

ECE 341 - Homework #13

t-Tests. Due Wednesday, June 10th

Please make the subject "ECE 341 HW#13" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Test of a Single Population: Full-House in Draw Poker

The calculated odds of a full house in 5-card draw are $p = 0.013245$. Verify whether this is / is not correct with a probability of 90%

1) Run a Monte Carlo simulation to determine the odds of getting a full-house in 5-card draw

- Each simulation goes through 10,000 hands (# of full houses in 1,000 hands of poker)
- Run the simulation 5 times
- data = { x1, x2, x3, x4, x5 }

From this, determine the 90% confidence interval for the actual odds of getting a full-house with 5-card draw.

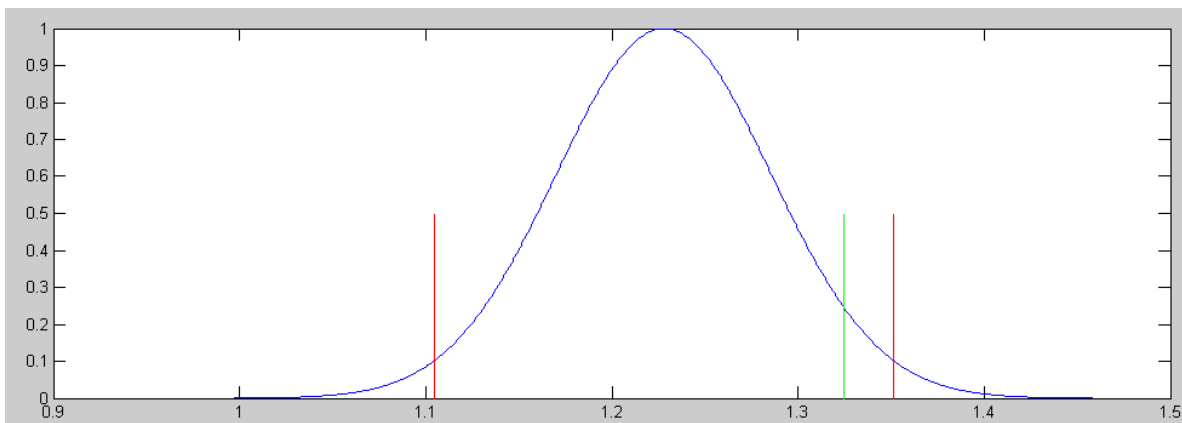
- if $p = 0.013245$ is in this interval, you cannot reject this answer with a probability of 90%

Data: Calculations from Test #1 did not account for flushes or straights. Doing likewise with a MonteCarlo simulation results in...

4-of-kind	full house	3-of-kind	2-pair	pair
26	111	794	1327	5221
17	142	743	1455	5124
31	130	791	1371	5124
20	116	768	1326	5190
28	115	758	1352	5241

```
FH = [111,142,130,116,115]
FH = 111 142 130 116 115
x = mean(FH)/100
x = 1.2280%
s = std(FH)/100 / sqrt(5)
s = 0.0577%
[x - 2.132*s, x + 2.132*s]
1.1049% 1.3511%
```

The computed odds are within the 90% confidence interval for getting a full house (no flushes)



pdf for getting a full-house with 90% confidence interval (red) and computed odds (green)

2) The height three people can jump is recorded (units = meters)

A: 0.413, 0.370, 0.345, 0.328, 0.424, 0.276, 0.494, 0.306, 0.419, 0.405
B: 0.390, 0.411, 0.543, 0.370, 0.425, 0.387, 0.556, 0.557, 0.603, 0.497
C: 0.649, 0.605, 0.628, 0.603, 0.645, 0.593, 0.637, 0.687, 0.635, 0.687

What is the 90% confidence interval for A? (two tails)

From StatTrek, the t-score corresponding to 5% tails and 9 degrees of freedom is

$$t = 1.833$$

A = [0.413, 0.370, 0.345, 0.328, 0.424, 0.276, 0.494, 0.306, 0.419, 0.405];

x = mean(A)

x = 0.3780

s = std(A)

s = 0.0654

[x - 1.833*s, x + 1.833*s]

0.2582 0.4978

It is 90% likely that A's jump will in in the interval (0.2582, 0.4978) meters

What is minimum height A will jump 90% of the time? (one tail)

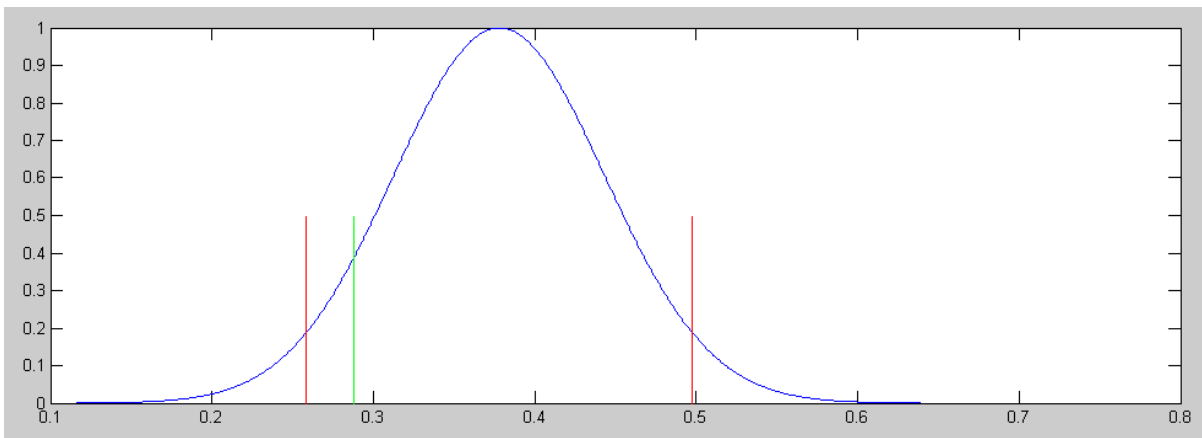
From StatTrek, the t-score corresponding to 10% tails and 9 degrees of freedom is

$$t = 1.383$$

x - 1.383*s

ans = 0.2876

It is 90% likely that A's jump will be at least 0.2876 meters



90% Confidence Intervals for A's jump: 2-Sided (red) and One-Sided (green)

Test of Two Populations

3) For the data set in problem #2:

A: 0.413, 0.370, 0.345, 0.328, 0.424, 0.276, 0.494, 0.306, 0.419, 0.405
B: 0.390, 0.411, 0.543, 0.370, 0.425, 0.387, 0.556, 0.557, 0.603, 0.497
C: 0.649, 0.605, 0.628, 0.603, 0.645, 0.593, 0.637, 0.687, 0.635, 0.687

What is the probability that A will jump higher than B the next time they jump?

A = [0.413, 0.370, 0.345, 0.328, 0.424, 0.276, 0.494, 0.306, 0.419, 0.405];
B = [0.390, 0.411, 0.543, 0.370, 0.425, 0.387, 0.556, 0.557, 0.603, 0.497];

Let $W = A - B$

$$\begin{aligned} X_w &= \text{mean}(A) - \text{mean}(B) \\ &= -0.0959 \end{aligned}$$

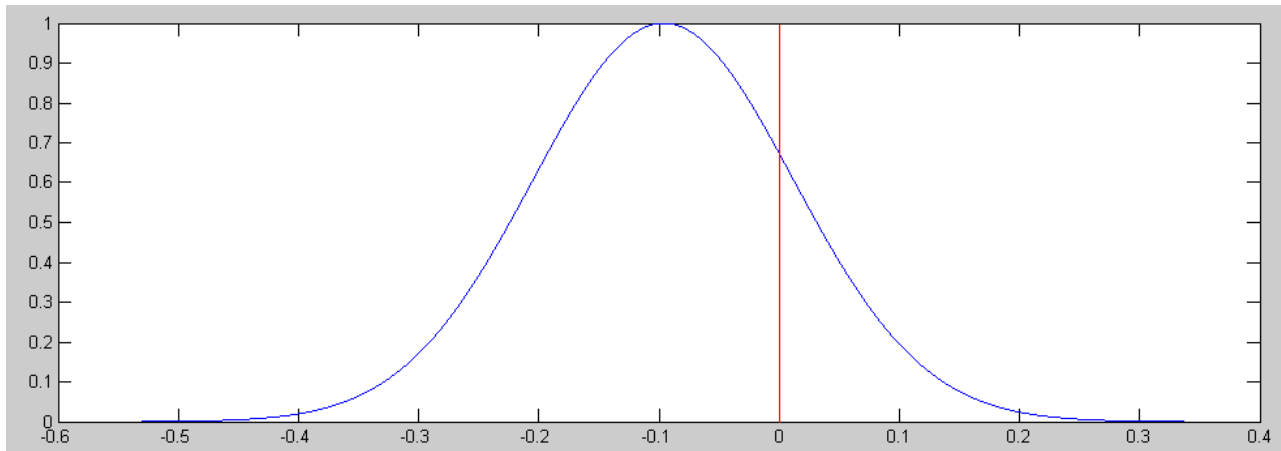
$$\begin{aligned} S_w &= \sqrt{\text{var}(A) + \text{var}(B)} \\ &= 0.1084 \end{aligned}$$

$$t = (X_w - 0) / S_w$$

$$t = -0.8845$$

From stat-trek, this corresponds to a probability of 0.1997

There is a 19.97% chance that A will jump higher than B next jump



pdf of $W = A - B$. The area to the right of zero is the probability that $A > B$

What is the probability that B's average is larger than A's average?

Let $W = A - B$

$$X_w = \text{mean}(A) - \text{mean}(B)$$

$$= -0.0959$$

$$S_w = \sqrt{\text{var}(A)/10 + \text{var}(B)/10}$$

$$= 0.0343$$

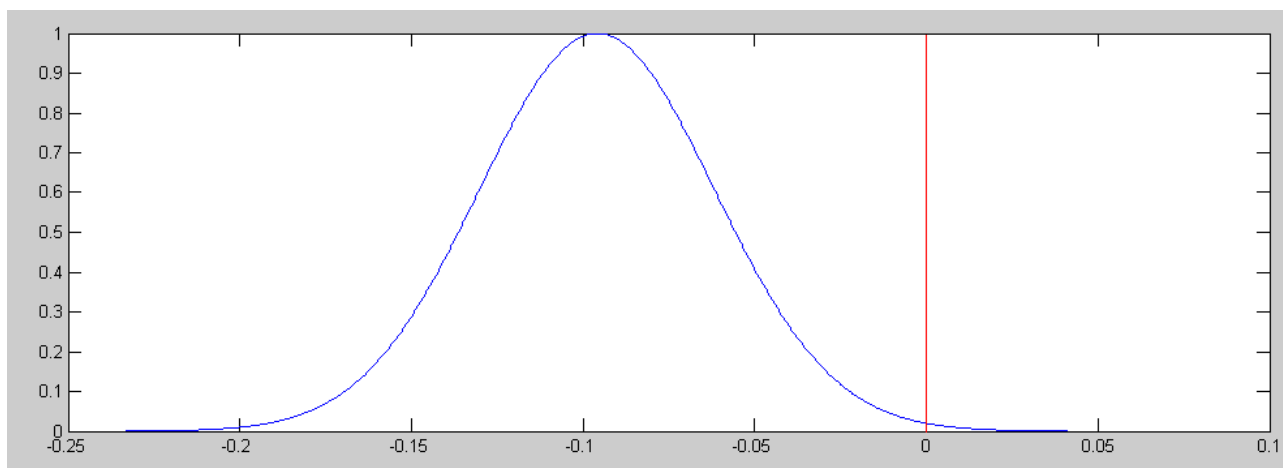
$$t = X_w / S_w$$

$$t = -2.7970$$

From stat-trek, this corresponds to a probability of 0.0104

There is a 1.04% chance that A's average is larger than B's.

There is a 98.96% chance that B's average is larger than A's.



pdf for $W = \text{mean}(A) - \text{mean}(B)$, The area to the right of zero is the probability that $A > B$

The reflex time of a person before and after drinking 2 shots is measured

Trial	Person A		Person B		Person C	
	sober	2 drinks	sober	2 drinks	sober	2 drinks
#1	0.2253	0.2559	0.1924	0.2721	0.2419	0.3012
#2	0.1923	0.3488	0.1893	0.2197	0.1976	0.2556
#3	0.1854	0.244	0.2081	0.2438	0.3063	0.2451

4) What is the probability that A has a faster reaction time than B?

Option 1: Consider sober vs. sober

- good: consistent experiment should give a lower variance (easier to see small differences)
- bad: ignores half of the data

A = [0.2253, 0.1923, 0.1854];

B = [0.1924, 0.1893, 0.2018];

Let $W = A - B$

$X_w = \text{mean}(A) - \text{mean}(B)$

= 0.0065

$S_w = \sqrt{\text{var}(A)/3 + \text{var}(B)/3}$

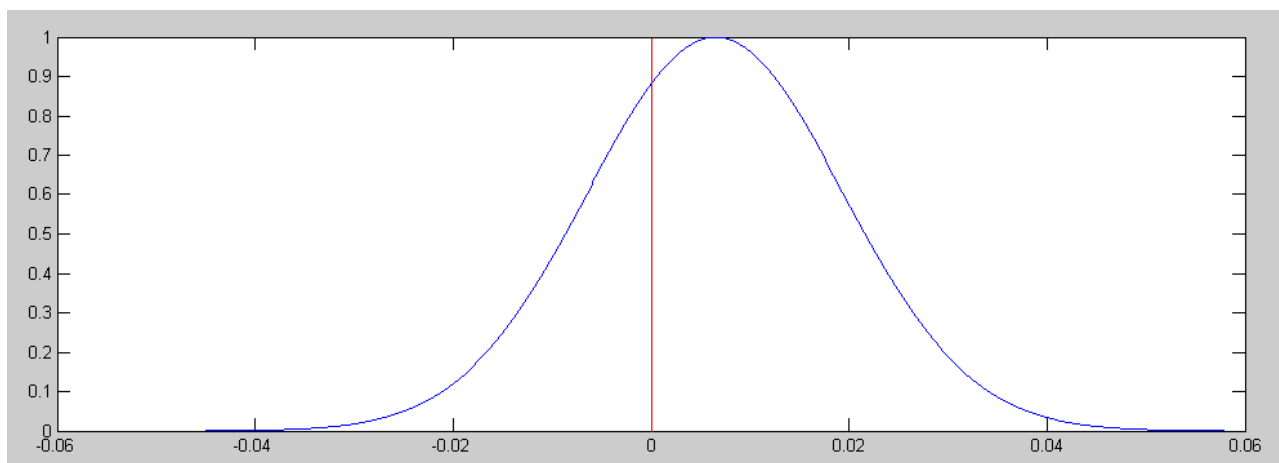
= 0.0129

$t = X_w / S_w$

$t = 0.5049$

From StatTrek, this corresponds to a probability of 0.6681

It is 66.81% likely that A has a larger (worse) reaction time than B



pdf of $W = A - B$. Area to the right is the probability that $A > B$ (slower reflex)

Option 2: Use all of the data (sober and 2 drinks)

- good: uses all of the data
- bad: mixes experiment, resulting in larger variations

```
A = [0.2253, 0.1923, 0.1854, 0.2559, 0.3488, 0.244];  
B = [0.1924, 0.1893, 0.2018, 0.2721, 0.2197, 0.2438];
```

```
Xw = mean(A) - mean(B)
```

```
= 0.0221
```

```
Sw = sqrt( var(A)/6 + var(B)/6 )
```

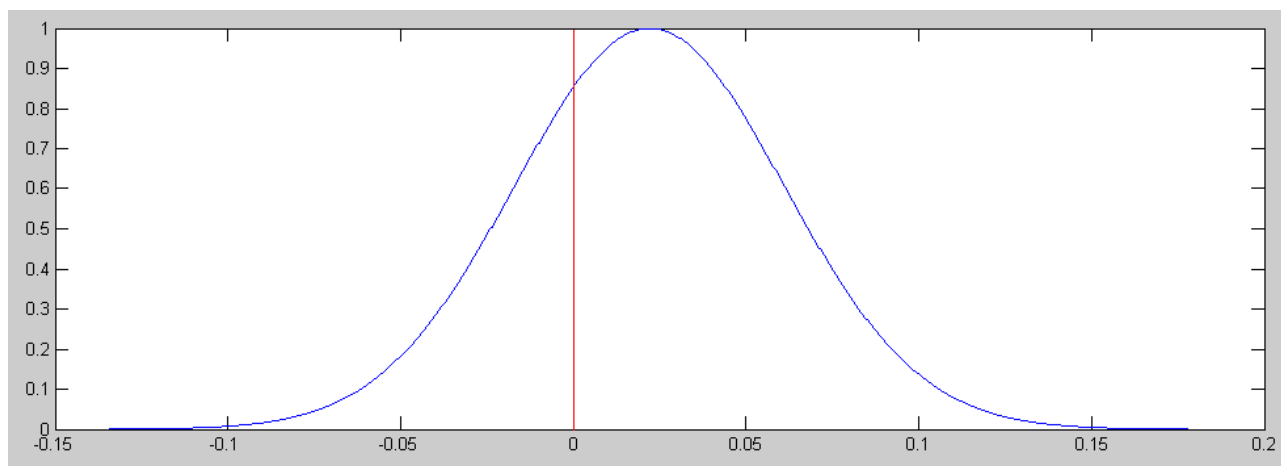
```
= 0.0276
```

```
t = Xw / Sw
```

```
t = 0.8004
```

From StatTrek, this corresponds to a probability of 0.7730

It is 77.30% likely that A has a larger (worse) reaction time than B



pdf of $W = A - B$. Area to the right is the probability that $A > B$

5) What is the probability that your reaction time after drinking 2 shots increases?

Trial	Person A		Person B		Person C	
	sober	2 drinks	sober	2 drinks	sober	2 drinks
#1	0.2253	0.2559	0.1924	0.2721	0.2419	0.3012
#2	0.1923	0.3488	0.1893	0.2197	0.1976	0.2556
#3	0.1854	0.244	0.2081	0.2438	0.3063	0.2451

Option #1: Group all sober and 2-drings together

- good: uses all the data
- bad: increased variation due to different people in each group

```
sober = [0.2253,0.1923,0.1854,0.1924,0.1893,0.2018,0.2419,0.1976,0.3063]
drink2 = [0.2559,0.3488,0.244,0.2721,0.2197,0.2438,0.3013,0.2556,0.2451]
```

```
Xw = mean(drink2) - mean(sober)
    = 0.0504
```

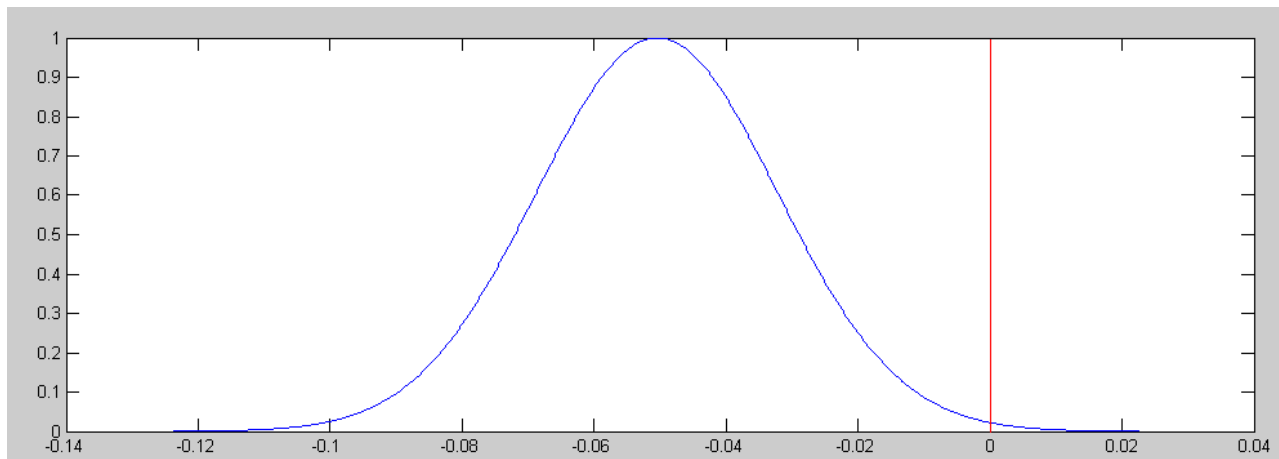
```
Sw = sqrt(var(drink2)/9 + var(sober)/9)
    = 0.0183
```

```
t = Xw / Sw
```

```
t = 2.7570
```

From StatTrek, this gives $p = 0.9889$

It is 98.89% likely that your reaction time increases after 2 drinks



pdf of $W = \text{sober} - 2 \text{ drinks}$. The area to the right is the probability that sober > 2 drinks (slower reflex)

6) Hector airport has been recording weather in Fargo since 1942.

http://www.bisonacademy.com/ECE111/Code/Fargo_Weather_Monthly_Avg.txt

Determine the probability that (April, 2000 - 2020) is warmer than (April, 1942 - 1962)

April 2020 was 39.649F (average)

<https://ndawn.ndsu.nodak.edu/station-info.html?station=23>

```
April = DATA(:,5);
size(April)

ans =

    78     1

April = [April;39.649];
X = April(1:21);
Y = April(59:79);

Xw = mean(Y) - mean(X)

    =    1.2452

Sw = sqrt( var(Y)/21 + var(X)/21 )

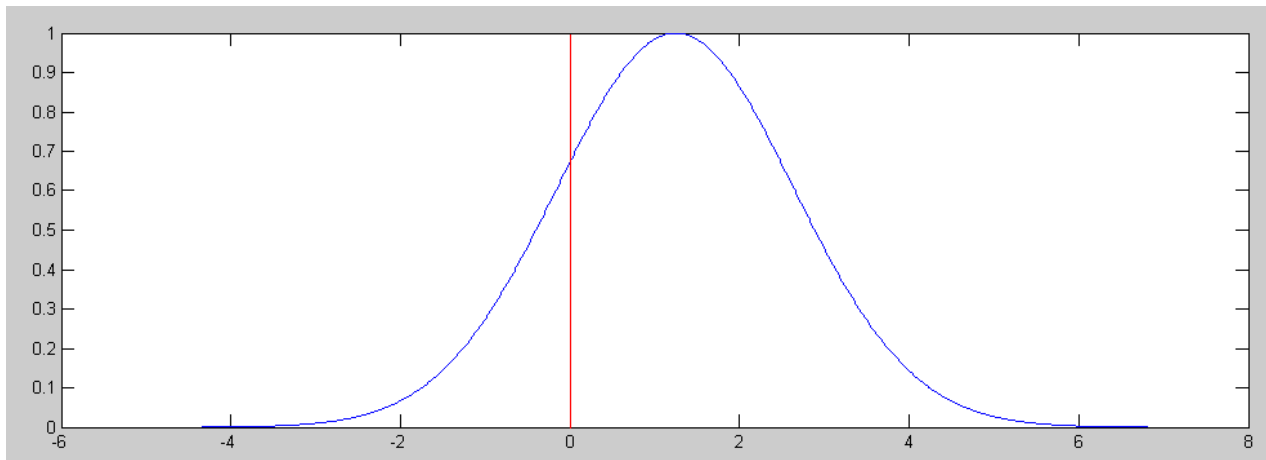
    =    1.3964

t = Xw / Sw

t =    0.8917
```

From StatTrek, this corresponds to a t-score of 0.8084

It is 80.84% likely that the last 21 years were warmer than 60 years ago



pdf of $W = (2020-2000) - (1942-1962)$
Area to the right of zero is the probability that it got warmer