ECE 341 - Homework #8

Queueing Theory & Normal Distributions. Due Tuesday, June 1st

Please make the subject "ECE 341 HW#8" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Queueing Theory

Assume you are running a fast-food restraunt.

- The time between customers arriving at a restaraunt is an exponential distribution with a mean of 100 seconds.
- The time it takes to serve each customer is an exponential distribution with a mean of 50 seconds.

1) Run a single Monte-Carlo simulation for this restaraunt over the span of one hour.

- Give the formula for each column in you simulation
- What is the longest waiting time for a customer in your simulation?
- What is the largest queue over the span of one hour?

Arrival Times:

pdf:

$$p(t) = 0.01 \cdot \exp(-0.01t) \cdot u(t)$$

cdf:

$$c(t) = \int p(t) \cdot dt$$

$$c(t) = (1 - \exp(-0.01t))u(t)$$

Solving for t

$$t_a = -100 \ln(1 - c_a)$$

Service Times

$$t_s = -50 \ln(1 - c_s)$$

Generating random times for arrival and service:

```
>> ca = rand(100,1);
>> ta = -100*log(1-ca);
>> cs = rand(100,1);
>> ts = -50*log(1-cs);
>> [ca(1:5),ta(1:5),cs(1:5),ts(1:5)]

ans =

     p(ta)      ta         p(ts)      ts
    0.1622   17.6955    0.6443  103.3719
    0.7943  158.1261    0.3786   47.5795
    0.3112   37.2826    0.8116  166.9084
    0.5285   75.1906    0.5328   76.1053
    0.1656   18.1101    0.3507   43.1902

>>
```

- d4 = d3 + b4 *arrival time*
- e4 = min(d4, f3) *service time*
- f4 = e4 + c4 *finish time*
- g4 = sum((f3>d4)*1 + (f2>d4)*1 + (f1>d4)*1) *queue size*
- h4 = e4 - d4 *wait time*

a	b	c	d	e	f	g	h
Customer	ta	ts	Arrival Time	Service Time	Finish Time	Queue Size	Wait Time
0	0	0	0	0	0	0	0
1	17.7	103.4	17.7	17.7	121.1	0	0
2	158.1	47.6	175.8	175.8	223.4	0	0
3	37.3	166.9	213.1	223.4	390.3	1	10.3
4	75.2	76.1	288.3	390.3	466.4	1	102
5	18.1	43.2	306.4	466.4	509.6	2	160
6	92.1	279.7	398.5	509.6	789.3	2	111.1
7	30.5	208.7	429	789.3	998	3	360.3
8	106.2	79.9	535.2	998	1,077.9	2	**462.8**
9	116.9	97.4	652.1	1,077.9	1,175.3	3	425.8
10	137.9	88.4	790	1,175.3	1,263.7	3	385.3
11	59.9	23.3	849.8	1,263.7	1,287	4	413.9
12	8.8	35.8	858.6	1,287	1,322.9	5	428.4
13	26	63.7	884.6	1,322.9	1,386.5	**6**	438.3
14	244.6	26.2	1,129.2	1,386.5	1,412.7	5	257.3
15	16.5	186	1,145.7	1,412.7	1,598.7	6	267
16	174.8	21.7	1,320.5	1,598.7	1,620.4	4	278.2
17	77.3	25.6	1,397.8	1,620.4	1,646	3	222.6
18	555.6	18.7	1,953.3	1,953.3	1,972.1	0	0
19	8.1	25.8	1,961.5	1,972.1	1,997.9	1	10.6
20	58.5	57.2	2,019.9	2,019.9	2,077.1	0	0
21	11.3	37.3	2,031.2	2,077.1	2,114.4	1	45.9
22	326.7	256.9	2,358	2,358	2,614.8	0	0
23	0.5	56.2	2,358.4	2,614.8	2,671.1	1	256.4
24	149.1	20.4	2,507.6	2,671.1	2,691.5	2	163.5
25	170	235.3	2,677.5	2,691.5	2,926.8	1	14
26	203	390	2,880.6	2,926.8	3,316.7	1	46.2
27	8.8	57.8	2,889.4	3,316.7	3,374.5	2	427.3
28	51	11.8	2,940.4	3,374.5	3,386.3	2	434.1
29	30.1	29.8	2,970.5	3,386.3	3,416.2	3	415.8
30	161	52.5	3,131.5	3,416.2	3,468.7	4	284.7
31	56.5	90.4	3,188	3,468.7	3,559.1	5	280.7
32	241.5	30.4	3,429.5	3,559.1	3,589.5	2	129.6
33	20.1	92.3	3,449.6	3,589.5	3,681.8	3	139.9
34	30.6	124.2	3,480.2	3,681.8	3,806	3	201.6

Normal Distribution

The low for the month has been measured at Hector Airport since 1942. The mean and standard deviations are:

Month	May	June	July	Aug	Sept	Oct
Mean	27.4013F	40.2179F	46.2949F	43.2321F	30.5526F	19.3462F
st dev	4.4236F	3.9924F	3.9481F	4.1435F	4.8050F	5.1265F

http://www.bisonacademy.com/ECE111/Code/Fargo_Weather_Monthly_Low.txt

2) What is the probability that we will have a killing frost (temperature drops below 30F) in

- August
- September
- October

August:

$$z = \left(\frac{30-43.2321}{4.1435}\right) = -3.1934$$

p = 0.001 *from StatTrek*

There is a 0.1% chance of a killing frost in August

September

$$z = \left(\frac{30-30.5526}{4.8050}\right) = -0.1150$$

p = 0.454 *from StatTrek*

There is a 45.4% chance of a killing frost in September

October

$$z = \left(\frac{30-19.346}{5.1265}\right) = 2.0782$$

p = 0.981 *from StatTrek*

There is a 98.1% chance of a killing frost in October

Rainfall

The rainfall in Fargo each month (in inches) is

Month	May	June	July	Aug	Sept	Oct
Mean	2.6549	3.5025	2.9668	2.6529	2.1344	1.694
st dev	1.6536	2.1054	1.9505	1.7339	1.4913	1.4619

3) What is the probability that we will get more than 6 inches of rain in June?

$$z = \left(\frac{3.5025-6}{2.1054}\right) = -1.1862$$

p = 0.118 *from StatTrek*

There is an 11.8% chance we'll get more than 6" or rain in June

4) What is the probability that the total rainfall for these six months will be less than 6"?

The sum of normal distrubutions is a normal distribution (central limit theorem)

- The mean is the sum of the means
- The variance is the sum of the variances

Mean = 15.6055

Variance:

$$1.6536^2 + 2.1054^2 + 1.9505^2 + 1.7339^2 + 1.4913^2 + 1.4619^2 = 18.3391$$

Standard Deviation

$$s = \sqrt{18.3391} = 4.2824$$

The z-score for 6" of rain over six months is

$$z = \left(\frac{6-18.3391}{4.2824}\right) = -2.8813$$

p = 0.002 (from StatTrek)

There is a 0.2% chance of getting less than 6" of rain over these six months

500:1 odds against, i.e. a 500-year droubt