

ECE 341 - Homework #1

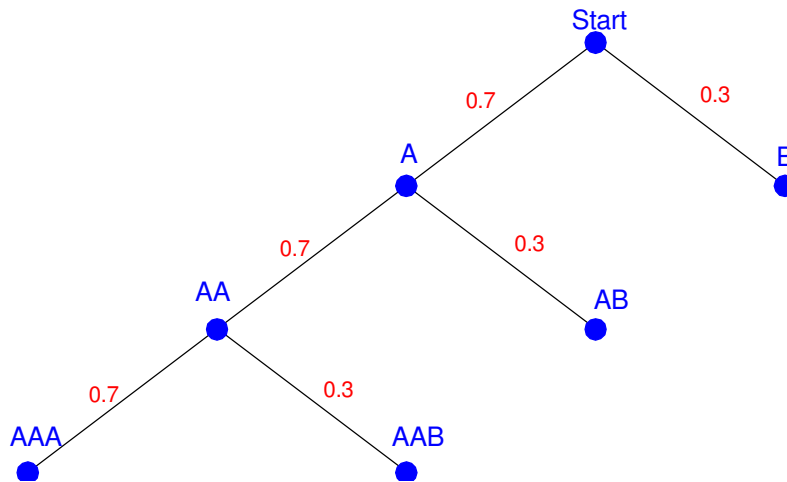
Tree Diagrams and Enumeration. Due Wednesday, May 19th

Please make the subject "ECE 341 HW#1" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

1) Two teams, A and B, are playing a best of 5 game series.

- The series is over once one team gets 3 points.
- The probability of A winning any given game is 0.7.
- B starts out with 2 points (odds)

Draw the tree diagram for all possible outcomes of the series.



2) List all possible combinations of rolling a 4-sided die (d4) and a 6-sided die (d6) (enumeration).

		6-sided die					
		1	2	3	4	5	6
4-sided die	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)

Also determine the probability $X \in \{1..6\}$ where X is the difference between the two numbers (largest - smallest).

Score		6-sided die					
		1	2	3	4	5	6
4-sided die	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2

0 Points: (1,1), (2,2), (3,3), (4,4)

- odds: 4/24

1 point: (1,2), (2,1), (2,3), (3,2), (3,4), (4,3), (4,5)

- odds = 7/24

2 points: (1,3), (2,4), (3,1), (3,5), (4,2), (4,6)

- odds = 6/24

3 points: (1,4), (2,5), (3,6), (4,1)

- odds = 4/24

4 points: (1,5), (2,6)

- odds = 2/24

5 points: (1,6)

- odds = 1/24

Two players, A and B, are playing a game of dice.

- Player A rolls a d4 and a d6 and takes the difference between the two numbers (i.e. problem #2)
- Player B rolls a 6-sided die and subtracts one ($d6 - 1$).

Player A wins on ties.

3) What is the conditional probability

- Player A wins given B's score is 3 (B rolled a 4)

A has to score 3, 4, or 5 points

$$p(\text{A winning}) = p(3) + p(4) + p(5)$$

$$p(\text{A}) = 4/24 + 2/24 + 1/24$$

$$p(\text{A}) = 7/24$$

A has a 7/24 chance of winning given that B's score is 3

4) What is the probability that player A will win any given game?

points	0	1	2	3	4	5
p(A)	4/24	7/24	6/24	4/24	2/24	1/24
p(B)	1/6	1/6	1/6	1/6	1/6	1/6

A scores 0 points:

$$p(\text{A}=0) * p(\text{B} \leq 0) = (4/24) * (1/6) = 4/144$$

A scores 1 point

$$p(\text{A}=1) * p(\text{B} \leq 1) = (7/24) * (2/6) = 14/144$$

A scores 2 points

$$p(\text{A}=2) * p(\text{B} \leq 2) = (6/24) * (3/6) = 18/144$$

A scores 3 points

$$p(\text{A} = 3) * p(\text{B} \leq 3) = (4/24) * (4/6) = 16/144$$

A scores 4 points

$$p(\text{A} = 4) * p(\text{B} \leq 4) = (2/24) * (5/6) = 10/144$$

A scores 5 points

$$p(\text{A} = 5) * p(\text{B} \leq 5) = (1/24) * (6/6) = 6/144$$

Adding them up gives you 68/144 (0.4722)

A has a 47.22% chance of winning this game

Monte Carlo Simulations & Enumeration with Farkle:

5) Determine the probability of rolling 6 dice and getting 4 of a kind (xxxx y z, xyz are different numbers) using a Monte Carlo simulation with 1 million rolls of the dice. (note: xxxx yy counts as 3 pair not 4 of a kind)

Code:

```
% Lecture #1: Monte Carlo
% Roll six dice
% Count how many times you get 4 of a kind
% xxx a b c
% xxx aa b
tic
Pair4 = 0;
for games = 1:1e5
    Dice = ceil( 6*rand(1,6) );
    Dice = sort(Dice);
    % check for pairs
    N = zeros(1,6);
    for i=1:6
        N(i) = sum(Dice == i);
    end
    [N,b] = sort(N, 'descend');
if ( (N(1) == 4) & (N(2) < 2) )
        Pair4 = Pair4 + 1;
    end
end

% probability:
disp('4 of a kind odds')
Pair4 / 1e5

toc
```

Monte-Carlo Simulation Results:

```
4 of a kind odds
ans =    0.0371
Elapsed time is 3.401550 seconds.
```

6) Determine the probability of rolling 6 dice and getting 4 of a kind using enumeration (exhaustive search).

Code:

```
% Lecture #1: Enumeration
% Roll six dice
% Count how many times you get 4 of a kind
% xxxx a b

tic
Pair4 = 0;

for d1 = 1:6
    for d2 = 1:6
        for d3 = 1:6
            for d4 = 1:6
                for d5 = 1:6
                    for d6 = 1:6
                        Dice = [d1,d2,d3,d4,d5,d6];
                        % check for pairs
                        N = zeros(1,6);
                        for i=1:6
                            N(i) = sum(Dice == i);
                        end
                        [N,b] = sort(N, 'descend');
                        if ( (N(1) == 4) & (N(2) < 2) )
                            Pair4 = Pair4 + 1;
                        end
                    end
                end
            end
        end
    end
end

% probability:
disp('4 of a kind odds')
Pair4 / (6^6)
toc
```

Results:

4 of a kind odds

ans = 0.0386

Elapsed time is 1.372184 seconds.

Monte Carlo Simulations & Enumeration with 4-Card Poker

7) Determine the probability of being dealt 3 of a kind in 4-card poker (you're dealt only 4 cards)

- 52-card deck, deal 4 cards.
- The five cards are xxx y where x, y are different values (ace to king)

Code:

```
% Lecture 1: Monte Carlo 4-Card Stud
% Probability of 3 of a kind
tic
Pair3 = 0;

for i0 = 1:1e5

    X = rand(1,52);
    [a,Deck] = sort(X);
    Hand = Deck(1:4);
    Value = mod(Hand,13) + 1;
    Suit = floor(Hand/13) + 1;

    N = zeros(1,13);
    for n=1:13
        N(n) = sum(Value == n);
    end

    [N,a] = sort(N, 'descend');

    if ((N(1) == 3)*(N(2) < 2)) Pair3 = Pair3 + 1; end

end

disp('3 of a kind');
[Pair3] / 1e5
toc
```

Result:

```
3 of a kind

ans =    0.0095

Elapsed time is 10.636327 seconds.
```

8) Determine the probability of being dealt a 3 of a kind in 4-card poker using enumeration (exhaustive search).

```
% lecture 1: Enumeration
% 4-Card Stud
% Probability of 3 of a kind
tic
Pair3 = 0;

for c1=1:52
    for c2 = c1+1:52
        clc
        disp([c1,c2])
        for c3 = c2+1:52
            for c4 = c3+1:52
                Hand = [c1,c2,c3,c4];
                Value = mod(Hand,13) + 1;
                Suit = floor(Hand/13) + 1;

                N = zeros(1,13);
                for n=1:13
                    N(n) = sum(Value == n);
                end

                [N,a] = sort(N, 'descend');
                if (N(1) == 3) Pair3 = Pair3 + 1; end
            end
        end
    end
end

Pair3 / NchooseM(52,4)
toc
```

Result

```
ans =      0.0092

Elapsed time is 22.304311 seconds.
```

note: This matches the Monte Carlo results (0.0095)

```
>> Pair3

Pair3 =      2496
```

Sidelight: There are 2496 ways to get 3 of a kind in 4-card poker.