ECE 341 - Test #1

Combinations, Permitations, and Discrete Probability

Open-Book, Open Notes. Calculators & Tarot cards allowed. Chegg or other people *not* allowed.

1. Enumeration (dice)

Let X be the sum of two 6-sided dice. Determine the probability that X is divisible by 3 using enumeration.

		Die #1					
		1	2	3	4	5	6
	1	2	**3**	4	5	**6**	7
Die #2	2	**3**	4	5	**6**	7	8
	3	4	5	**6**	7	8	**9**
	4	5	**6**	7	8	**9**	10
	5	**6**	7	8	**9**	10	11
	6	7	8	**9**	10	11	**12**

Twelve results are divisible by 3.

The odds of rolling a number that's divisible by three is

$$p = \left(\frac{12}{36}\right) = \left(\frac{1}{3}\right)$$

2. Combinations and Permutations (cards)

In 8-card stud,

- 3 cards are placed face up in the middle, and
- Each player is dealt 5 cards.

Each player can then make the best hand they can with these 8 cards.

a) How many hands are possible in 8-card stud?

- *How many ways can you deal 8 cards from a 52-card deck. Order doesn't matter.*

$$N = \binom{52}{8} = 752,538,150$$

b) Determine the probabililty of having 2-pair in 8-card stud.

- *Hand = (aa bb cdef) or*
- *Hand = (aa bb cc de) or*
- *Hand = (aa bb cc dd)*

where each letter is a different value.

Hand = aa bb cdef

(13 cards choose 2 for a&b)(4 a's in deck, choose 2)(4 b's choose 2)(11 choose 4 for cdef)(4 c's choose 1)...

$$M = \binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{4}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}$$

$$M_1 = 237,219,840$$

Hand = aa bb cc de

(13 choose 3 for abc)(4 a's choose 2)(4 b's choose 2)(4 c's choose 2)(10 values choose 2 for de)(4c1)(4c1)

$$M = \binom{13}{3}\binom{4}{2}\binom{4}{2}\binom{4}{2}\binom{10}{2}\binom{4}{1}\binom{4}{1}$$

$$M_2 = 44,478,720$$

Hand = aa bb cc dd

(13 choose 4 for abcd)(4 choose 2)(4 choose 2)(4 choose 2)(4c2)

$$M = \binom{13}{4}\binom{4}{2}\binom{4}{2}\binom{4}{2}\binom{4}{2}$$

$$M_3 = 926,640$$

p(2-pair) is

$$p = \left(\frac{M_1+M_2+M_3}{\binom{52}{8}} \right) = 0.37556$$

Check: Running a Monte Carlo simulation with 100,000 hands results in 37,786 two-pair hands (p = 0.37786)

3. Binomial Distribution

Let

M be your birth month (1..12) plus 2

Determine the probability of rolling M ones when rolling sixteen 5-sided dice (p = 1/5)

M birth month plus 2 (4..15)	probability of M ones with 16 die rolls p = 1/5
5+2 = 7	$p = \binom{16}{7}\left(\frac{1}{5}\right)^7\left(\frac{4}{5}\right)^9 = 0.01965$

p(7 ones in 16 rolls) = $\binom{16}{7}\left(\frac{1}{5}\right)^7\left(\frac{4}{5}\right)^9$

p = 0.01965

4. Convolution

Determine by hand (i.e. show your work - Matlab doesn't count) the product of the following polynomials using convolution.

$$Y = (2 + Mx + Dx^2)(3 + 4x)$$

where

- M is your birth month (1..12) and
- D is your birth date (1..31)

M birth month (1..12)	D birth date (1..31)	Y(x)
5	**14**	

k = -1	k=0	k = 1	k = 2	k = 3	result
	2	5x	$14x^2$		6
4x	3				
	6				

k = -1	k=0	k = 1	k = 2	k = 3	result
-	2	5x	$14x^2$	-	$23x$
-	4x	3	-	-	
-	8x	15x	-	-	

k = -1	k=0	k = 1	k = 2	k = 3	result
-	2	5x	$14x^2$	-	$62x^2$
-	-	4x	3	-	
		$20x^2$	$42x^2$	-	

k = -1	k=0	k = 1	k = 2	k = 3	result
-	2	5x	$14x^2$	-	$56x^3$
-	-	-	4x	3	
-	-	-	$56x^3$	-	

Result:

$$6 + 23x + 62x^2 + 56x^3$$

5. Geometric & z-Transforms

Let

- X be the number of rolls of an 8-sided die until you get a one with the following moment-generating function:

  $$X = \left(\frac{1/8}{z-7/8}\right)$$

- Y be the number of rolls of an 4-sided die until you get a one with the following moment-generating function:

  $$Y = \left(\frac{1/4}{z-3/4}\right)$$

Determine the pdf for W = X + Y using z-transforms

(the number of times you have to roll an 8 sided die until you get a 1, then roll a 4 sided die until you get a 1)

$$W = \left(\frac{1/8}{z-7/8}\right)\left(\frac{1/4}{z-3/4}\right)$$

$$z^2 W = \left(\frac{z/32}{(z-7/8)(z-3/4)}\right)z$$

$$z^2 W = \left(\left(\frac{0.21875}{z-7/8}\right) + \left(\frac{-0.1875}{z-3/4}\right)\right)z$$

$$z^2 w(k) = \left(0.21875\left(\frac{7}{8}\right)^k - 0.1875\left(\frac{3}{4}\right)^k\right)u(k)$$

$$w(k) = \left(0.21875\left(\frac{7}{8}\right)^{k-2} - 0.1875\left(\frac{3}{4}\right)^{k-2}\right)u(k-2)$$