ECE 341 - Test #2

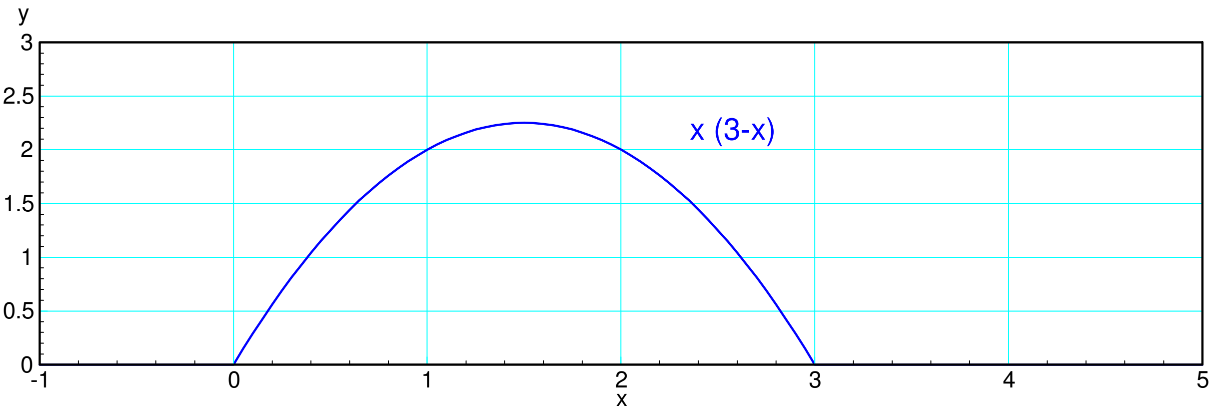
Continuous Probability

Open-Book, Open Notes. Calculators, Matlab, Tarot cards. Chegg and other people **not** allowed

1) Continuous PDF

Let

$$y = \begin{cases} \alpha \cdot x(3-x) & 0 < x < 3 \\ 0 & elsewhere \end{cases}$$



a) Determine the scalar, α, so that this is a valid pdf (i.e. the total area = 1.0000)

$$area = \alpha \int_0^3 (3x - x^2)dx = 1$$

$$\alpha \cdot \left(\tfrac{3}{2}x^2 - \tfrac{1}{3}x^3\right)_0^3 = 1$$

$$\alpha\left(\tfrac{27}{2} - \tfrac{27}{3}\right) = 1$$

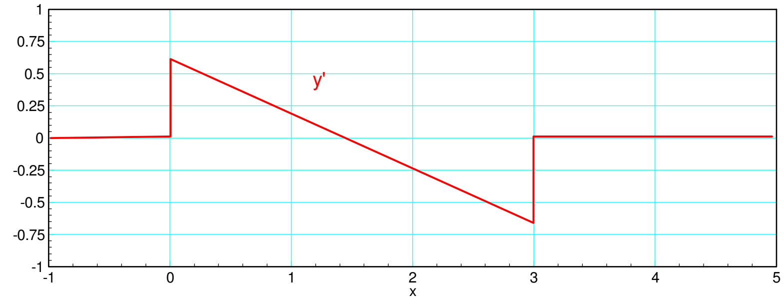
$$\alpha = \tfrac{2}{9}$$

b) Determine the moment generating function (i.e. LaPlace transform)

$$y = \tfrac{2}{9}(3x - x^2)$$

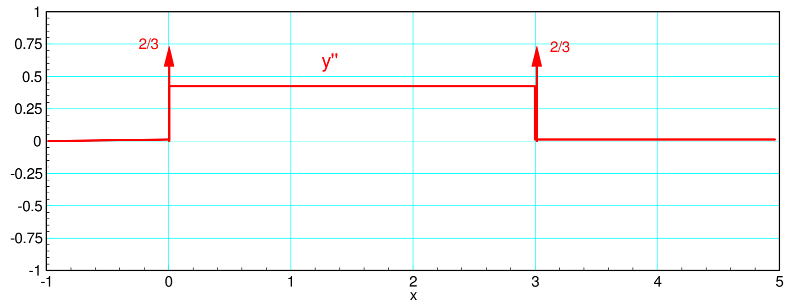
One derivative:

$$y' = \tfrac{2}{9}(3 - 2x)$$



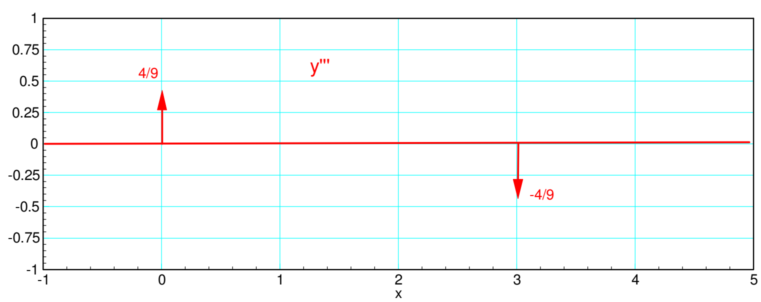
Two derivatives

$$y'' = \tfrac{2}{9}(3\delta(x) - 2 + 3\delta(x-3))$$



Three derivatives

$$y''' = \tfrac{2}{9}(-2\delta(x) + 2\delta(x-3))$$



The LaPlace transforms is then

$$Y(s) = \left(\frac{2}{3s^2}\right)(1 + e^{-3s}) + \left(\frac{4}{9s^3}\right)(-1 + e^{-3s})$$

2) Uniform Distribuitions

Let A and B be continuous uniform distributions

- A = uniform over the interval of (0, 13)
- B = uniform over the interval of (0, m) where x is your birth month (1..12),
- X = A + B

Use moment generating functions to determine the pdf for X (i.e. LaPlace Transforms)

$$A = \left(\frac{1}{13s}\right)(1 - e^{-13s})$$

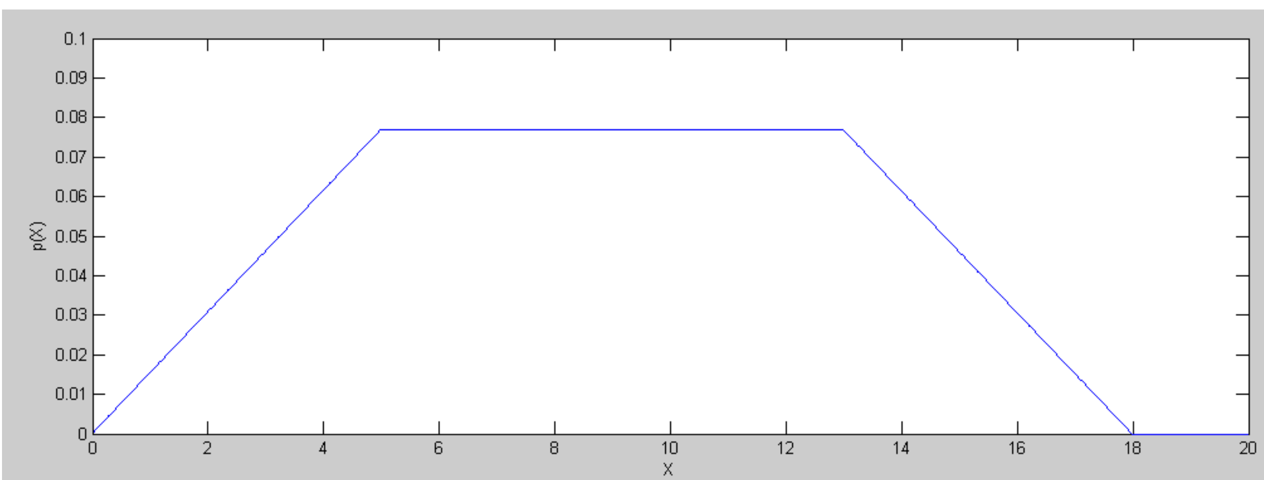
$$B = \left(\frac{1}{5s}\right)(1 - e^{-5s})$$

$$X = AB = \left(\frac{1}{13s}\right)(1 - e^{-13s}) \cdot \left(\frac{1}{5s}\right)(1 - e^{-5s})$$

$$X = \left(\frac{1}{65s^2}\right)(1 - e^{-5s} - e^{-13s} + e^{-18s})$$

Taking the inverse LaPlace transform

$$x(t) = \left(\frac{1}{65}\right)(tu(t) - (t-5)u(t-5) - (t-13)u(t-13) + (t-18)u(t-18))$$



3) Geometric & Gamma PDF

Let A, B, and C be continuous exponential distributions:

- A has a mean of 13
- B has a mean of m (m is your birth month (1..12)), and
- C has a mean of d (d is your birth date (1..31))

(note: if you have a repeated root, add one to m or d)

Determine the pdf of Y = A + B + C using moment generating functions (LaPlace transforms)

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$$A(s) = \left(\frac{1/13}{s+1/13}\right)$$

$$B(s) = \left(\frac{1/5}{s+1/5}\right)$$

$$C(s) = \left(\frac{1/14}{s+1/14}\right)$$

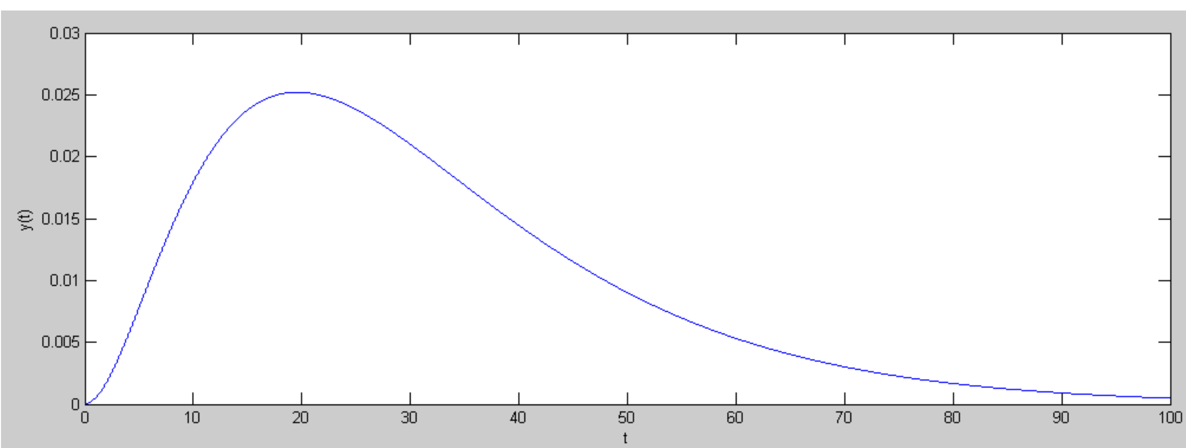
$$Y = ABC = \left(\frac{\frac{1}{13}}{s+\frac{1}{13}}\right)\left(\frac{\frac{1}{5}}{s+\frac{1}{5}}\right)\left(\frac{\frac{1}{14}}{s+\frac{1}{14}}\right)$$

Using partial fractions

$$Y = \left(\frac{-1.6250}{s+\frac{1}{13}}\right) + \left(\frac{0.0694}{s+\frac{1}{5}}\right) + \left(\frac{1.5556}{s+\frac{1}{14}}\right)$$

Taking the inverse LaPlace transform gives the pdf

$$y = -1.6250e^{-t/13} + 0.0694e^{-t/5} + 1.5556e^{-t/14} \qquad \text{for } t > 0$$



4) Central Limit Theorem

Let A, B, and C be continuous uniform distributions

- A = uniform over the interval of (0, 5)
- B = uniform over the interval of (0, m) where m is your birth month (1..12),
- C = uniform over the interval of (0, d) where d is your birth date (1..31), and
- Y = A + B + C

a) Find the mean and standard deviation of Y

	range	mean	variance
A	(0,5)	2.5	25/12
B	(0,5)	2.5	25/12
C	(0,14)	7.0	196/12
Y = A+B+C		12.0	246/12

Y has

- a mean of 12.00
- a variance of 20.50
- a standard deviation of 4.5277

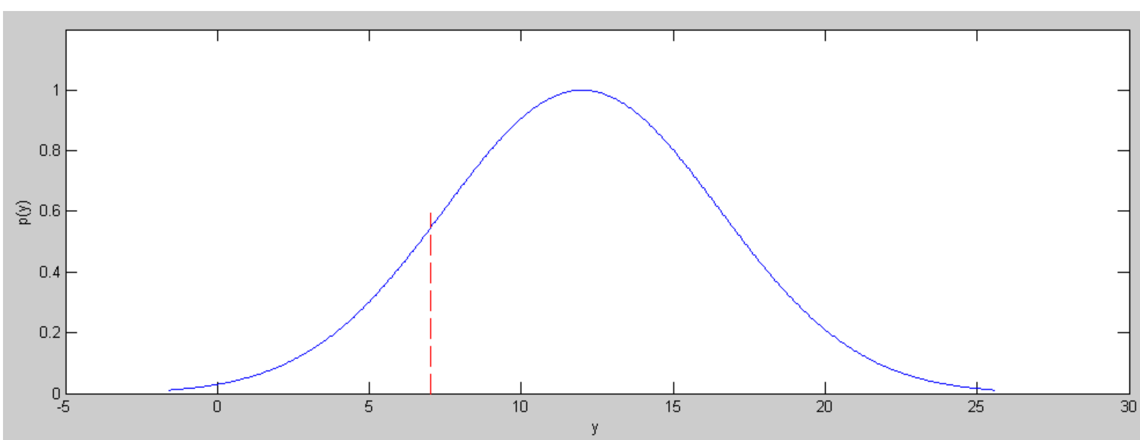
b) Use a normal approximation to Y to determine the

- z-score corresponding to Y=7 and
- The probability that Y > 7

$$z = \left(\frac{12-7}{4.5277}\right) = 1.1043$$

From StatTrek, this corresponds to a probability of 0.865

There is an 86.5% chance that Y will be more than 7



normalized pdf for y: area to the right of 7 is 0.865

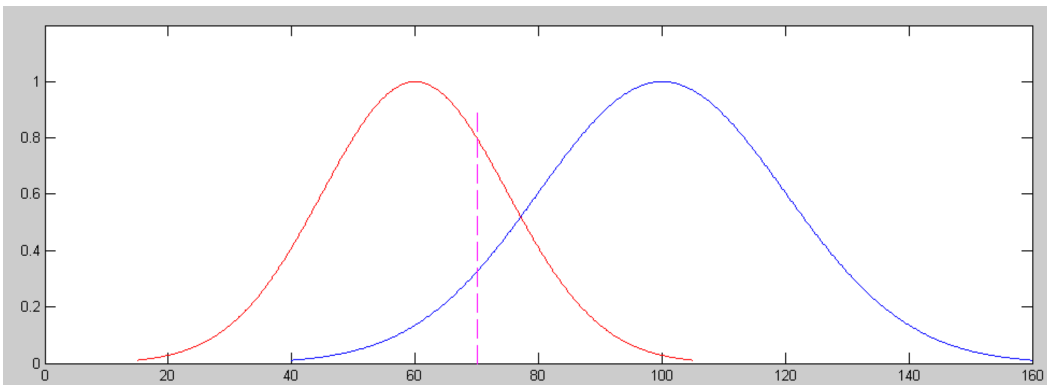
5) Testing with Normal pdf

x is selected at random from population A or B. Assume A and B have normal distributions:

	mean	standard deviation
A (negative)	60	15
B (positive)	100	20

A threshold is used to classify x:

- If x < 70, it is assigned to population A
- If x > 70, it is assigned to population B.



Normalized pdf for A (red) and B (blue)

a) What is the probability of a false positive?

- x is from population A but is assigned to population B
- (area of the right tail for the red curve above)

The z-score is

$$z = \left(\frac{70-60}{15}\right) = 0.6667$$

From StatTrek, this corresponds to a probability of 0.253

There is a 25.3% chance of a false positive

b) What is the probabilty of a false negative?

- x is from population B but is assigned to population A
- (area of the left tail for the blue curve aboce)

The z-score is

$$z = \left(\frac{100-70}{20}\right) = 1.500$$

From StatTrek, this corresponds to a probability of 0.067

There is a 6.7% chance of a false negative