ECE 341 - Test #1

Combinations, Permitations, and Discrete Probability

Open-Book, Open Notes. Calculators, Matlab, Tarot cards allowed. Just not other people.

1. Enumeration and Dice

Let

$$M = \left(\frac{\text{birth month+14}}{5}\right) \text{ rounded down} \quad \text{(for example, February results in M = (2+14)/5 = 3.2 = 3)}$$

$$N = \left(\frac{\text{birth date + 30}}{10}\right) \text{ rounded down} \quad \text{(for example, the 14th results in N = (14+30)/10 = 4.4 = 4)}$$

M			N			
Jan - May	June - Oct	Nov - Dec	1-9	10-19	20-29	30-31
3	4	5	3	4	5	6

Assume you are rolling two dice:

- d1 = 1..M
- d2 = 1..N

Let Y be the difference betwen the two rolls

Determine through enumeration the probability that Y = {0..5}

M	N	p(Y=0)	p(Y=1)	p(Y=2)	p(Y=3)	p(Y=4)
3	4	3	5	3	1	0

y			N		
		1	2	3	4
M	1	0	1	2	3
	2	1	0	1	2
	3	2	1	0	1

2. Combinations and Permutations

Using combinations and permutations, calculate the odds of a full house (xxx yy) in 7-card stud poker

- You are dealt 7 cards
- One card value has three of a kind (xxx)
- Another card has two of a kind (yy)
- The other two cards could be anything except x (which would be 4 of a kind)
- *also except yy*

The number of ways to deal 7 cards is

$$N = \binom{52}{7} = 133,784,560$$

The number of hands that are full house are

hand = xxx yy ab or xxx yyy a

xxx yy ab

(13 values pick 1 for x)(4 x's, pick 3)(12 values left pick 1 for y)(4 y's, pick 2)(44 cards pick 2 for ab)

$$M = \binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}\binom{44}{2} = 3,541,824$$

xxx yyy a

(13 values pick 2 for xy)(4 x's pick 3)(4 y's pick 3)(44 cards pick 1 for a)

$$M_2 = \binom{13}{2}\binom{4}{3}\binom{4}{3}\binom{44}{1} = 54,912$$

$$M = M_1 + M_2 = 3,596,736$$

The odds then are

$$\boxed{p = \frac{M}{N} = 0.02688}$$

From a Monte-Carlo simulation in matlab,

p = 0.0262 (100,000 hands)

3. Binomial Distribution

Let

$$M = \left(\frac{\text{birth month+14}}{5}\right)$$ rounded down   (for example, February results in M = (2+14)/5 = 3.2 = 3)

$$N = \left(\frac{\text{birth date + 30}}{10}\right)$$ rounded down (for example, the 14th results in N = (14+30)/10 = 4.4 = 4)

Assume

- N-sided dice (rolls numbers 1..N)
- You roll 10 of these N-sided dice
- Y = the number of 1's and 2's on these ten dice.

What is the probability that Y = M?

M # successes	N N sided dice	p(y=M) M rolls or 1 or 2 on with 10 die rolls
3	**4**	**0.11719**

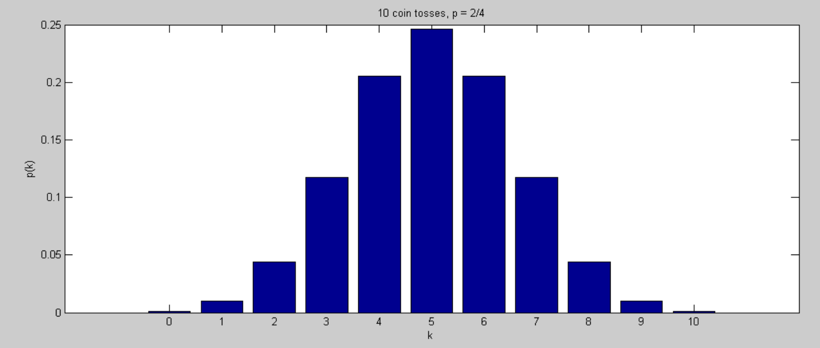
$$p = \left(\frac{2}{4}\right) = \left(\frac{1}{2}\right) \qquad q = 1 - p = \left(\frac{1}{2}\right)$$

$$p(3) = \begin{pmatrix} 10 \\ 3 \end{pmatrix} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^7 = 0.11719$$

You can also solve in Matlab

```
>> d1 = [0.5,0.5]';
>> d2 = conv(d1,d1);
>> d4 = conv(d2,d2);
>> d8 = conv(d4,d4);
>> d10 = conv(d8,d2);
>> k = [0:10]';
>> bar(k,d10)
>> title('10 coin tosses, p = 2/4');
>> xlabel('k');
>> ylabel('p(k)');
>> [k,d10]
```

```
     k        p(k)
       0     0.0010
  1.0000     0.0098
  2.0000     0.0439
  3.0000     0.1172
  4.0000     0.2051
  5.0000     0.2461
  6.0000     0.2051
  7.0000     0.1172
  8.0000     0.0439
  9.0000     0.0098
 10.0000     0.0010
```



4. Uniform Distribution and Convolution

Let

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$$N = \left(\frac{\text{birth date + 30}}{10}\right) \text{ rounded down} \quad \text{(for example, the 14th results in N = (14+30)/10 = 4.4 = 4)}$$

Assume

- N-sided dice (rolls numbers 1..N)
- You roll M of these N-sided dice
- Y = the sum of all M dice

a) Determine the pdf for Y: the sum of all of the dice

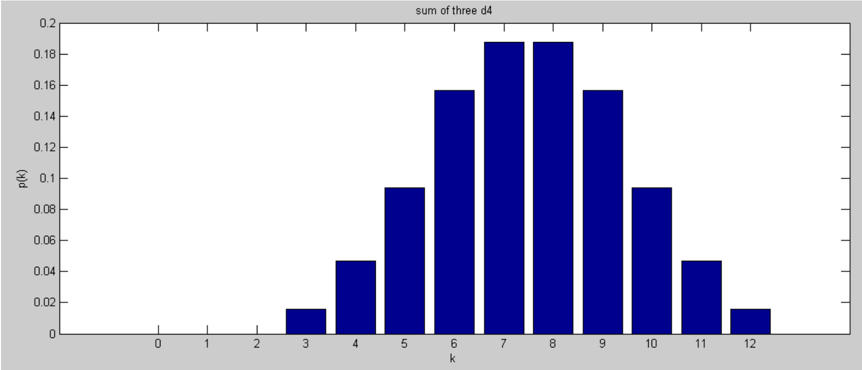
b) Determine the probability that the sum is 7 or less.

M	N	p(y = x)	p(y <= 7)
3	**4**	**see below**	**0.500**

a) Using Matlab

```
>> d1 = [0,1,1,1,1]' / 4;
>> d2 = conv(d1,d1);
>> d3 = conv(d1,d2);
>> k = [0:12]';
>> [k,d3]

         k        p(k)
           0         0
      1.0000         0
      2.0000         0
      3.0000    0.0156
      4.0000    0.0469
      5.0000    0.0938
      6.0000    0.1563
      7.0000    0.1875
      8.0000    0.1875
      9.0000    0.1563
     10.0000    0.0938
     11.0000    0.0469
     12.0000    0.0156
```



```
>> bar(k,d3)
>> % probability of 7 or less
>> sum(d3(1:8))

ans =    0.5000
```

5. Geometric & Pascal Distribution

Let

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$$N = \left(\frac{\text{birth date + 30}}{10}\right)$$ rounded down (for example, the 14th results in N = (14+30)/10 = 4.4 = 4)

Let

- d1 is an M-sided die (rolls numbers 1..M)
- d2 is an N-sided die (rolls the numbers 1..N)

Let Y be

- The number of times you have to roll d1 to get a 1 or 2, plus
- The number of times you have to roll d2 to get a 1.

Determine the explicit fumction for y(x) using z-transforms

- partial credit of you solve for the pdf of y(x) using a different method

M	N	p(y = k)
3 p = 2/3	**4** p = 1/4	$y(k) = \left(-0.4\left(\frac{1}{3}\right)^{k-1} + 0.4\left(\frac{3}{4}\right)^{k-1}\right)u(k-1)$

The moment generating functions are

$$M = \left(\frac{2/3}{z-1/3}\right)$$

$$N = \left(\frac{1/4}{z-3/4}\right)$$

$$Y = \left(\frac{2/3}{z-1/3}\right)\left(\frac{1/4}{z-3/4}\right)$$

using partial fractions

$$zY = \left(\left(\frac{-0.4}{z-1/3}\right) + \left(\frac{0.4}{z-3/4}\right)\right)z$$

$$zy(k) = \left(-0.4\left(\frac{1}{3}\right)^{k} + 0.4\left(\frac{3}{4}\right)^{k}\right)u(k)$$

$$y(k) = \left(-0.4\left(\frac{1}{3}\right)^{k-1} + 0.4\left(\frac{3}{4}\right)^{k-1}\right)u(k-1)$$

Solving using matlab (partial credit)

```
>> k = [0:50]';
>> d1 = (2/3) * (1/3).^(k-1);
>> d1(1) = 0;
>> d2 = (1/4) * (3/4).^(k-1);
>> d2(1) = 0;
>> y = conv(d1,d2);
>> y = y(1:51);
>> bar(k,y)
>> xlim([0,30])
>> xlabel('k');
>> ylabel('p(k)');
>>
>> [k,y]

        k         p(k)
          0          0
     1.0000          0
     2.0000     0.1667
     3.0000     0.1806
     4.0000     0.1539
     5.0000     0.1216
     6.0000     0.0933
     7.0000     0.0706
     8.0000     0.0532
     9.0000     0.0400
    10.0000     0.0300
      :           :
```