ECE 341 - Homework #7

Uniform and Exponential Distributions. Due Friday, May 28th

Please make the subject "ECE 341 HW#7" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Uniform Distributions

Let

- a be a sample from A, a uniform distribution over the range of (0, 2)
- b be a sample from B, a uniform distribution over the range of (0, 3)
- c be a sample from C, a uniform distribution over the range of (0, 4)

1) Determine the pdf for a + b using moment generating funcitons (i.e. LaPlace transforms)

$$A = \frac{1}{2s}(1 - e^{-2s})$$

$$B = \frac{1}{3s}(1 - e^{-3s})$$

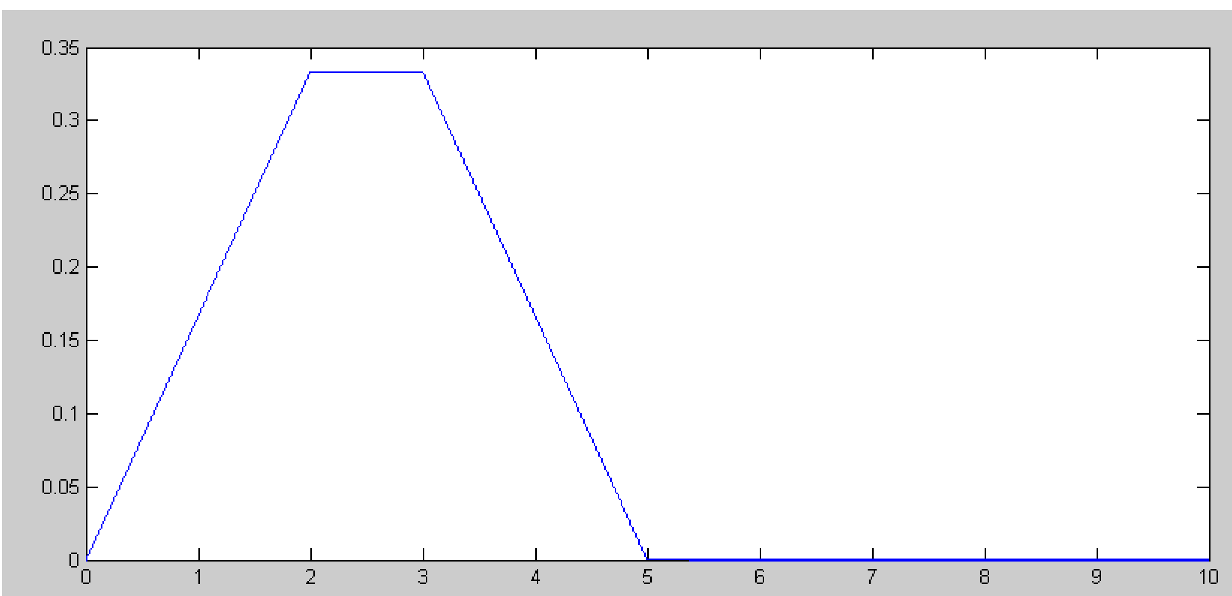
$$Y = AB = \left(\frac{1}{6s^2}\right)(1 - e^{-2s})(1 - e^{-3s})$$

$$Y = \left(\frac{1}{6s^2}\right)(1 - e^{-2s} - e^{-3s} + e^{-5s})$$

Taking the inverse LaPlace transform

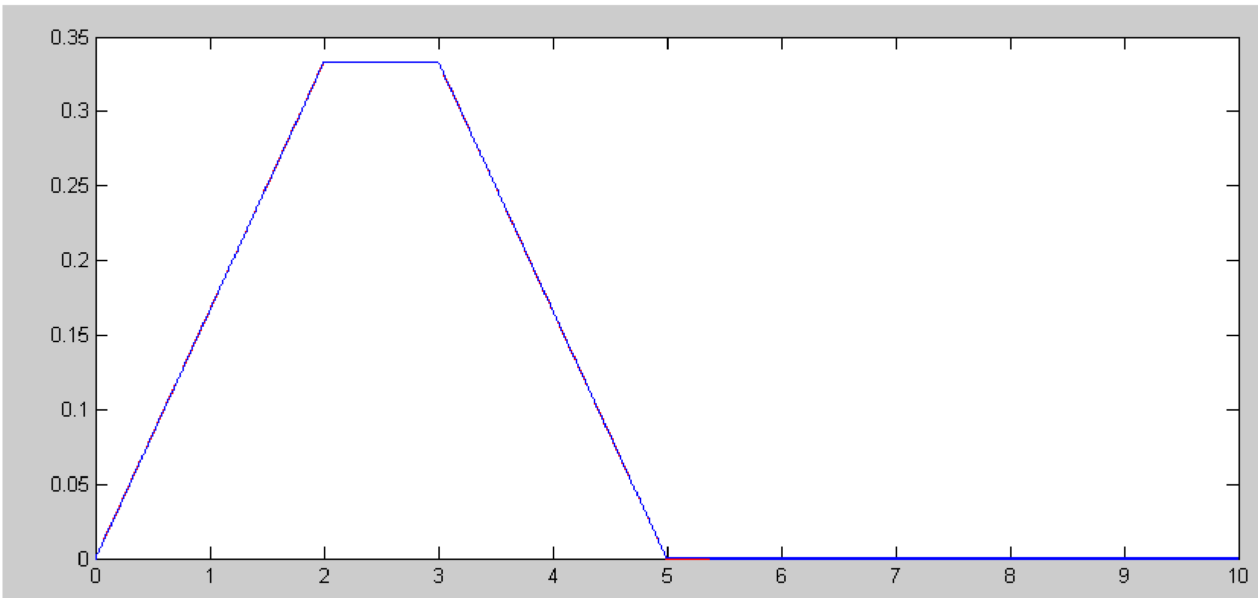
$$y(t) = \frac{1}{6}(t \cdot u(t) - (t-2)u(t-2) - (t-3)u(t-3) + (t-5)u(t-5))$$

```
>> t = [0:0.001:10]';
>> y = (t/6  -  (1/6)*(t-2).*(t>2) - (1/6)*(t-3).*(t>3) + (1/6)*(t-5).*(t>5));
>> plot(t,y)
```

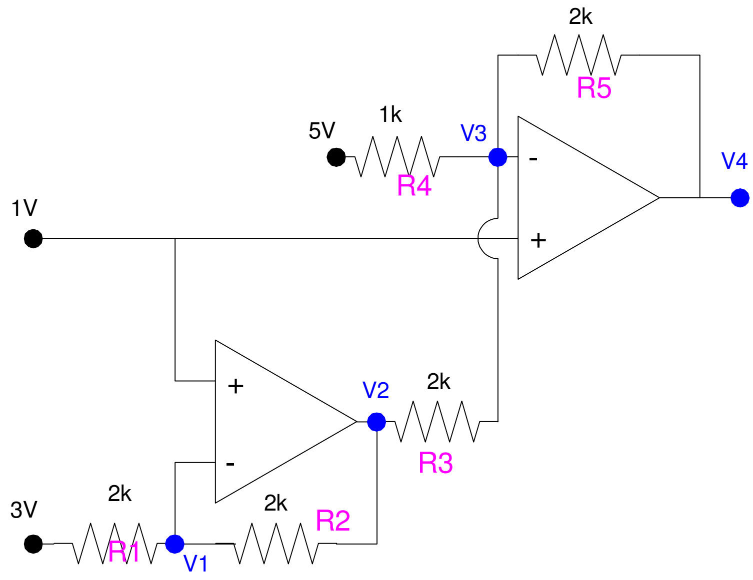


2) Determine the pdf for a + b using convolution (by hand or Matlab)

```
>> dt = 0.001;

>> A = 0.5 * (t<2);
>> B = 1/3 * (t<3);
>> AB = conv(A,B) .* dt;

>> plot(t,AB(1:length(t)),'r',t,y,'b')
>>
```



3) Assume each resistor has a tolerance of 5% (i.e. a uniform distribution over the range of (0.95, 1.05) of the nominal value. Determine the mean and standard deviation for the voltage at V4 for the following circuit.



$$V_1 = 1$$

$$V_3 = 1$$

$$\left(\frac{V_1 - 3}{R_1}\right) + \left(\frac{V_1 - V_2}{R_2}\right) = 0$$

$$\left(\frac{V_3 - 5}{R_4}\right) + \left(\frac{V_3 - V_2}{R_3}\right) + \left(\frac{V_3 - V_4}{R_5}\right) = 0$$

Grouping terms

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_1 - \left(\frac{1}{R_2}\right) V_2 = \left(\frac{3}{R_1}\right)$$

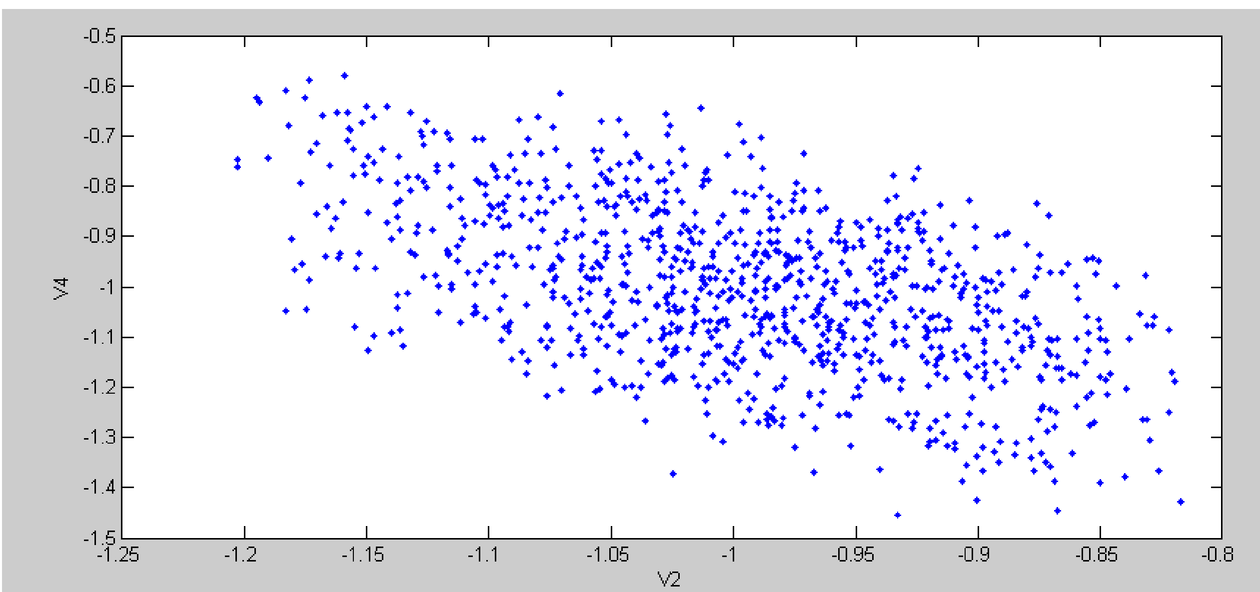
$$\left(\frac{1}{R_4} + \frac{1}{R_3} + \frac{1}{R_5}\right) V_3 - \left(\frac{1}{R_3}\right) V_2 - \left(\frac{1}{R_5}\right) V_4 = \left(\frac{5}{R_4}\right)$$

In matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \left(\frac{1}{R_1} + \frac{1}{R_2}\right) & \left(\frac{-1}{R_2}\right) & 0 & 0 \\ 0 & \left(\frac{-1}{R_3}\right) & \left(\frac{1}{R_4} + \frac{1}{R_3} + \frac{1}{R_5}\right) & \left(\frac{-1}{R_5}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ \left(\frac{3}{R_1}\right) \\ \left(\frac{5}{R_4}\right) \end{bmatrix}$$

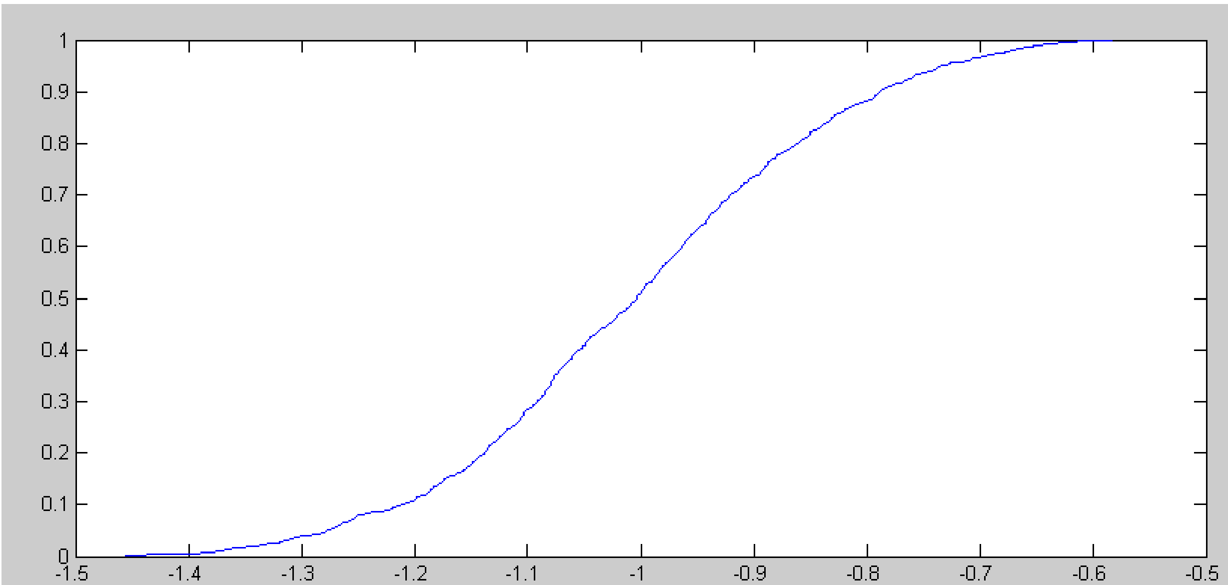
In Matlab

```
result = [];

for i=1:1000
   R1 = (1 + 0.05*(2*rand-1)) * 2000;
   R2 = (1 + 0.05*(2*rand-1)) * 2000;
   R3 = (1 + 0.05*(2*rand-1)) * 2000;
   R4 = (1 + 0.05*(2*rand-1)) * 2000;
   R5 = (1 + 0.05*(2*rand-1)) * 2000;
   A = [1,0,0,0;0,0,1,0;1/R1+1/R2,-1/R2,0,0;0,-1/R3,1/R4+1/R3+1/R5,-1/R5];
   B = [1;1;3/R1;5/R4];
   V = inv(A)*B;
   result = [result ; V'];
end



>> mean(result)
      V1         V2          V3         V4
    1.0000    -0.9961     1.0000    -1.0040

>> std(result)

    V1       V2            V3       V4
     0     0.0836           0     0.1626

>> plot(result(:,2),result(:,4),'.')
>> xlabel('V2');
>> ylabel('V4');
```



To see the cdf of V4, sort and plot

```
>> V4 = result(:,4);
>> plot(sort(V4))
>> V4 = sort(V4);
>> size(V4)

ans =

        1000           1

>> p = [1:1000]' / 1000;
>> plot(p,V4);
>> plot(V4,p);
>>
```



cdf for V4

Exponential Distributions

Let

- d be a sample from D, an exponential distribution with a mean of 4
- e be a sample from E, an exponential distribution with a mean of 10
- f be a sample from F, an exponential distribution with a mean of 20

4) Use moment generating functions to determine the pdf for X = d + e

$$D = \left(\frac{0.25}{s+0.25}\right)$$

$$E = \left(\frac{0.1}{s+0.1}\right)$$

$$X = \left(\frac{0.25}{s+0.25}\right)\left(\frac{0.1}{s+0.1}\right)$$

do partial fractions

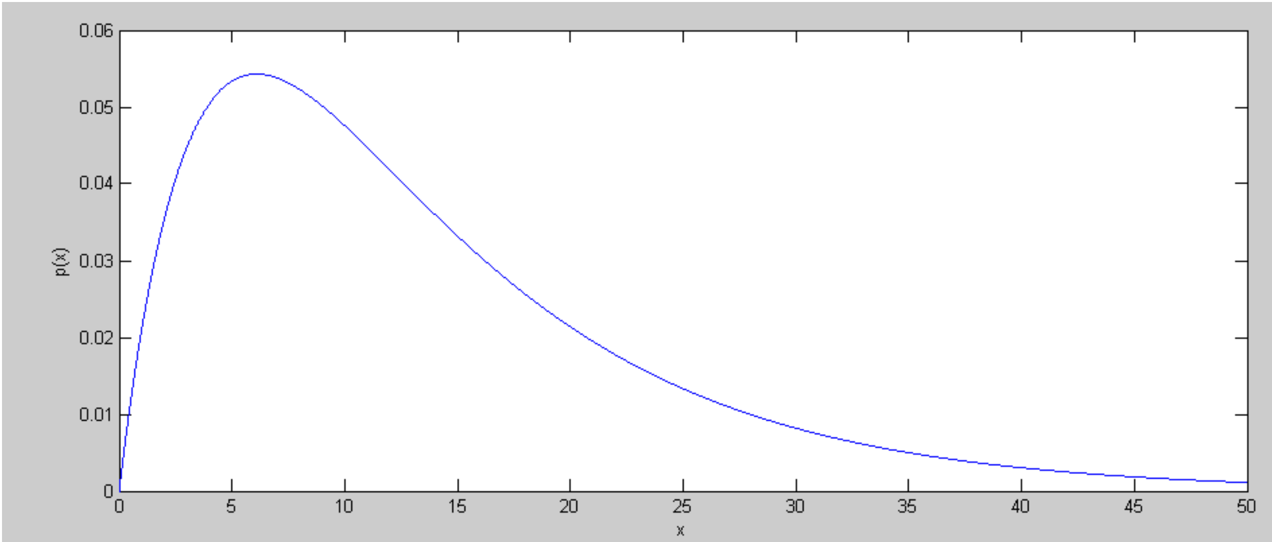
$$X = \left(\frac{-0.1667}{s+0.25}\right) + \left(\frac{0.1667}{s+0.1}\right)$$

$$x(t) = 0.1667e^{-0.1t} - 0.1667e^{-0.25t}$$

Plotting in Matlab

```
>> t = [0:0.001:50]';
>> x = -0.1667*exp(-0.25*t) + 0.1667*exp(-0.1*t);
>> plot(t,x);
>> dt = 0.001;
>> sum(x)*dt

ans =  0.9890
```

check: all probabilities add to one (slightly less due to truncating at 50)



5) Use moment generating functions to determine the pdf for the sum: Y = d + e + f

$$F = \left(\frac{0.05}{s+0.05}\right)$$

$$Y = \left(\frac{0.25}{s+0.25}\right)\left(\frac{0.1}{s+0.1}\right)\left(\frac{0.05}{s+0.05}\right)$$

$$Y = \left(\frac{0.041667}{s+0.25}\right) + \left(\frac{-0.1667}{s+0.1}\right) + \left(\frac{0.125}{s+0.05}\right)$$

$$y(t) = 0.041667e^{-0.25t} - 0.1667e^{-0.1t} + 0.125e^{-0.05t}$$

In Matlab

```
>> t = [0:0.001:100]';
>> y = 0.041667*exp(-0.25*t) - 0.16667*exp(-0.1*t) + 0.125*exp(-0.05*t);
>> sum(y)*dt

ans =    0.9832
```

check: the area is one (slightly less due to truncating at 100)

```
>> plot(t,y)
>> xlabel('y');
>> ylabel('p(y)')
```