ECE 341 - Homework #8

Queueing Theory & Normal Distributions. Due Tuesday, June 2nd

Please make the subject "ECE 341 HW#8" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Queueing Theory

Assume you are running a fast-food restraunt.

- The time between customers arriving at a restaraunt is an exponential distribution with a mean of 60 seconds.
- The time it takes to serve each customer is an exponential distribution with a mean of 40 seconds.

1) Run a single Monte-Carlo simulation for this restaraunt over the span of one hour.

- Give the formula for each column in you simulation
- What is the longest waiting time for a customer in your simulation?
- What is the largest queue over the span of one hour?

Matlab code to generate randome arrival and serving times

```
DATA = [];
for i=1:100
 p = rand;
 Tarr = -60*log(1-p);
 p = rand;
 Tser = -40*log(1-p);
 DATA = [DATA ; [i, Tarr, Tser]];
end
round(DATA)
```

Result

```
     1    32     9
     2    79    23
     3    28    14
     4    15    56
     5     4    14
     6     7    73
     7     2     8
     8    26    66
```

Customer #11

- Had the longest wait (191 second)
- Had the largest queue (5 deep)

Equations:

E: E11 = E10+C11

F: F11=MAX(G10,E11)

G: G11=F11+D11

H: H11=F11-E11

I: I11 = (E11<G10)*1+(E11<G9)*1+(E11<G8)*1+(E11<G7)*1+(E11<G6)*1+(E11<G5)*1

B	C	D	E	F	G	H	I
Customer	**Arrival**	**Serve**	**t(Arr)**	**t(Serve)**	**t(done)**	**t(Wait)**	**Queue**
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
1	32	9	32	32	41	0	0
2	79	23	111	111	134	0	0
3	28	14	139	139	153	0	0
4	15	56	154	154	210	0	0
5	4	14	158	210	224	52	1
6	7	73	165	224	297	59	2
7	2	8	167	297	305	130	3
8	26	66	193	305	371	112	4
9	40	7	233	371	378	138	3
10	23	69	256	378	447	122	4
11	**0**	**6**	**256**	**447**	**453**	**191**	**5**
12	88	6	344	453	459	109	4
13	42	10	386	459	469	73	3
14	124	52	510	510	562	0	0
15	405	14	915	915	929	0	0
16	43	41	958	958	999	0	0
17	31	80	989	999	1,079	10	1
18	33	1	1,022	1,079	1,080	57	1
19	10	13	1,032	1,080	1,093	48	2
20	4	56	1,036	1,093	1,149	57	3
21	132	2	1,168	1,168	1,170	0	0
22	14	3	1,182	1,182	1,185	0	0
23	51	17	1,233	1,233	1,250	0	0
24	6	45	1,239	1,250	1,295	11	1
25	79	5	1,318	1,318	1,323	0	0
26	57	48	1,375	1,375	1,423	0	0
27	103	14	1,478	1,478	1,492	0	0
28	100	2	1,578	1,578	1,580	0	0
29	6	49	1,584	1,584	1,633	0	0
30	48	68	1,632	1,633	1,701	1	1
31	12	54	1,644	1,701	1,755	57	1

32	4	27	1,648	1,755	1,782	107	2
33	4	8	1,652	1,782	1,790	130	3
34	15	9	1,667	1,790	1,799	123	4
35	55	36	1,722	1,799	1,835	77	4
36	153	32	1,875	1,875	1,907	0	0
37	60	15	1,935	1,935	1,950	0	0
38	27	18	1,962	1,962	1,980	0	0
39	110	15	2,072	2,072	2,087	0	0
40	132	50	2,204	2,204	2,254	0	0
41	104	60	2,308	2,308	2,368	0	0
42	25	2	2,333	2,368	2,370	35	1
43	19	3	2,352	2,370	2,373	18	2
44	67	87	2,419	2,419	2,506	0	0
45	64	16	2,483	2,506	2,522	23	1
46	17	13	2,500	2,522	2,535	22	2
47	72	20	2,572	2,572	2,592	0	0
48	103	12	2,675	2,675	2,687	0	0
49	6	53	2,681	2,687	2,740	6	1
50	33	9	2,714	2,740	2,749	26	1
51	56	26	2,770	2,770	2,796	0	0
52	195	2	2,965	2,965	2,967	0	0
53	76	70	3,041	3,041	3,111	0	0
54	13	68	3,054	3,111	3,179	57	1
55	103	33	3,157	3,179	3,212	22	1
56	64	23	3,221	3,221	3,244	0	0
57	10	29	3,231	3,244	3,273	13	1
58	28	17	3,259	3,273	3,290	14	1
59	55	63	3,314	3,314	3,377	0	0
60	7	128	3,321	3,377	3,505	56	1
61	117	17	3,438	3,505	3,522	67	1
62	107	10	3,545	3,545	3,555	0	0
63	43	42	3,588	3,588	3,630	0	0

Normal Distribution

The low for the month has been measured at Hector Airport since 1942. The mean and standard deviations are:

Month	May	June	July	Aug	Sept	Oct
Mean	27.4013F	40.2179F	46.2949F	43.2321F	30.5526F	19.3462F
st dev	4.4236F	3.9924F	3.9481F	4.1435F	4.8050F	5.1265F

http://www.bisonacademy.com/ECE111/Code/Fargo_Weather_Monthly_Low.txt

2) What is the probability that we will have a killing frost (temperature drops below 30F) in

May: p = 0.722

$$z = \left(\frac{30 - 27.4013}{4.4236} \right) = 0.587463$$

From StatTrek, this corresponds to a probability of 0.722

June: p = 0.001

$$z = \left(\frac{30 - 40.2179}{3.9924} \right) = -2.5593$$

From StatTrek, this corresponds to a probability of 0.001

July: p < 0.0005

$$z = \left(\frac{30 - 46.2949}{3.9481} \right) = -4.1272$$

From StatTrek, this corresponds to a probability of 0.000 (meaning is is less than 0.0005 rounded down)

Rainfall

The rainfall in Fargo each month (in inches) is

Month	May	June	July	Aug	Sept	Oct
Mean	2.6549	3.5025	2.9668	2.6529	2.1344	1.694
st dev	1.6536	2.1054	1.9505	1.7339	1.4913	1.4619

3) What is the probability that we will get more than 10 inches of rain in the months of June, July, and August (combined)?

The sum will have a mean and variance equal to the sum

$$\mu = \mu_{june} + \mu_{july} + \mu_{aug}$$

$$\mu = 9.1222$$

$$\sigma^2 = \sigma^2_{\ june} + \sigma^2_{\ july} + \sigma^2_{\ a}$$

$$\sigma^2 = 11.2435$$

$$\sigma = 3.3531$$

The z-score for 10 inches is

$$z = \left(\frac{9.1222-10}{3.3531}\right) = -0.2617$$

From StatTrek, this corresponds to a probability of 0.397

There is a 39.7% chance Fargo will get more than 10" of rain this summer

4) What is the probability that we will get no rain over these 6 months

Month	May	June	July	Aug	Sept	Oct	Sum
Mean	2.6549	3.5025	2.9668	2.6529	2.1344	1.694	15.6055
st dev	1.6536	2.1054	1.9505	1.7339	1.4913	1.4619	4.2824*
Var	2.7344	4.4327	3.8045	3.0064	2.2240	2.1372	18.3391

* standard deviation is the root sum of squares of the rain for the summer

The z-score for 0" if rain is 0.007

$$z = \left(\frac{0-15.6055}{4.2824}\right) = -3.6441$$

There probability of no rain over 6 months is 0.001 (1000 : 1 odds against)