ECE 341 - Test #2

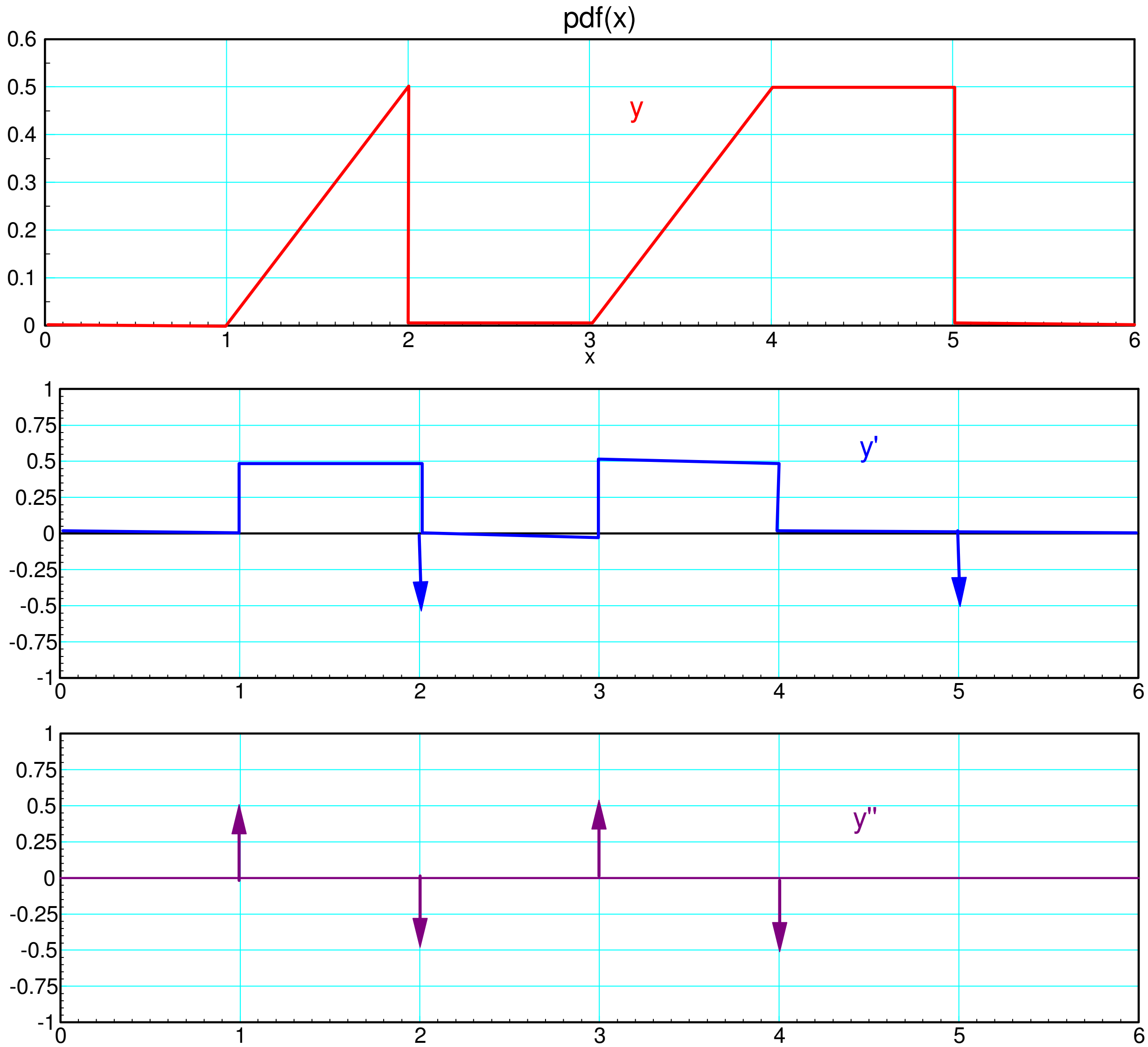
Continuous Probability

Open-Book, Open Notes. Calculators, Matlab, Tarot cards allowed. Just not other people.

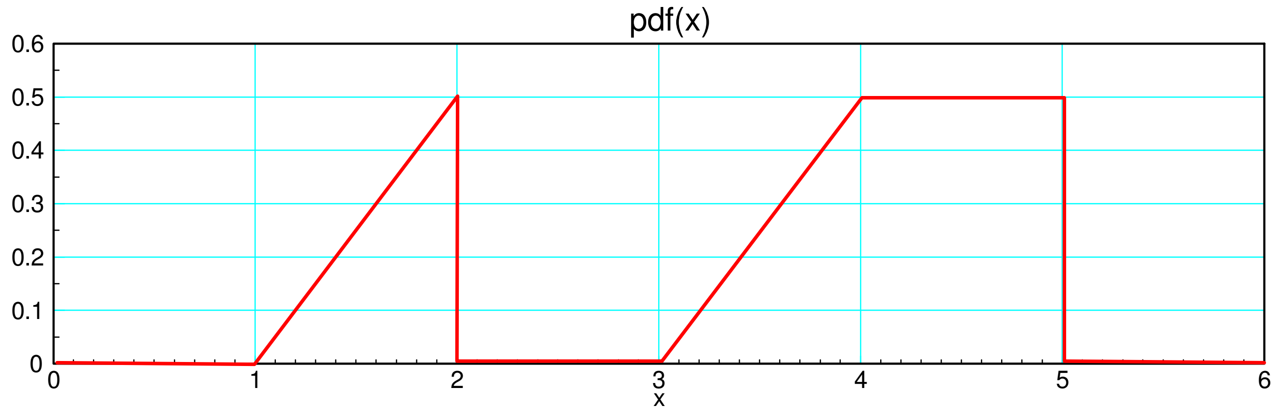
1. Continuous PDF

Determine the moment generating function (i.e. LaPlace transform) that corresponds to the following pdf.

$$Y(s) = \left(\frac{1}{s}\right)(-0.5e^{-2s} - 0.5e^{-5s}) + \left(\frac{1}{s^2}\right)(0.5e^{-s} - 0.5e^{-2s} + 0.5e^{-3s} - 0.5e^{-4s})$$



2. Continuous CDF



a) Determine the cumulative density function (cdf) that corresponds to the above pdf.

pdf:

$$pdf(x) = \begin{cases} 0 & x < 1 \\ 0.5(x-1) & 1 < x < 2 \\ 0 & 2 < x < 3 \\ 0.5(x-3) & 3 < x < 4 \\ 0.5 & 4 < x < 5 \\ 0 & x > 5 \end{cases}$$

cdf (integrate)

$$cdf(x) = \begin{cases} 0 & x < 1 \\ 0.25x^2 - 0.5x + 0.25 & 1 < x < 2 \\ 0.25 & 2 < x < 3 \\ 0.25x^2 - 1.5x + 2.5 & 3 < x < 4 \\ 0.5x - 1.5 & 4 < x < 5 \\ 1 & x > 5 \end{cases}$$

b) In Matlab, generate 5 random numbers between 0 and 1 (rand function).

- Determine x that corresponds to each random number using the cdf.

Matlab Code:

```
function [ p ] = cdf( x )

   if(x<1)
      p = 0;
   elseif(x<2)
      p = 0.25*x*x - 0.5*x + 0.25;
   elseif(x<3)
      p = 0.25;
   elseif(x<4)
      p = 0.25*x*x - 1.5*x + 2.5;
   elseif(x<5)
      p = 0.5*x-1.5;
   else
      p = 1;
   end
end
```

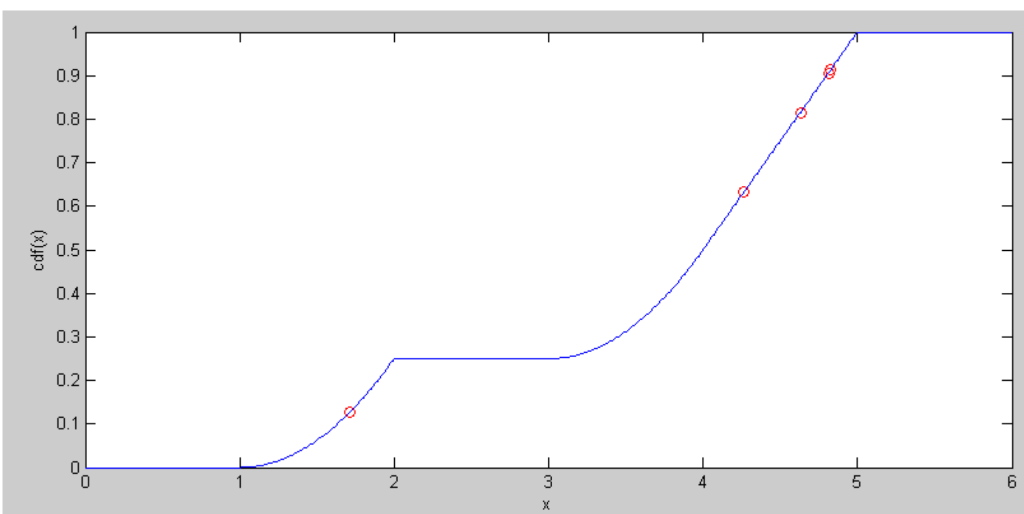
Calling Routine

```
dx = 0.001;
p = rand(5,1);
for i=1:5

   x = 0;
   while(cdf(x) < p(i))
       x = x + dx;
   end
   disp([p(i), x]);
end
```

Result

```
    cdf(x)      x
    0.8147    4.6300
    0.9058    4.8120
    0.1270    1.7130
    0.9134    4.8270
    0.6324    4.2650
```



3. Uniform Distribution

Let

- A be a uniform distribution over the range of (0, 4),
- B be a uniform distribution over the range of (0, 5), and
- C be a uniform distribution over the range of (0,6).
- Y be the sum: $Y = A + B + C$

a) Determine the pdf of Y

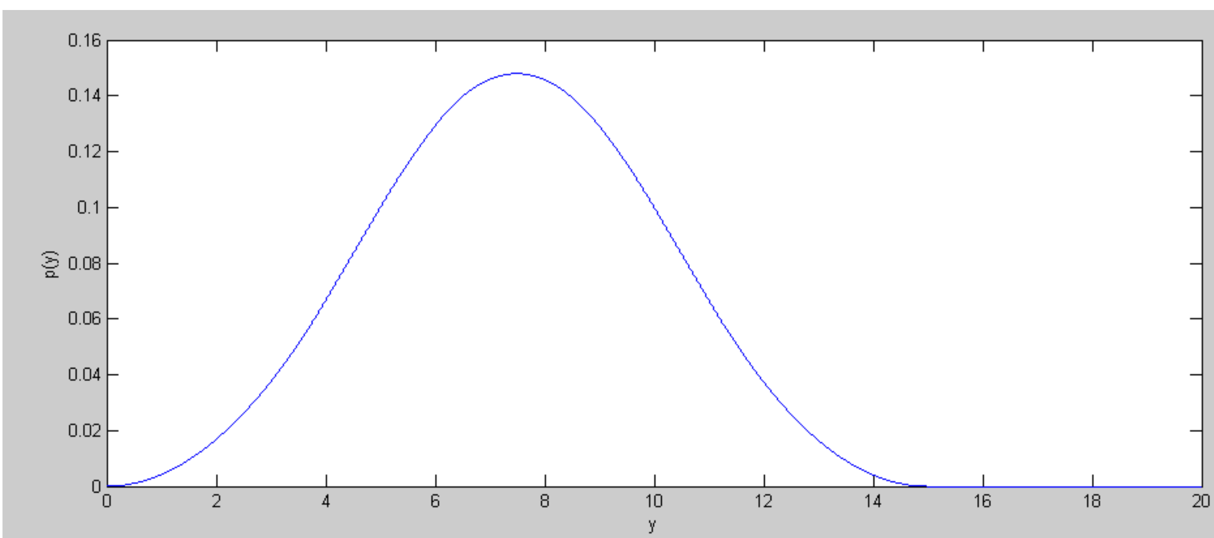
In matlab

```
>> dx = 0.001;
>> x = [0:dx:20]';
>> A = (x < 4) /4;
>> B = (x < 5) /5;
>> C = (x < 6) /6;
>> AB = conv(A,B) * dx;
>> Y = conv(AB, C) * dx;
>> size(x)

ans =

       20001           1

>> Y = Y(1:20001);
>> plot(x,Y)
>> xlabel('y');
>> ylabel('p(y)');
>>
```



b) Determine the probability that $Y > 13$

```
>> 13/dx

ans =       13000

>> sum(Y(13000:20001))*dx
```

ans =   0.0111

4. Central Limit Theorem

Let

- A be a uniform distribution over the range of (0, 4),
- B be a uniform distribution over the range of (0, 5), and
- C be a uniform distribution over the range of (0, 6).
- Y be the sum: Y = A + B + C

a) Determine the mean and standard deviation of Y

$$\mu_a = 2$$

$$\mu_b = 2.5$$

$$\mu_c = 3$$

$$\mu_y = \mu_a + \mu_b + \mu_c = 7.5$$

$$\sigma_a^2 = \frac{4^2}{12}$$

$$\sigma_b^2 = \frac{5^2}{12}$$

$$\sigma_c^2 = \frac{6^2}{12}$$

$$\sigma_y^2 = \sigma_a^2 + \sigma_b^2 + \sigma_c^2 = \frac{77}{12}$$

$$\sigma_y = \sqrt{\frac{77}{12}} = 2.533$$

b) Using a normal approximation, determine the probability that Y > 13

- note: For a uniform distribution over the range of (a,b)

$$\mu = \left(\frac{a+b}{2}\right), \qquad \sigma = \frac{b-a}{\sqrt{12}}$$

$$z = \left(\frac{13-7.5}{2.533}\right) = 2.171$$

From StatTrek, this corresponds to a probability of 1.5%

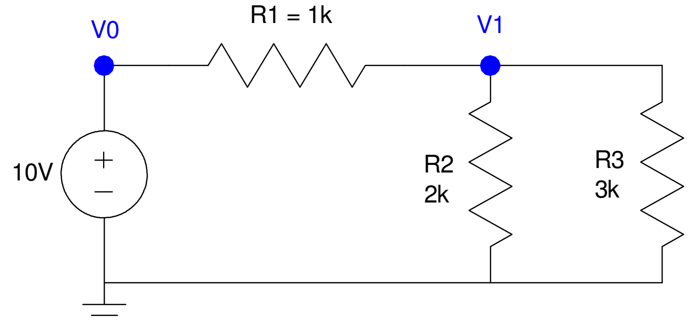
mean(Y)	std(Y)	p(Y < 13)
7.5	**2.53**	**1.5%**

5. Testing with Normal Distributions

Assume each resistor has 5% tolerance:

$$R = (1 + 0.05x)R_0$$

where x is a uniform distribution over the range of (-1, 1).



a) Determine V1 as a function of {R1, R2, R3}

$$R_{23} = \left(\frac{R_2 R_3}{R_2 + R_3}\right)$$

$$V_1 = \left(\frac{R_{23}}{R_1 + R_{23}}\right) 10V$$

b) Run a Monte Carlo simulation to solve for V1 with 100 random values for {R1, R2, R3}

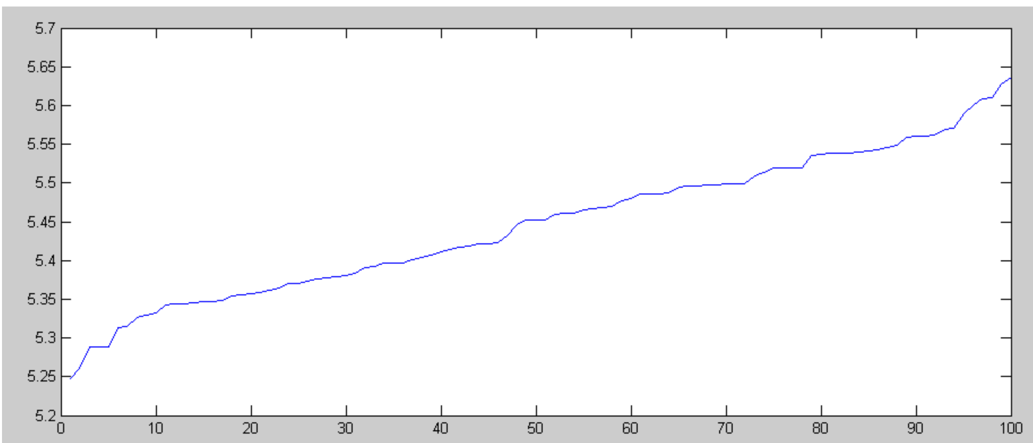
```
V1 = [];

for i=1:100

   R1 = 1000 * (1 + 0.05*(rand*2-1));
   R2 = 2000 * (1 + 0.05*(rand*2-1));
   R3 = 3000 * (1 + 0.05*(rand*2-1));

   R23 = 1/(1/R2 + 1/R3);
   V1 = [V1 ; (R23 / (R1 + R23)) * 10];
end

plot(sort(V1))
```



c) Determine the mean and standard deviation of V1

```
x = mean(V1)
s = std(V1)

x =    5.4460
s =    0.0903
```

d) Determine the 90% confidence interval for V1 using a normal approximation

mean(V1)	std(V1)	90% confidence interval for V1
5.446V	**0.0903V**	**5.2976V < V1 < 5.5945V**

5% tails corresponds to a z-score of 1.645

```
>> x = mean(V1)

x =    5.4460

>> s = std(V1)

s =    0.0903

>> x - 1.645*s

ans =    5.2976

>> x + 1.645*s

ans =    5.5945

>>
```