ECE 341 - Homework #4

Binomial and Uniform Distributions.

Binomial Distribution

Assume you toss a coin with a probability of a heads being 0.6

$$X(z) = \left(\frac{0.4z+0.6}{z} \right)$$

1) Determine the probability of tossing 6 heads in 8 tosses

$$p = \binom{8}{6} (0.6)^6 (0.4)^2$$

$$p = 0.209$$

2) Determine the probability distribution when tossing this same coin 8 times

Option 1: Compute

$$p(n) = \binom{8}{n} (0.6)^n (0.4)^{8-n}$$

Option 2: Use Matlab and convolution

```
x1 =     0.4000     0.6000

>> x2 = conv(x1,x1)

x2 =     0.1600     0.4800     0.3600

>> x4 = conv(x2,x2)

x4 =     0.0256     0.1536     0.3456     0.3456     0.1296

>> x8 = conv(x4,x4)
```

n=0	n=1	n=2	n=3	n=4	n=5	**n=6**	n=7	n=8
0.0007	0.0079	0.0413	0.1239	0.2322	0.2787	**0.2090**	0.0896	0.0168

NOAA has been keeping track of world weather for the past 142 years. 19 of last 20 years have been in the 20 hottest years on record.

3a) What is the probability of any given year being one of the 20 hottest on record (i.e. what is p?)

$$p = \left(\frac{20}{142} \right)$$

3b) What is the probability of 19 of the last 20 years being the hottest on record?

$$p(19) = \binom{20}{19} \left(\frac{20}{142} \right)^{19} \left(\frac{122}{142} \right)^{1}$$

$$p(19) = 1.1513 \cdot 10^{-15}$$

$$p(19) = 0.000\ 000\ 000\ 000\ 001\ 151\ 3$$

There *is* a chance this is just random fluctuation.

Uniform Distribution

Assume a fair six-sided die:

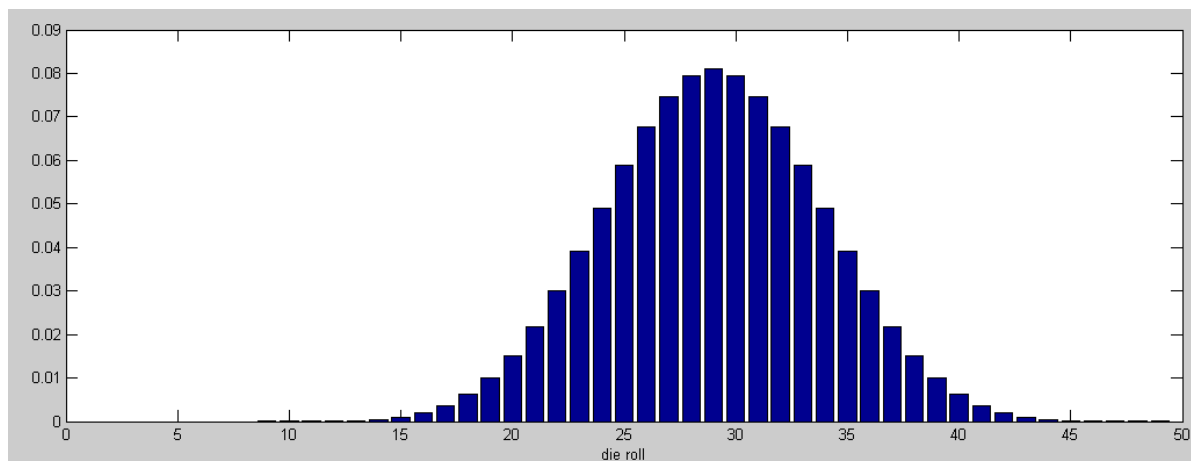$$Y(z) = \left(\frac{1}{6}\right)\left(\frac{z^5 + z^4 + z^3 + z^2 + z + 1}{z^6}\right)$$

4) Assume you roll 8 dice (8d6 or a level-8 fireball). Determine the
   - pdf
   - mean, and
   - standard deviation

Using colvolution in Matlab

```
>> d6 = [0 1 1 1 1 1 1]' / 6;
>> d6x2 = conv(d6,d6);
>> d6x4 = conv(d6x2,d6x2);
>> d6x8 = conv(d6x4,d6x4);
>> bar(d6x8);
>> xlabel('die roll')
```


The mean and variance is then

$$\mu = 8 \cdot 3.5 = 28$$

$$\sigma^2 = 8 \cdot 2.9167 = 23.333$$
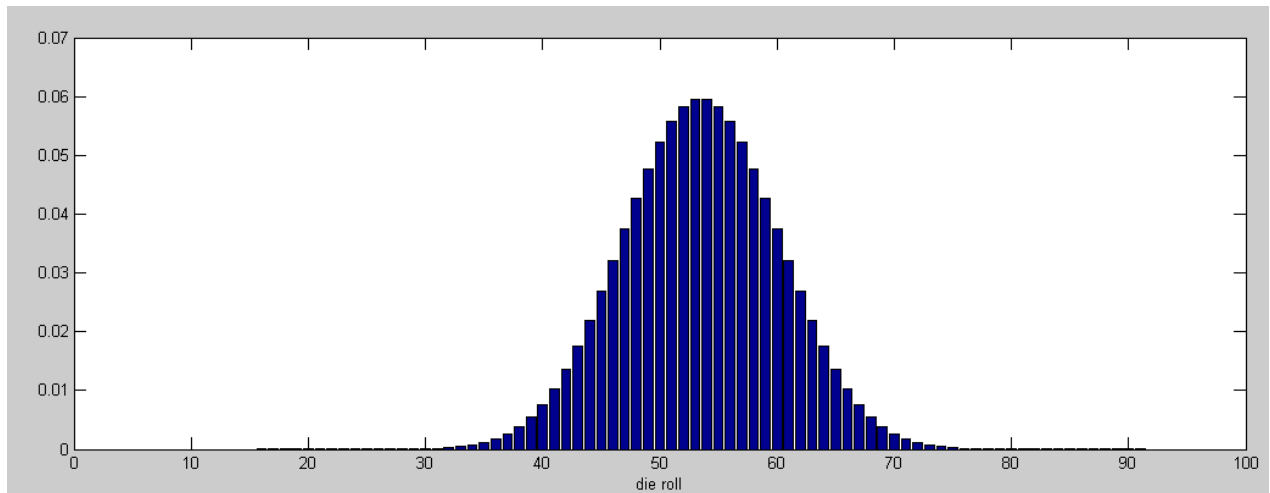
$$\sigma = 4.8305$$

Another way to compute the mean and variance is

```
>> k = [0:48]';
>> x = sum(d6x8 .* k)

x =    28.0000

>> s2 = sum( (k-x).^2 .* d6x8 )

s2 =    23.3333
```

5) Assume you roll 15 dice (15d6 or a level-15 fireball). Determine the
   - pdf
   - mean, and
   - standard deviation

```
>> d6 = [0 1 1 1 1 1 1]' / 6;
>> d6x2 = conv(d6,d6);
>> d6x4 = conv(d6x2,d6x2);
>> d6x8 = conv(d6x4,d6x4);
>> d6x12 = conv(d6x8,d6x4);
>> d6x14 = conv(d6x12,d6x2);
>> d6x15 = conv(d6x14,d6);
>> bar(d6x15);
>> xlabel('die roll')
>>
```


The mean and standard devation are then

$$\mu = 15 \cdot 3.5 = 52.5$$

$$\sigma^2 = 15 \cdot 2.9167 = 43.7505$$

$$\sigma = 6.6144$$

You can also use the definitions of mean and variance:

```
>> k = [0:90]';

>> x = sum(d6x15 .* k)

x =    52.5000

>> s2 = sum( (k-x).^2 .* d6x15 )

s2 =    43.7500
```