ECE 341 - Homework #3

Dice Games and z-Transform. Due Friday, May 21st

Please make the subject "ECE 341 HW#3" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Farkle

1) Compute the odds of rolling a straight when rolling six dice

dice = 1, 2, 3, 4, 5, 6

The number of rolls possible are

$$N = 6^6 = 46,656$$

For a straignt,

M = (6 numbers, pick 1)(5 remaining numbers, pick 1)(4 remaining numbers, pick 1)...

$$M = \binom{6}{1}\binom{5}{1}\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{1}{1} = 6! = 720$$

The odds are then

$$p = \left(\frac{720}{46,656}\right) = 0.01543$$

The odds of rolling a straight are 64.8 : 1 against

2) Compute the odds of rolling three pair (x and y are different. z can be any value including x or y)

dice = xx yy zz or xx xx yy

xx yy zz

M = (6 numbers, pick 3)(6 spots for x, choose 2)(4 spots for y, choose 2)(2 spots for z, choose 2)

$$M = \binom{6}{3}\binom{6}{2}\binom{4}{2}\binom{2}{2} = 1800$$

xx xx yy

M = (6 numbers choose 2)(6 spots for x choose 4)(2 spots for y choose 2)

$$M = \binom{6}{2}\binom{6}{4}\binom{2}{2} = 225$$

The odds are then

$$p = \left(\frac{1800+225}{46,656}\right) = 0.043402$$

The odds are 23.04 : 1 against

z-Transforms

3) Find the inverse z-transform

$$X = \left(\frac{0.01z^2}{(z-1)(z-0.9)(z-0.8)} \right)$$

Pull out a z and use partial fractions

$$X = \left(\frac{0.01z}{(z-1)(z-0.9)(z-0.8)} \right) z$$

$$X = \left(\frac{0.5}{z-1} + \frac{-0.9}{z-0.9} + \frac{0.4}{z-0.8} \right) z$$

$$X = \left(\frac{0.5z}{z-1} + \frac{-0.9z}{z-0.9} + \frac{0.4z}{z-0.8} \right)$$

Converting back

$$x(k) = 0.5 - 0.9(0.9)^k + 0.4(0.8)^k \qquad \text{for k >= 0}$$

4) Find the inverse z-transform

$$X = \left(\frac{0.02(z+1)^3}{(z-1)(z-0.9)(z-0.8)} \right)$$

Pull out a z and use partial fractions

$$zX = \left(\frac{0.02(z+1)^3}{(z-1)(z-0.9)(z-0.8)} \right) z$$

$$zX = \left(\left(\frac{8}{z-1} \right) + \left(\frac{-13.71}{z-0.9} \right) + \left(\frac{5.832}{z-0.8} \right) \right) z$$

Take the inverse-z transform

$$zx(k) = \left(8 - 13.71(0.9)^k + 5.832(0.8)^k \right) u(k)$$

Divide by z (delay by one sample)

$$x(k) = \left(8 - 13.71(0.9)^{k-1} + 5.832(0.8)^{k-1} \right) u(k-1)$$

5) A new Nissan Leaf costs $31,760. Assume you take out a 36-month loan at 2.59% interest per year (0.2158% per month). What will be the monthly payments? Solve using z-transforms.

Let x(k) be how much you owe at month k

$$x(k+1) = 1.002158 \cdot x(k) - p + 31760 \cdot \delta(k)$$

Take the z-transform. Assume you start making payments at k=1 (rather than at month #0)

$$zX = 1.002158X - p\left(\frac{1}{x-1}\right) + 31760$$

$$(z - 1.002158)X = -p\left(\frac{1}{z-1}\right) + 31760$$

put over a common denominator

$$(z - 1.002158)X = \left(\frac{31760(z-1)-p}{z-1}\right)$$

$$X = \left(\frac{31760(z-1)-p}{(z-1)(z-1.002158)}\right)$$

Take the inverse z transform. Start with partial fraction expansion

$$zX = \left(\frac{31760(z-1)-p}{(z-1)(z-1.002158)}\right)z$$

$$zX = \left(\left(\frac{463.392p}{z-1}\right) + \left(\frac{31760-463.392p}{z-1.002158}\right)\right)z$$

$$zx(k) = \left(463.392p + 31760 \cdot (1.002158)^k - 463.392p \cdot (1.002158)^k\right)u(k)$$

$$x(k) = \left(463.392p + 31760 \cdot (1.002158)^{k-1} - 463.392p \cdot (1.002158)^{k-1}\right)u(k-1)$$

You can also write this as

$$x(k) = \left(31760 \cdot (1.002158)^{k-1} + 463.392p \cdot (1 - 1.002158^{k-1})\right)u(k-1)$$

At k = 37 (36 payments), the balance is zero

$$x(37) = 0 = 34,322.87 - 37.3933p$$

$$p = 917.886$$

Your monthly payments will be $917.886