ECE 341 - Homework #12

Markov Chains & Absorbing States, t-Test with a Single Population. Summer 2023

Markov Chains & Absorbing States

Assume player A and B are playing a match. For each game

- A has a 50% chance of winning (+1 point)
- There is a 15% chance of a tie (+0 points), and
- A has a 35% chance of losing (-1 point).

The match is over once you reach +3 points (A wins) or -3 points (B wins).

When the match starts, both players start out at 0 points (no odds).

1) Using matrix multiplication, determine the probability

- That A wins after 10 games
- That A wins the series (infinite number of games)

The state-transistion matrix is

$$\begin{bmatrix} p3(k+1) \\ p2(k+1) \\ p1(k+1) \\ e(k+1) \\ m1(k+1) \\ m2(k+1) \\ m3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.15 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.35 & 0.15 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.35 & 0.15 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.35 & 0.15 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.35 & 0.15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.35 & 1 \end{bmatrix} \begin{bmatrix} p3(k) \\ p2(k) \\ p1(k) \\ e(k) \\ m1(k) \\ m2(k) \\ m3(k) \end{bmatrix}$$

In Matlab

```
>> a1 = [1,0.5,0,0,0,0,0];
>> a2 = [0,0.15,0.5,0,0,0,0];
>> a3 = [0,0.35,0.15,0.5,0,0,0];
>> a4 = [0,0,0.35,0.15,0.5,0,0];
>> a5 = [0,0,0,0.35,0.15,0.5,0];
>> a6 = [0,0,0,0,0.35,0.15,0];
>> a7 = [0,0,0,0,0,0.35,1];

>> A = [a1;a2;a3;a4;a5;a6;a7]

    1.0000    0.5000         0         0         0         0         0
         0    0.1500    0.5000         0         0         0         0
         0    0.3500    0.1500    0.5000         0         0         0
         0         0    0.3500    0.1500    0.5000         0         0
         0         0         0    0.3500    0.1500    0.5000         0
         0         0         0         0    0.3500    0.1500         0
         0         0         0         0         0    0.3500    1.0000
```

```
>> X0 = [0;0;0;1;0;0;0]

     0
     0
     0
     1
     0
     0
     0

>> A^10 * X0

    0.4958     Probability that A wins after 10 games
    0.0633
    0.0890
    0.0886
    0.0623
    0.0310
    0.1701     Probability that B wins after 10 games

>> A^100 * X0

    0.7446     Probability that A wins after 100 games
    0.0000
    0.0000
    0.0000
    0.0000
    0.0000
    0.2554     Probability that B wins after 100 games
```

2) Using z-transforms, determine the probabilty

- That A wins after 10 games
- That A wins the series (infinite number of games)

```
>> C = [1,0,0,0,0,0,0];
>> D = 0;
>> G = ss(A, X0, C, D, 1);
>> zpk(G)

                0.125 (z+0.2683) (z-0.5683)
----------------------------------------------------------
(z-1) (z-0.8746) (z-0.5683) (z+0.5746) (z+0.2683) (z-0.15)
```

Multiply by z and do a partial fraction expansion

$$A(z) = \left(\left(\frac{0.7446}{z-1} \right) + \left(\frac{-0.8301}{z-0.8746} \right) + \left(\frac{0}{z-0.5683} \right) + \left(\frac{0.0434}{z+0.5746} \right) + \left(\frac{0}{z+0.2683} \right) + \left(\frac{0.0420}{z-0.15} \right) \right) z$$

$$A(z) = \left(\frac{0.7446z}{z-1} \right) + \left(\frac{-0.8301z}{z-0.8746} \right) + \left(\frac{0z}{z-0.5683} \right) + \left(\frac{0.0434z}{z+0.5746} \right) + \left(\frac{0z}{z+0.2683} \right) + \left(\frac{0.0420z}{z-0.15} \right)$$

Take the inverse z-transform

$$a(k) = 0.7446 - 0.8301(0.8746)^k + 0.0434(-0.5746)^k + 0.0420(0.15)^k$$

At k = 10:

$$a(10) = 0.4958$$

As k goes to infinity

$$a(100) = 0.7446$$

These match the results from problem #1

Test of a Single Population: 2-pair in 5-Card Stud

The calculated odds of drawing 2-pair in 5-card stud are 21.03 : 1 odds against

- You should draw 2-pair 4755.11 times in 100,000 hands

3) Run a Monte Carlo simulation to determine the odds of being dealt 2-pair in 5-card stud

- Each simulation goes through 100,000 hands (# of hands that are 2-pair with 100,000 hands of poker)
- Run the simulation 3 times
- data = { x1, x2, x3 }

From this, determine the 90% confidence interval for the actual odds of getting a 2-pair with 5-card stud.

- if p = 4.7539% is in this interval, you cannot reject this answer with a probability of 90%

4 of a kind	Full House	3 of a kind	2 Pair	Pair
26	165	2074	**4661**	42221
23	164	2149	**4783**	42272
20	149	2096	**4653**	42386

```
>> N = [4661, 4783, 4653]

N =        4661        4783        4653

>> x = mean(N)

x =        4699

>> s = std(N) / sqrt(3)

s =   42.0634
```

Note: Since we're looking at a population (the population's true mean), divide the standard deviation by the square root of the sample size

With 2 degrees of freedom the t-score that corresponds to 5% tails is t = 2.9000

```
>> LOW = x - 2.92 * s

LOW =  4.5762e+003

>> HIGH = x + 2.92 * s

HIGH =  4.8218e+003
```

The 90% confidence interval for the actual odds of drawing 2-pair is

4576.2 < hands < 4821.8 **p = 0.9**

Note: 4755 is in this range

4) Repeat problem #1 with 10 simulations of 100,000 hands.

- With 10 simulations, what is the 90% confidence interval for the actual odds of being dealt 2-pair?

4 of a kind,	Full House,	3 of a kind,	**2 Pair**,	Pair
26	145	2141	**4773**	42363
17	150	2172	**4781**	42122
20	167	2110	**4705**	42389
23	149	2155	**4857**	42545
17	155	2072	**4810**	41992
29	142	2108	**4811**	42195
28	151	2104	**4809**	42269
16	144	2081	**4760**	42347
37	127	2149	**4752**	42524
14	146	2059	**4728**	42092

```
>> N = [4773,4781,4705,4857,4810,4811,4809,4760,4752,4728]
>> x = mean(N)

x =  4.7786e+003

>> s = std(N) / sqrt(10)

s =   14.2105

>> LOW = x - 1.8331*s

LOW =  4.7526e+003

>> HIGH = x + 1.8331*s

HIGH =  4.8046e+003
```

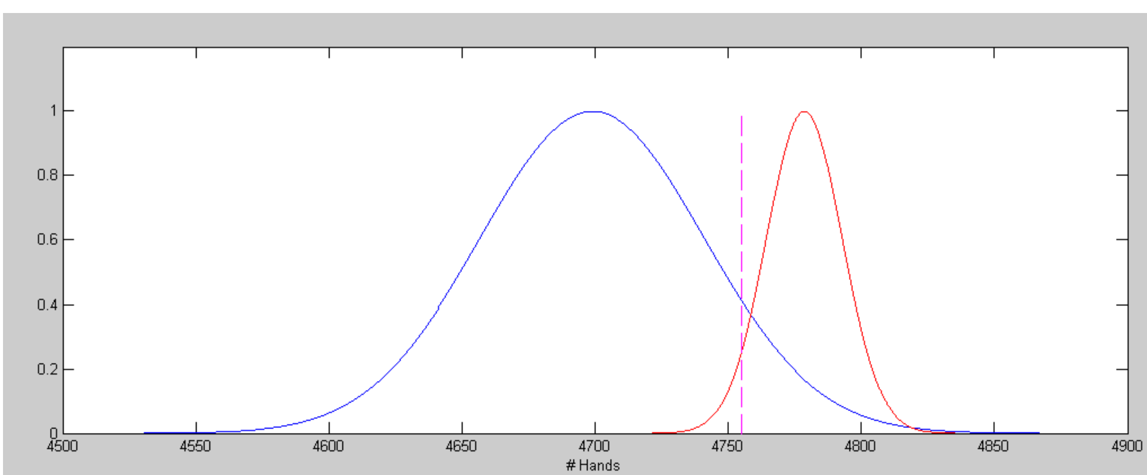
With a sample size of ten, the 90% confidence interval for the number 2-pairs you draw in 100,000 hands is

4752.6 < hands < 4804.6   p = 0.9

Note that the actual odds are within this rance (hands = 4755.11)

Also note that the range is tighter than before

- As the sample size goes to infinity, you eventually zero in on the actual odds of being dealt 2-pair



pdf for the number of 2-pair with 100,000 hands dealt
Blue: Sample Size = 3,   Red: Sample Size = 10

Let Y be the sum of rolling two 4-sided dice (2d4) plus three 6-sided dice (3d6) plus four 8-sided dice.

Y = 2d4 + 3d6 + 4d8

```
d4 = ceil( 4*rand(2,1) );
d6 = ceil( 6*rand(3,1) );
d8 = ceil( 8*rand(4,1) );
Y = sum(d4) + sum(d6) + sum(d8);
```

Monte-Carlo

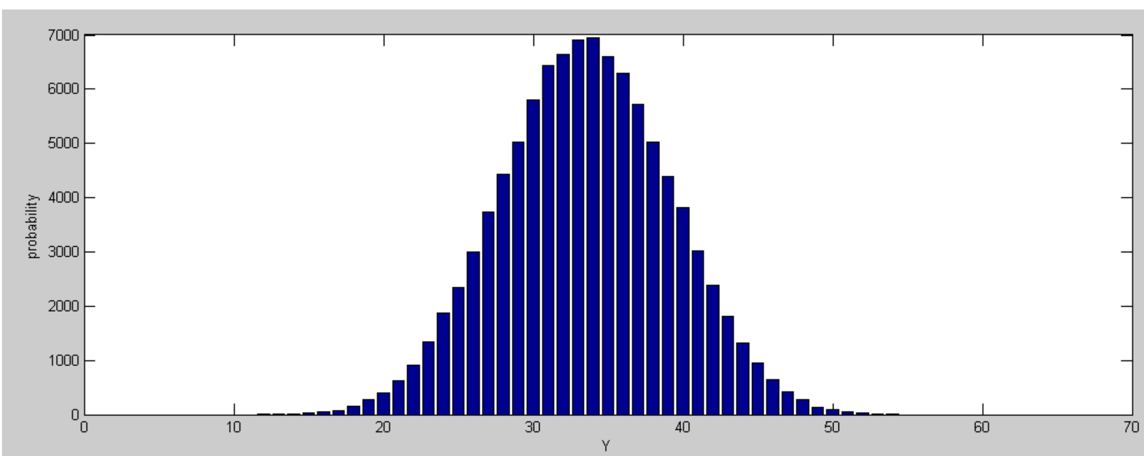
Problem 5: Use a Monte-Carlo simulation to determine

- The probability that Y > 39.5
- The number, a, such that 5% of the rolls are less than a
- The number, b, such that 5% of the rolls are more then b

note: The 90% confidence interval for Y is then

a < Y < b with p = 90%

```
RESULT = zeros(1,60);
for i=1:1e5
   d4 = ceil( 4*rand(2,1) );
   d6 = ceil( 6*rand(3,1) );
   d8 = ceil( 8*rand(4,1) );
   Y = sum(d4) + sum(d6) + sum(d8);
   RESULT(Y) = RESULT(Y) + 1;
   end
bar(RESULT) / 1e5
xlabel('Y');
ylabel('probability')
```



The probability of rolling 40 or more is the sum of the bars from 40+

```
>> sum(RESULT(40:60))/1e5

ans =     0.1494
```

There is a 14.94% chance of rolling 40 or more.

The right tail with an area of 5% is (trial and error)

```
>> sum(RESULT(42:60))/1e5

ans =    0.0811
```

`>> sum(RESULT(43:60))/1e5`

`ans =    0.0572`

```
>> sum(RESULT(44:60))/1e5

ans =    0.0391
```

There is a 5% chance of rolling 43 or more

The left tail with an area of 5% is (trial and error)

```
>> sum(RESULT(1:23))/1e5

ans =    0.0386
```

`>> sum(RESULT(1:24))/1e5`

`ans =    0.0573`

```
>> sum(RESULT(1:25))/1e5

ans =    0.0807
```

There is a 5% chance of rolling 24 or less

The 90% confidence interval (90% of the time you'll roll values in this range)

$24 < \text{roll} < 43$ $\qquad$ $p = 0.9$

or, since this is a discrete function

$24.5 < \text{roll} < 42.5$ $\qquad$ $p = 0.9$

t-Test with a Single Population

Problem 6: Take four measurements of Y

```
DATA = [];
for i=1:4
   d4 = ceil( 4*rand(2,1) );
   d6 = ceil( 6*rand(3,1) );
   d8 = ceil( 8*rand(4,1) );
   Y = sum(d4) + sum(d6) + sum(d8);
   DATA = [DATA, Y];
   end
```

From this data, determine

- The mean and standard deviation
- The probability that Y > 39.5, and
- The 90% confidence interval for Y

```
DATA =    29    38    37    36

>> x = mean(DATA)

x =    35

>> s = std(DATA)

s =    4.0825
```

p(Y > 39.5): Find the t-score

```
>> t = (39.5 - x) / s

t =    1.1023
```

Convert to a probability using a t-table (or StatTrek)

p = 17.541%	t-test, sample size = 4
p = 14.94%	Monte-Carlo, sample size = 100,000

```
>> LOW = x - 2.3534*s

LOW =   25.3923

>> HIGH = x + 2.3524*s

HIGH =   44.6036
```

The 90% confidence interval is

25.39 < roll < 44.60	**t-test, sample size = 4**
24.5 < roll < 42.5	**Monte-Carlo, sample size = 100,000**

Problem 7: Take ten measurements of Y

From this data, determine

- The mean and standard deviation
- The probability that Y > 39.5, and
- The 90% confidence interval for Y

```
DATA = [];
for i=1:10
   d4 = ceil( 4*rand(2,1) );
   d6 = ceil( 6*rand(3,1) );
   d8 = ceil( 8*rand(4,1) );
   Y = sum(d4) + sum(d6) + sum(d8);
   DATA = [DATA, Y];
   end

DATA =    44    38    24    36    30    29    28    31    31    42

>> x = mean(DATA)

x =   33.3000

>> s = std(DATA)

s =    6.4472
```

p(y > 39.5): Determine the t-score

```
>> t = (39.5 - x) / s

t =    0.9617
```

Convert to a probability using a t-table or StatTrek (9 dof)

p = 0.1807

The odds of rolling more than 39.5 are

18.07%	**t-test, sample size = 10**
14.94%	**Monte-Carlo, sample size = 100,000**

The 90% confidence interval. With 9 degrees of freedom, 5% tails corresponds to a t-score of 1.8331

```
>> LOW = x - 1.8331*s

LOW =   21.4816

>> HIGH = x + 1.8331*s

HIGH =   45.1184
```

The 90% confidence interval is

21.48 < roll < 45.11	**t-test, sample size = 10**
24.5 < roll < 42.5	**Monte-Carlo, sample size = 100,000**