ECE 341 - Homework #7

Uniform and Exponential Distributions.

Uniform Distributions

Let

- a be a sample from A, a uniform distribution over the range of (0, 3)
- b be a sample from B, a uniform distribution over the range of (0,4)

1) Determine the pdf for y = a + b using moment generating funcitons (i.e. LaPlace transforms)

$$A = \left(\frac{1}{3s}\right)(1 - e^{-3s})$$

$$B = \left(\frac{1}{4s}\right)(1 - e^{-4s})$$

$$Y = \left(\left(\frac{1}{3s}\right)(1 - e^{-3s})\right)\left(\left(\frac{1}{4s}\right)(1 - e^{-4s})\right)$$

Doing some algebra

$$Y = \left(\frac{1}{12s^2}\right)(1 - e^{-3s} - e^{-4s} + e^{-7s})$$

take the inverse LaPlace transform, the pdf of A+B is

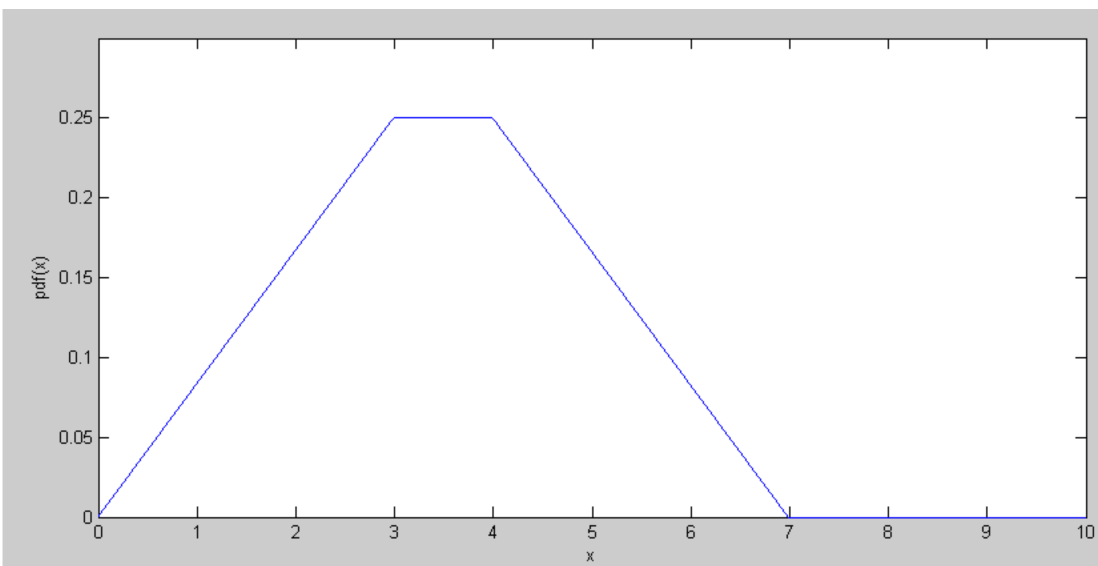
$$y(x) = \left(\frac{1}{12}\right)((x)u(x) - (x-3)u(x-3) - (x-4)u(x-4) + (x-7)u(x-7))$$

or

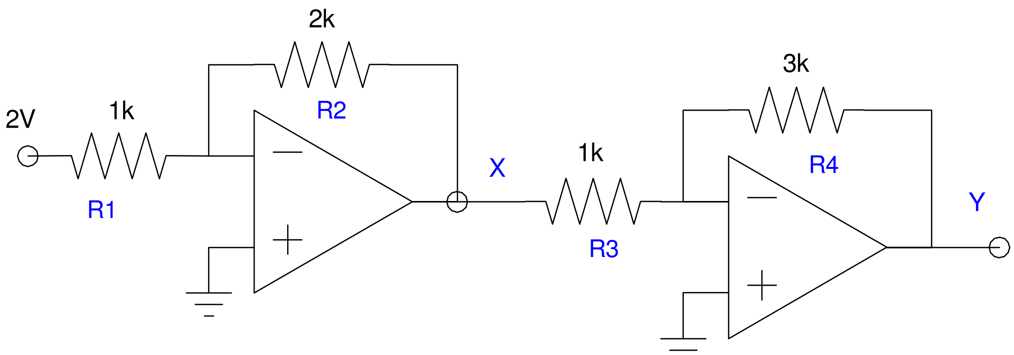
$$y(x) = \begin{cases} 0 & x < 0 \\ \left(\frac{x}{12}\right) & 0 < x < 3 \\ \left(\frac{3}{12}\right) & 3 < x < 4 \\ \left(\frac{7-x}{12}\right) & 4 < x < 7 \\ 0 & x < 7 \end{cases}$$

2) Determine the pdf for a + b using convolution (by hand or Matlab)

```
>> t = [0:0.01:10]';
>> dt = 0.01;
>> t = [0:dt:10]';
>> A = (1/3)*(t>=0).*(t<3);
>> B = (1/4)*(t>=0).*(t<4);
>> Y = conv(A,B)*dt;
>> plot(t,Y)
??? Error using ==> plot
Vectors must be the same lengths.

>> Y = Y(1:length(t));
>> plot(t,Y)
>> xlabel('x'):
??? xlabel('x'):
                |
Error: Expression or statement is incomplete or
incorrect.

>> xlabel('x');
>> ylabel('pdf(x)');
>> ylim([0,0.3])
>>
```



3) Assume each resistor has a tolerance of 5% (i.e. a uniform distribution over the range of (0.95, 1.05) of the nominal value. Determine the mean and standard deviation for the voltage at Y for the following circuit using a Monte Carlo simulation.



The output is

$$Y = \left(\frac{-R_2}{R_1}\right)\left(\frac{-R_4}{R_3}\right)2V$$

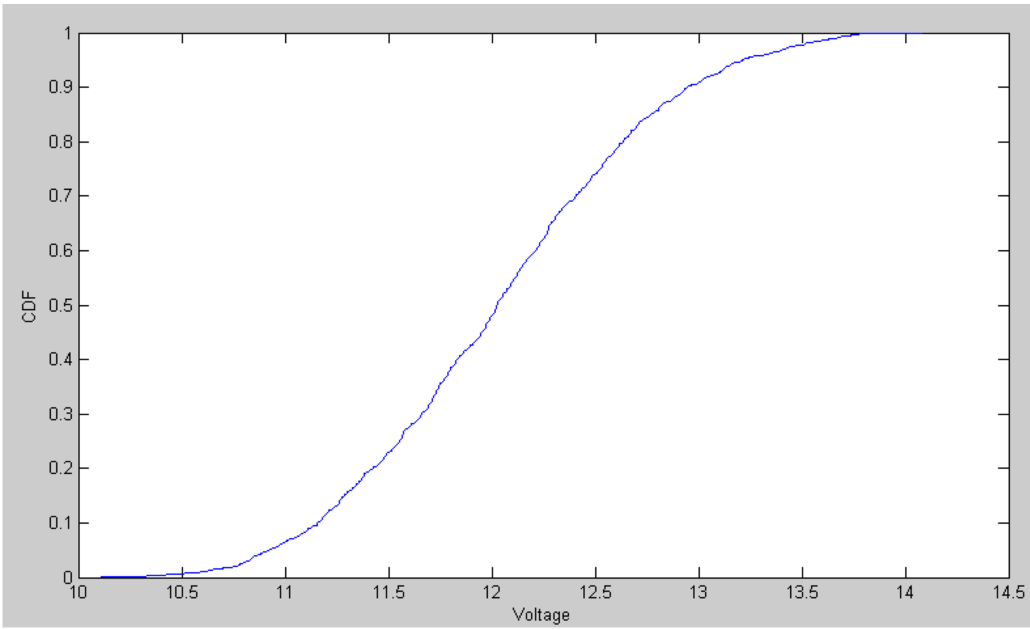
$$Y = 2\left(\frac{R_2 R_4}{R_1 R_3}\right)$$

In Matlab

```
y = [];

for i=1:1000
   R1 = 1000 * (1 + (2*rand-1)*0.05);
   R2 = 2000 * (1 + (2*rand-1)*0.05);
   R3 = 1000 * (1 + (2*rand-1)*0.05);
   R4 = 3000 * (1 + (2*rand-1)*0.05);
   Y = 2*R2*R4 / (R1*R3);
   y = [y ; Y];
end


>> mean(y)

ans =   12.0434

>> std(y)

ans =    0.6916
```

Sidelight: Plot the cdf for this distribution

```
>> p = [1:1000]' / 1000;
>> plot(sort(y),p)
>> xlabel('Voltage');
>> ylabel('CDF');
```



Exponential Distributions

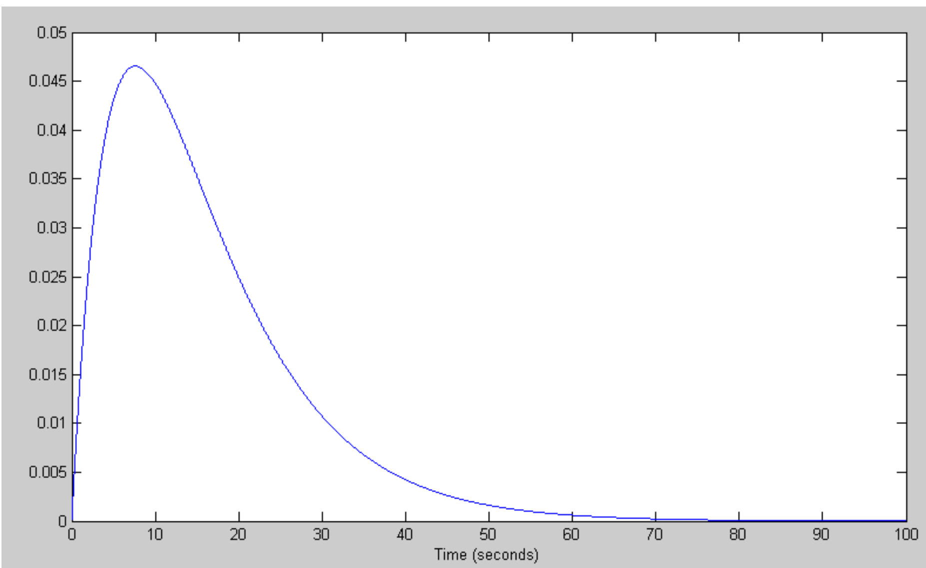
Let

- d be a sample from D, an exponential distribution with a mean of 6
- e be a sample from E, an exponential distribution with a mean of 10
- f be a sample from F, an exponential distribution with a mean of 12

4) Let X = d + e

a) Use convolution to find the pdf of X

```
>> dt = 0.01;
>> t = [0:dt:100]';
>> d = 1/6 * exp(-t/6);
>> e = 1/10 * exp(-t/10);
>> f = 1/12 * exp(-t/12);
>> X = conv(d,e) * dt;
>> Y = conv(X,f) * dt;
>> X = X(1:length(t));
>> Y = Y(1:length(t));
>> plot(t,X)
>> xlabel('Time (seconds)');

>> disp([t(1000),X(1000)])
    9.9900    0.0448
```



b) Use moment generating functions to find he pdf of X

$$X = \left(\frac{1/6}{s+1/6}\right)\left(\frac{1/10}{s+1/10}\right)$$

Doing partial fraction expansion and taking the inverse LaPlace transform

$$X = \left(\frac{-0.25}{s+1/6}\right) + \left(\frac{0.25}{s+1/10}\right)$$

$$x(t) = 0.25(e^{-t/10} - e^{-t/6})u(t)$$

A 10 seconds

$$x(10) = 0.04475$$

This matches what was calculated using convolutiopn

5) Use moment generating functions to determine the pdf for Y = d + e + f

a) Use convolution to find the pdf of Y

```
dt = 0.01;
t = [0:dt:100]';
d = 1/6 * exp(-t/6);
e = 1/10 * exp(-t/10);
f = 1/12 * exp(-t/12);
X = conv(d,e) * dt;
Y = conv(X,f) * dt;
X = X(1:length(t));
Y = Y(1:length(t));
plot(t,Y)
xlabel('Time (seconds)');

disp([t(1000),Y(1000)])

    9.9900    0.0220
```

b) Use moment generating functions to find he pdf of Y

$$Y = \left(\frac{1/6}{s+1/6}\right)\left(\frac{1/10}{s+1/10}\right)\left(\frac{1/12}{s+1/12}\right)$$

$$Y = \left(\frac{0.25}{s+1/6}\right) + \left(\frac{-1.25}{s+1/10}\right) + \left(\frac{1}{s+1/12}\right)$$

$$y(t) = (0.25e^{-t/6} - 1.25e^{-t/10} + e^{-t/12})u(t)$$

c) Check that the two answers match at t = 10 seconds.

$$y(10) = 0.02197$$

The answers match