.010103680208897
:		

Answers match what we got with convolution

6) Determine the cdf for the number of hands played using z-transforms.

$$cdf = \left(\frac{0.076255}{z-0.923745} \right) \left(\frac{0.0287146}{z-0.9712854} \right) \left(\frac{z}{z-1} \right)$$

Trick: Multiply by z^2 to make the numerator and denominator both 3rd order

$$z^2 \cdot cdf = \left(\frac{0.076255}{z-0.923745} \right) \left(\frac{0.0287146}{z-0.9712854} \right) \left(\frac{z}{z-1} \right) \cdot z^2$$

Pull out a z

$$z^2 \cdot cdf = \left(\left(\frac{0.076255}{z-0.923745} \right) \left(\frac{0.0287146}{z-0.9712854} \right) \left(\frac{z^2}{z-1} \right) \right) z$$

Do a partial fraction expansion

$$z^2 \cdot cdf = \left(\left(\frac{0.515399}{z-0.923745} \right) + \left(\frac{-1.51321}{z-0.9712854} \right) + \left(\frac{1}{z-1} \right) \right) z$$

Multiply through by z

$$z^2 \cdot cdf = \left(\frac{0.515399z}{z-0.923745} \right) + \left(\frac{-1.51321z}{z-0.9712854} \right) + \left(\frac{z}{z-1} \right)$$

Take the inverse z-transform

$$z^2 \cdot cdf = \left(0.515399(0.923745)^k - 1.51321(0.9712854)^k + 1 \right) u(k)$$

Divide by z^2 (delay two)

$$cdf = \left(0.515399(0.923745)^{k-2} - 1.51321(0.9712854)^{k-2} + 1 \right) u(k-2)$$

```
>> k = [0:99]';
>> cdf = (1 + 0.515399*(0.923745).^(k-2) - 1.51321*(0.9712854).^(k-2)).*(k>=2);
>> [k, cdf]
```

k	cdf (k)
0	0
1	0
2	0.0021890000000000
3	0.006338469121000
4	0.012237208847096
5	0.019692513132943
6	0.028528073951110
7	0.038582681953987
8	0.049709027621074
9	0.061772595188802
10	0.074650642248602
:	:
50	0.637723884153239

