ECE 341 - Test #3

Markov Chains and Data Analysis. Summer 2021

Open-Book, Open Notes. Calculators, Matlab, Tarot cards, StatTrek allowed. Just not other people.

1) Markov Chains:

Two people, A and B, are playing a game.

- A has a 20% chance of winning A gains +1 point on a win
- There is a 70% chance of a tie Neither A nor B score a point
- A has a 10% chance of losing A loses 2 points

If A reaches +2 points, A wins the match (win by 2)

If A reaces -2 points, B wins the match

1a) What is the state transition matrix (going from k games to k+1 games)

$$\begin{bmatrix} X_2(k+1) \\ X_1(k+1) \\ X_0(k+1) \\ X_{-1}(k+1) \\ X_{-2}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0.2 & 0 & 0 & 0 \\ 0 & 0.7 & 0.2 & 0 & 0 \\ 0 & 0 & 0.7 & 0.2 & 0 \\ 0 & 0.1 & 0 & 0.7 & 0 \\ 0 & 0 & 0.1 & 0.1 & 1 \end{bmatrix} \begin{bmatrix} X_2(k) \\ X_1(k) \\ X_0(k) \\ X_{-1}(k) \\ X_{-2}(k) \end{bmatrix}$$

1b) What is the probability that the match will end after 10 games (either A or B wins after 10 games)

In Matlab

```
>> A = [1,0.2,0,0,0 ; 0,0.7,0.2,0,0 ; 0,0,0.7,0.2,0 ; 0,0.1,0,0.7,0 ;
0,0,0.1,0.1,1]

    1.0000    0.2000         0         0         0
         0    0.7000    0.2000         0         0
         0         0    0.7000    0.2000         0
         0    0.1000         0    0.7000         0
         0         0    0.1000    0.1000    1.0000

>> A^10

    1.0000    0.6826    0.3880    0.1850         0
         0    0.0686    0.1006    0.1106         0
         0    0.0553    0.0686    0.1006         0
         0    0.0503    0.0553    0.0686         0
         0    0.1432    0.3875    0.5353    1.0000


>>
```

After 10 games

- There is a 38.80% chance that A has won
- There is a 38.75% chance that B has won

The odds that someone has won after 10 games is 77.55%

1c) What is the probability that A will eventually win the match?

```
>> A^100

ans =

    1.0000    0.7826    0.5217    0.3478         0
         0    0.0000    0.0000    0.0000         0
         0    0.0000    0.0000    0.0000         0
         0    0.0000    0.0000    0.0000         0
         0    0.2174    0.4783    0.6522    1.0000
```

A has an 52.17% chance of eventually winning the match with this format.

2) t-Test: One data set.

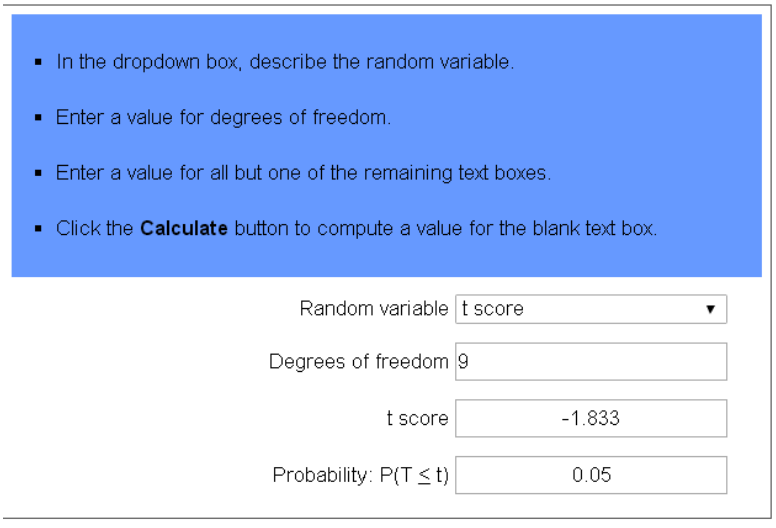
a) Generate 10 random numbers in Matlab

```
X = zeros(10,1);
for i=1:10
   X(i) = 100*sum(  rand(4,1) .^ 0.4 );
   end
>> X

  328.4937
  261.1877
  343.4287
  310.4198
  356.8023
  301.8121
  332.7129
  246.0043
  220.9325
  273.5838
```

b) Use a t-test to determine the 90% confidence interval for X

t-score	90% confidence interval for x
t = 1.833	**214.6189 to 380.4566**

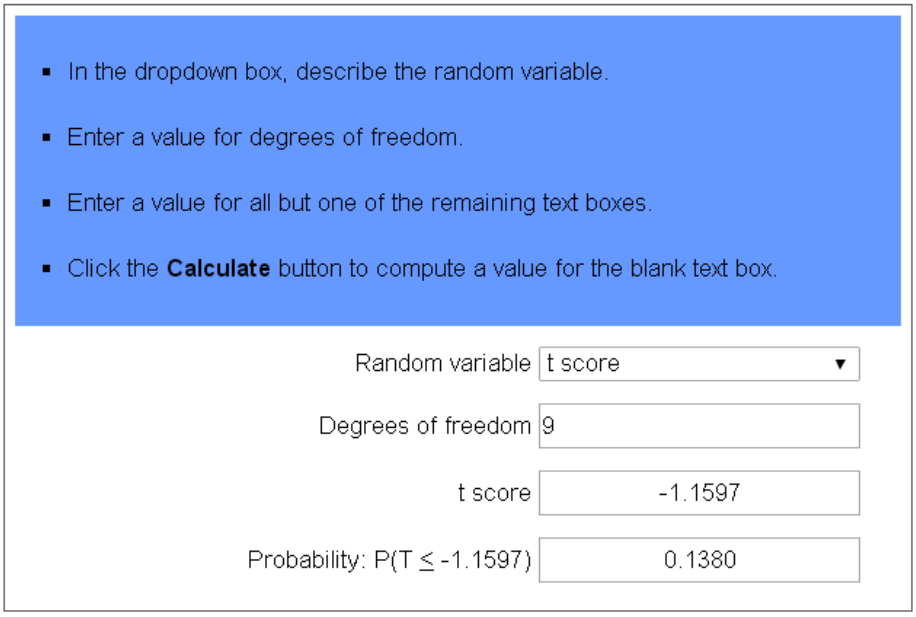
```
>> x = mean(X)
x =  297.5378

>> s = std(X)
s =   45.2367

>> high = x + 1.833*s
high =  380.4566

>> low = x - 1.833*s
low =  214.6189
```



c) Use a t-test to determine the probability that X > 350

t-score	p(X > 350)
1.1597	**p = 0.1380**

```
>> t = (350 - x) / s

t =    1.1597
```



3) t-Test (Two data sets):

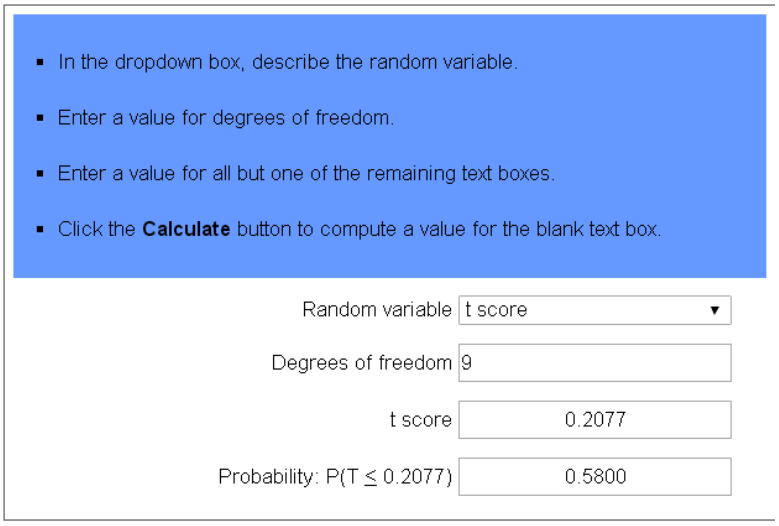
3a) Generate two sets of random numbersfor X and Y in Matlab (10 trials each)

```
X = zeros(10,1);
for i=1:10
   X(i) = 100*sum(  rand(4,1) .^ 0.4 );
   end

Y = zeros(10,1);
for i=1:10
   Y(i) = 90*sum(  rand(6,1) .^ 0.7 );
   end
>> [X,Y]

      X         Y
  321.0482  365.5862
  282.6372  280.6923
  321.9638  354.3522
  251.1578  245.2314
  299.0121  333.6652
  309.8946  286.7434
  317.7734  320.4592
  255.9728  300.4032
  311.7623  308.9908
  266.2294  271.1690
```

3b) If you generate an 11th value for X and Y, what is the probability that Y > X?

t-score	p(y(11) > x(11))
0.2777 varies with data	**p = 0.5800** 9 degrees of freedom (approx) **p = 0.5808** 13 degrees of freedom

```
>>Xx = mean(X)
   Xx =  293.7452

>> Sx = std(X)
   Sx =   27.5947

>> Xy = mean(Y)
   Xy =  306.7293

>> Sy = std(Y)
   Sy =   37.7347

>> Xw = Xy - Xx
   Xw =  12.9841

>> Sw = sqrt(Sx^2 + Sy^2)
   Sw =   46.7480

>> t = Xw / Sw
   t =   0.2777
```



3c) Based up 10 data points, what is the probability that the mean of Y is larger than the mean of X?

t-score	p(mean(Y) > mean(X))
t = 0.8783 varies with data	**p = 0.7987**

```
>> Xw = Xy - Xx

Xw =   12.9841

>> Sw = sqrt(Sx^2 /10 + Sy^2 /10)

Sw =   14.7830

>> t = Xw / Sw

t =    0.8783
```

- In the dropdown box, describe the random variable.
- Enter a value for degrees of freedom.
- Enter a value for all but one of the remaining text boxes.
- Click the **Calculate** button to compute a value for the blank text box.

Random variable	t score ▾
Degrees of freedom	9
t score	0.8783
Probability: P(T ≤ 0.8783)	0.7987

4) Chi-Squared Test:

The following Matlab code generated 100 random values for X:

```
RESULT = zeros(1,5);

for i=1:100
    d5 = ceil( 5*(rand ^ 0.9) );
    RESULT(d5) = RESULT(d5) + 1;
    end

RESULT

RESULT =      12    28    26    17    17
```

It is conjectured that X has a uniform distribution over the range of (1, 5)

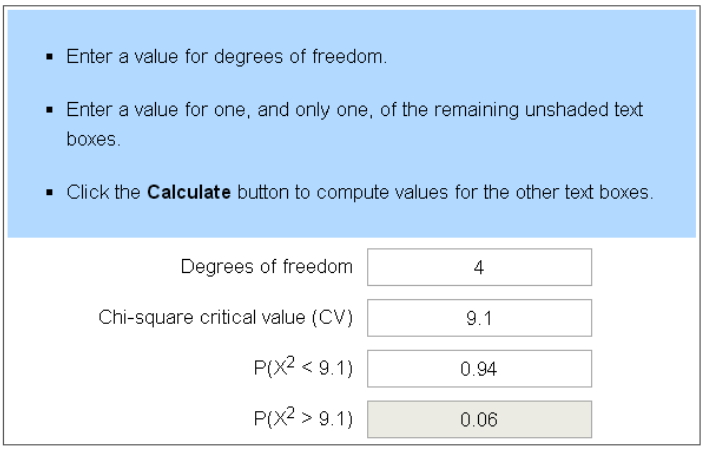
4a) Generate 100 values for X and give the result (give the number of times you rolled each number)

1	2	3	4	5
12	**28**	**26**	**17**	**17**

4b) Determine if X does or does not have a uniform distribution (i.e. is a fair die) using a Chi-squared test.

chi-squared critical value	p(d5 is not a uniform distribution)
9.10	**p = 0.94**

Roll	p	np	N	chi-squared
1	1/5	20	12	3.2
2	1/5	20	28	3.2
3	1/5	20	26	1.8
4	1/5	20	17	0.45
5	1/5	20	17	0.45
			Total	**9.1**



5) F-Test (Three data sets):

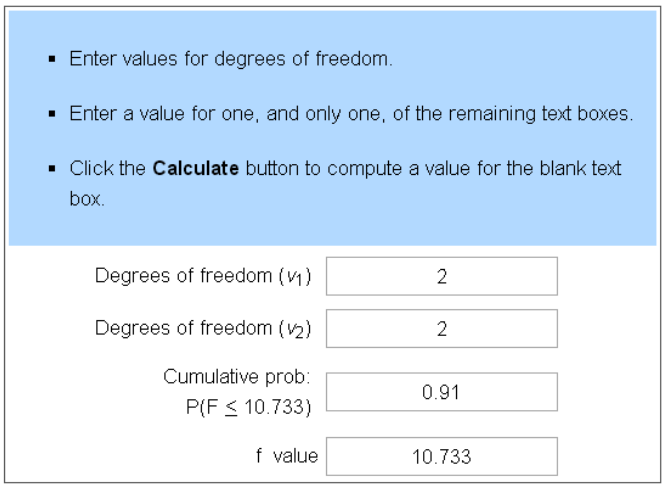
The reaction time of three people are measured:

Person	A	B	C
Reaction Times	0.2253	0.1924	0.2419
	0.1923	0.1893	0.1976
	0.1854	0.2018	0.3063

5a) What is the probability that the variance of A is different than the variance of B? (F-test)

F-score	p(var(A) != var(B))
F = 10.7333	**p = 0.91**

```
A = [ 0.22530; 0.1923 ; 0.1854 ];
B = [ 0.1924 ; 0.1893 ; 0.2018 ];
C = [ 0.2419 ; 0.1976 ; 0.3063 ];

F = var(A) / var(B)
   F =   10.7333
```



5b) What is the probability that all three people have the same average reaction time using an ANOVA test?

MSSb	MSSw	F-score	p(means are different)
0.0026	**0.0012**	**F = 2.2533**	**p = 0.81**

```
A = [ 0.22530; 0.1923 ; 0.1854 ];
B = [ 0.1924 ; 0.1893 ; 0.2018 ];
C = [ 0.2419 ; 0.1976 ; 0.3063 ];

Na = length(A);
Nb = length(B);
Nc = length(C);
N = Na + Nb + Nc

k = 3;

G = mean([A;B;C])

MSSb = ( Na*(mean(A)-G)^2 + Nb*(mean(B)-G)^2 + Nc*(mean(C)-G)^2 ) / (k-1)
MSSw = ( (Na-1)*var(A) + (Nb-1)*var(B) + (Nc-1)*var(C) ) / (N - k)
F = MSSb / MSSw

N =     9

G =    0.2147

MSSb =    0.0026

MSSw =    0.0012

F =    2.2533
```

- Enter values for degrees of freedom.
- Enter a value for one, and only one, of the remaining text boxes.
- Click the **Calculate** button to compute a value for the blank text box.

Degrees of freedom (v_1)	2
Degrees of freedom (v_2)	6
Cumulative prob: P(F ≤ 2.2533)	0.81
f value	2.2533