

ECE 341 - Homework #4

Binomial and Uniform Distributions.

Binomial Distribution

1) Poker: Odds of pair or high-card is $p=0.923745$. What are the odds of getting nothing better than a pair in 10 hands of poker?

$$odds = \binom{n}{m} p^m (1-p)^{n-m}$$

$$odds = \binom{10}{10} (0.923745)^{10} (0.076255)^0$$

$$odds = 0.452398$$

There is a 45.2% chance you'll see nothing better than a pair in ten hands of poker

In Matlab

```
>> p = 0.923745;
>> Odds = nCm(10,10) * p^10 * (1-p)^0

Odds =    0.4524
```

This uses a function nCm

```
function [X] = nCm(n,m)
    X = factorial(n) / factorial(m) / factorial(n-m);
end
```

2) Odds of a flush are $p=0.001965$. What are the odds of getting 3 or more flushes in 10 hands of poker?

$$odds = \binom{n}{m} p^m (1-p)^{n-m}$$

Solve this as odds of 0, 1, or 2 flushes in 10 hands

```
>> p = 0.001965;
>> n = 10;
>> m = 0;
>> % Zero flushes in ten hands

>> X0 = nCm(10,0) * p^0 * (1-p)^10

X0 =      0.9805

>> % One flush in ten hands
>> X1 = nCm(10,1) * p^1 * (1-p)^9

X1 =      0.0193

>> Two flushes in ten hands
>> X2 = nCm(10,2) * p^2 * (1-p)^8

X2 =      1.7104e-004

>> % Odds of 0, 1, or 2 flushes in ten hands
>> format long
>> X012 = X0 + X1 + X2

X012 =      0.999999098871677

>> Odds of 3 or more flushes in ten hands
>> X3_10 = 1 - X012

X3_10 =      9.011283234050538e-007
```

3) Odds of a flush are $p=0.001965$. What are the odds of 5 flushes in 1024 hands of poker?

In Matlab

```
>> p = 0.001965;  
>> n = 1024;  
>> m = 5;  
>> x = nCm(n,m) * p^m * (1-p)^(n-m)  
  
x = NaN
```

1024! is out of range.

```
>> nCm(1024,5)  
  
ans = NaN
```

To solve this, use the Poisson approximation for a binomial distribution

$$\lambda = np$$

$$\lambda = 2.01216$$

Then

$$f(x) \approx \frac{1}{x!} \cdot \lambda^x \cdot e^{-\lambda}$$

In Matlab

```
>> lambda = n*p  
  
lambda = 2.0121600000000000  
  
>> f = (1 / factorial(5)) * lambda^5 * exp(-lambda)  
  
f = 0.036750337007643
```

There is about a 3.67% chance of getting five flushes in 1024 hands of poker

Mendel's Experiment

4) Mendel came up with the idea of dominant and recessive genes. If you cross two hybrid plants (Gg x Gg)

- 75% of the offspring should display the dominant trait (success: GG, Gg, or gG) while
- 25% of the offspring should display the recessive trait (failure: gg).

Assume you run this experiment of crossing two hybrid plants, producing 40 offspring. What are the odds of getting *exactly* 30 offspring which display the dominant trait?

This is a binomial distribution

$$odds = \binom{n}{m} p^m (1-p)^{n-m}$$

$$odds = \binom{40}{30} (0.75)^{30} (0.25)^{10}$$

In Matlab

```
>> Odds = nCm(40,30) * (0.75)^30 * (0.25)^10
```

```
Odds = 0.144364346356257
```

There is a 14.43% chance that you'll get exactly 30 dominant and 10 recessive plants. This *could* happen. If it happens experiment after experiment, you know that the data is being fudged.

Uniform Distribution

5) Let player A roll ten dice:

- $A = 3d6 + 3d8 + 4d12$

Determining the odds that

- $A = 60$
- $A > 60$

Use Matlab and convolution

```
d6 = [0,1,1,1,1,1,1] / 6;  
d8 = [0,1,1,1,1,1,1,1,1] / 8;  
d12 = [0,1,1,1,1,1,1,1,1,1,1,1,1,1,1] / 12;  
  
d6x2 = conv(d6,d6);  
d6x3 = conv(d6x2,d6);  
  
d8x2 = conv(d8,d8);  
d8x3 = conv(d8x2,d8);  
  
d12x2 = conv(d12,d12);  
d12x4 = conv(d12x2, d12x2);  
  
d6x3_d8x3 = conv(d6x3, d8x3);  
  
A = conv(d6x3_d8x3, d12x4);  
  
npt = length(A);  
disp('p(A == 60)')  
disp(A(61))  
disp('p(A > 60)')  
disp(sum(A(62:npt)))
```

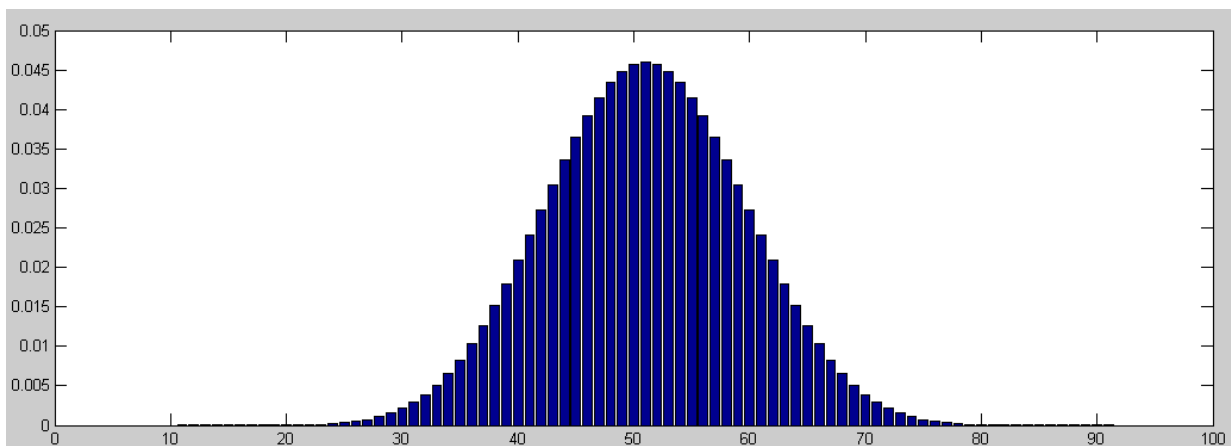
Results

- note: $A(1)$ is $p(\text{sum equals zero})$
- (index is off by one)

```
p(A == 60)  
0.0240
```

```
p(A > 60)  
0.1105
```

Just for fun, the pdf if A is (bar(A))



6) Let player B roll ten dice

- $B = 10d10$

Determine the odds that

- $B = 60$
- $B > 60$

Matlab Code

```
d10 = [0,1,1,1,1,1,1,1,1,1,1] / 10;  
  
d10x2 = conv(d10, d10);  
d10x4 = conv(d10x2, d10x2);  
d10x8 = conv(d10x4, d10x4);  
d10x10 = conv(d10x8, d10x2);  
  
B = d10x10;  
  
npt = length(B);  
disp('p(B == 60)')  
disp(B(61))  
disp('p(B > 60)')  
disp(sum(B(62:npt)))
```

Results

p(B == 60)
0.0375

p(B > 60)
0.2751

Just for fun, the pdf for B is

`bar(B)`

