ECE 341 - Homework #5

Geometric, Pascal. Due Tuesday, May 25th

Please make the subject "ECE 341 HW#5" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

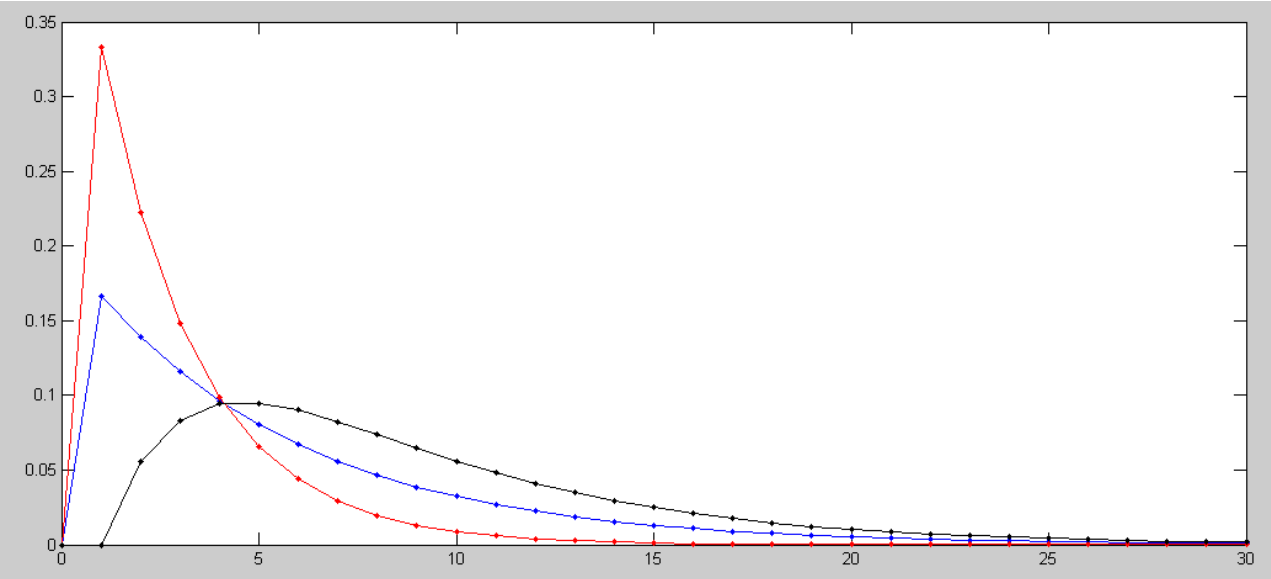
Problem 1 & 2) Let

- A be the number of times you roll a 6-sided die until you roll a 1 (p = 1/6)
- B be the number of times you roll an 6-sided die until you roll a 1 or 2 (p = 1/3)

1) Determine the pdf of A + B using convolution.

In Matlab

```
>> p = 1/6;
>> N = [0:30]';
>> A = p * (1-p).^N;
>> bar(A)
>> plot(N,A,'.')
>> A = p * (1-p).^(N-1) .* (N>0);

>> p = 1/3;
>> B = p * (1-p).^(N-1) .* (N>0);

>> Y = conv(A,B);
>> plot(N,A,'b.-',N,B,'r.-',N,Y(1:31),'k.-')
>> sum(A)
ans =    0.9958

>> sum(B)
ans =    1.0000

>> sum(Y)
ans =    0.9958
```



pdf of A (blue), B (red), and Y (black)

2) Determine the pdf of A + B using z-transforms.

$$A = \left(\frac{1/6}{z-5/6}\right)$$

$$B = \left(\frac{1/3}{z-2/3}\right)$$

$$Y = AB = \left(\frac{1/6}{z-5/6}\right)\left(\frac{1/3}{z-2/3}\right)$$

Take the inverse z-transform.

$$z^2 Y = \left(\frac{1/18z}{\left(z-\frac{5}{6}\right)\left(z-\frac{2}{3}\right)}\right)z$$

$$z^2 Y = \left(\left(\frac{0.2778}{z-5/6}\right) + \left(\frac{-0.2222}{z-2/3}\right)\right)z$$

$$z^2 y(k) = \left(0.2778\left(\frac{5}{6}\right)^k - 0.2222\left(\frac{2}{3}\right)^k\right)u(k)$$

$$y(k) = \left(0.2778\left(\frac{5}{6}\right)^{k-2} - 0.2222\left(\frac{2}{3}\right)^{k-2}\right)u(k-2)$$

Checking

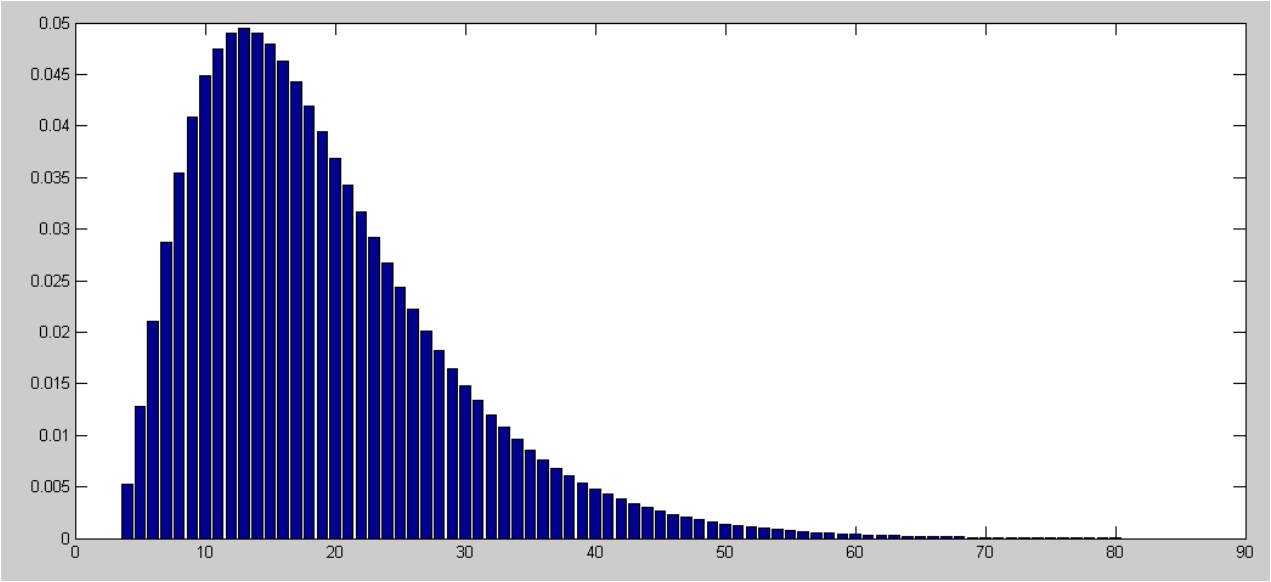
```
>> k = [0:10]';
>> y = (0.2778*(5/6).^(k-2) - 0.2222*(2/3).^(k-2)) .* (k>=2);
>> [Y(1:10),y(1:10)]

         0         0
         0         0
    0.0556    0.0556
    0.0833    0.0834
    0.0941    0.0942
    0.0949    0.0949
    0.0901    0.0901
    0.0824    0.0824
    0.0735    0.0735
    0.0645    0.0645
```

Problem 3 & 4: Let

- A be the number of times you roll a 4-sided die until you roll a 1 (p = 1/4)
- B be the number of times you roll a 6-sided die until you roll a 1 (p = 1/6)
- C be the number of times you roll an 8-sided die until you roll a 1 (p = 1/8)

3) Determine the pdf of A + B + C using convolution.

```
>> N = [0:60]';
>> p = 1/4;
>> A = p * (1-p).^(N-1) .* (N>0);
>> p = 1/6;
>> B = p * (1-p).^(N-1) .* (N>0);
>> p = 1/8;
>> C = p * (1-p).^(N-1) .* (N>0);
>> AB = conv(A,B);
>> ABC = conv(AB,C);
>> bar(ABC(1:80))
>>
```



pdf for the number of rolls needed to get a '1' on a 4, 6, and 8 dided die (total rolls)

4) Determine the pdf of Y = A + B + C using z-transforms.

$$A = \left(\frac{0.25}{z-0.75}\right)$$

$$B = \left(\frac{0.1667}{z-0.8333}\right)$$

$$C = \left(\frac{0.125}{z-0.875}\right)$$

$$Y = \left(\frac{0.25}{z-0.75}\right)\left(\frac{0.1667}{z-0.8333}\right)\left(\frac{0.125}{z-0.875}\right)$$

Using partial fractions

$$z^3 Y = \left(\frac{0.005208z^2}{(z-0.75)(z-0.8333)(z-0.875)}\right)z$$

$$z^3 Y = \left(\left(\frac{0.28125}{z-0.75}\right) + \left(\frac{-1.041667}{z-0.8333}\right) + \left(\frac{0.765619}{z-0.875}\right)\right)z$$

Taking the inverse transform

$$z^3 y(k) = \left(0.28125(0.75)^k - 1.041667(0.8333)^k + 0.765619(0.875)^k\right)u(k)$$

$$y(k) = \left(0.28125(0.75)^{k-3} - 1.041667(0.8333)^{k-3} + 0.765619(0.875)^{k-3}\right)u(k-3)$$

checking... this matches the result from convolution

```
y = (0.28125*(3/4).^(k-3) - 1.04167*(5/6).^(k-3) + 0.76562*(7/8).^(k-3)).*(k>=3);
[y(1:10),ABC(1:10)]

    z-trans    convolution
         0              0
         0              0
         0              0
    0.0052         0.0052
    0.0128         0.0128
    0.0210         0.0210
    0.0287         0.0287
    0.0354         0.0354
    0.0408         0.0408
    0.0448         0.0448

>>
```