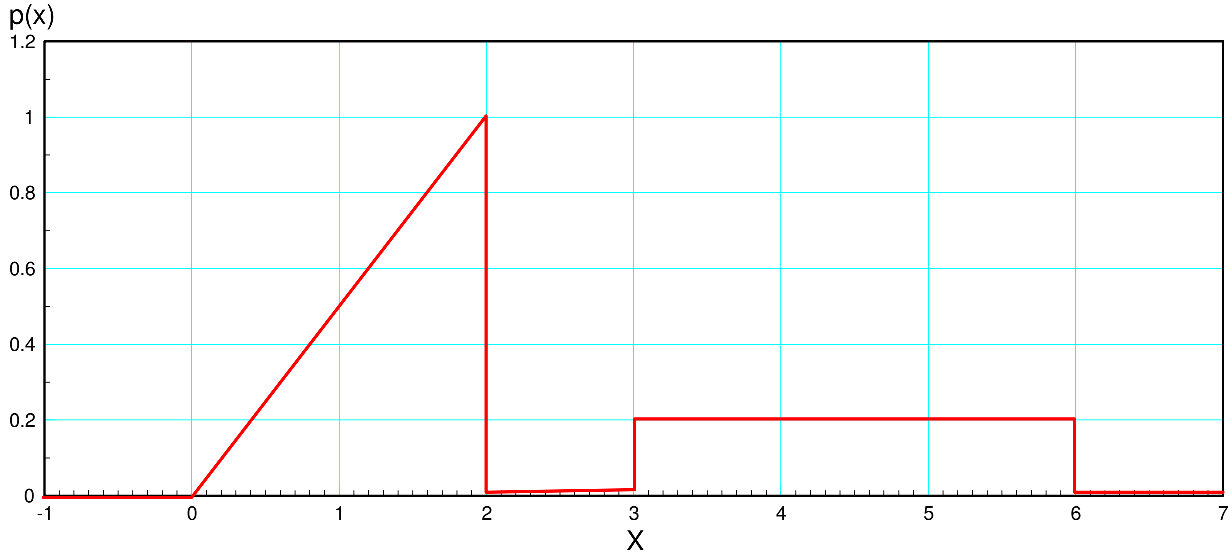
ECE 341 - Homework #6

LaPlace Transforms, Continuous Probability Density Functions.



1) Determine the scalar so that the above function is a valid pdf (i.e. the total area is 1.000)

The total area as drawn is 1.6 For the area to be 1.000,

$$k = \frac{1}{1.6} = 0.625$$

The pdf is then

The pdf is

$$pdf(x) = \begin{cases} 0 & x < 0 \\ 0.3125x & 0 > x > 2 \\ 0 & 2 < x < 3 \\ 0.125 & 3 < x < 6 \\ 0 & x > 6 \end{cases}$$

2) Determine the corresponding cdf

The cdf is the integral of the pdf

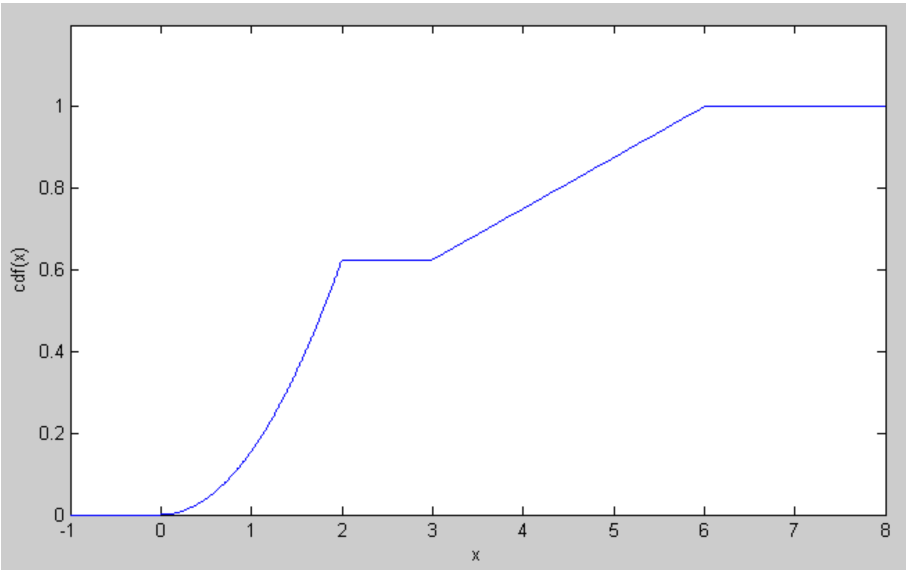
$$p = \begin{cases} 0 & x < 0 \\ 0.15625x^2 & 0 > x > 2 \\ 0.625 & 2 < x < 3 \\ 0.125x + 0.25 & 3 < x < 6 \\ 1 & x > 6 \end{cases}$$

Create a function for the cdf:

```
function [y] = cdf(x)
if(x < 0)
    y=0;
elseif(x<2)
    y = 0.15625*x*x;
elseif(x<3)
    y = 0.625;
elseif(x<6)
    y = 0.125*x + 0.25;
else
    y=1;
end
```

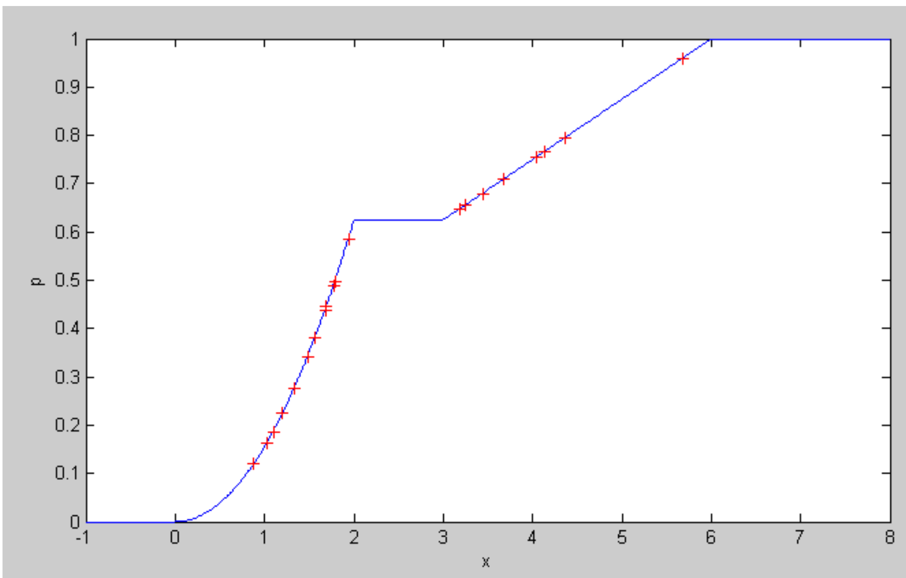
Checking:

```
x = [-1:0.01:8]';
y = 0*x;
for i=1:length(x)
   y(i) = cdf(x(i));
   end;
plot(x,y);
ylim([0,1.2]);
xlabel('x');
ylabel('cdf(x)');
```



3) Using Matlab, find 20 random values of x for the above pdf

```
x0 = zeros(20,1);
p0 = zeros(20,1);

for i=1:20
    p = rand
    x1 = -1;
    while(cdf(x1) < p)
        x1 = x1 + 0.01;
    end
    x0(i) = x1;
    p0(i) = p;
end

plot(x,y,'b-',x0,p0,'r+');
```



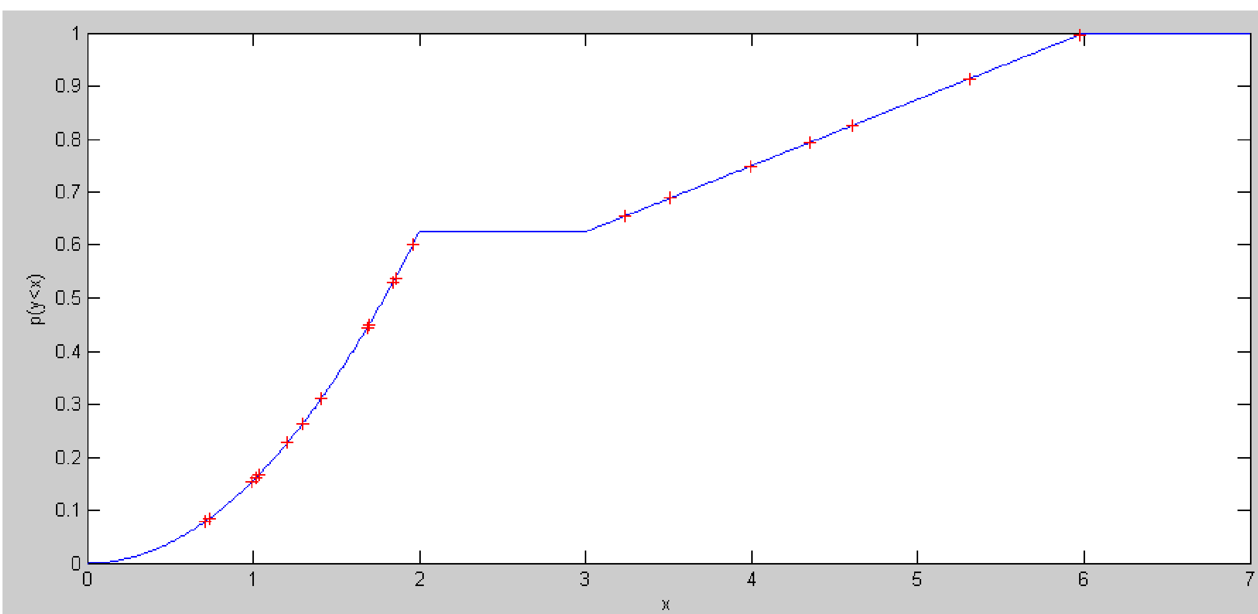
p	x
0.4387	1.6800
0.3816	1.5700
0.7655	4.1300
0.7952	4.3700
0.1869	1.1000
0.4898	1.7800
0.4456	1.6900
0.6463	3.1800
0.7094	3.6800
0.7547	4.0400

Problem 3) (take 2): Use interval halving

```
function [x] = pdf(p)

x0 = 0;
x2 = 10;
for i=1:20
  x1 = (x0+x2)/2;
  y = cdf(x1);
  if(y>p) x2 = x1;
  else x0=x1;
  end
end

x = (x0+x2)/2;
end
```

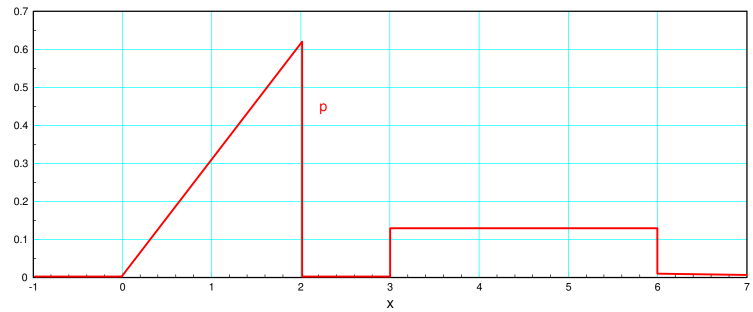
Calling Function

```
x0 = [0:0.01:7];
p0 = 0*x0;
for i=1:length(x0)
    p0(i) = cdf(x0(i));
end


x = zeros(20,1);
p = zeros(20,1);

for i=1:20
    p(i) = rand;
    x(i) = pdf(p(i));
end

plot(x0,p0,'b-',x,p,'r+');
```

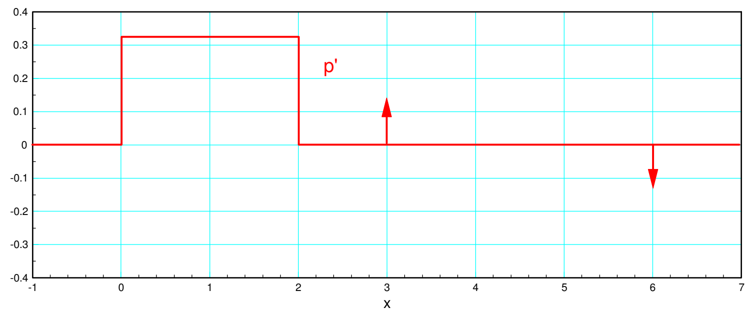


4) Find the moment generating function for p(x)

Start with p(x)

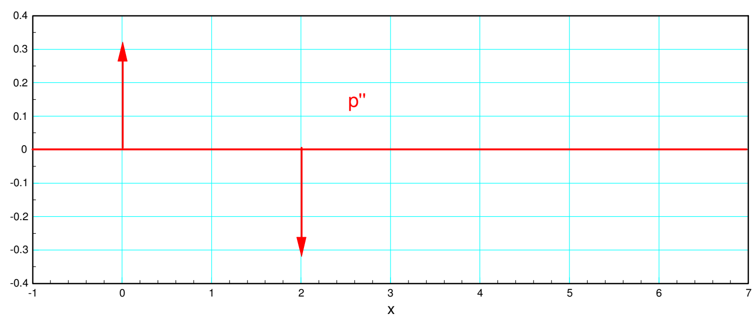


Take the derivative



$$sp = 0.125e^{-3s} - 0.125e^{-6s}$$

Take another derivative



$$s^2p = 0.3125 - 0.3125e^{-2s}$$

Putting them all together, the moment generating funciton is:

$$P(s) = \left(\frac{0.125}{s}\right)(e^{-3s} - e^{-6s}) + \left(\frac{0.3125}{s^2}\right)(1 - e^{-2s})$$