ECE 341 - Homework #7

Uniform and Exponential Distributions. Due Monday, June 1st

Please make the subject "ECE 341 HW#7" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Uniform Distributions

Let

- a be a sample from A, a uniform distribution over the range of (0, 1)
- b be a sample from B, a uniform distribution over the range of (0,6)
- c be a sample from C, a uniform distribution over the range of (0,10)

1) Determine the pdf for a + b using moment generating funcitons (i.e. LaPlace transforms)

$$A(s) = \left(\frac{1}{s}\right)(1 - e^{-s})$$

$$B(s) = \left(\frac{1}{6s}\right)(1 - e^{-6s})$$

$$Y = AB = \left(\left(\frac{1}{s}\right)(1 - e^{-s})\right)\left(\left(\frac{1}{6s}\right)(1 - e^{-6s})\right)$$

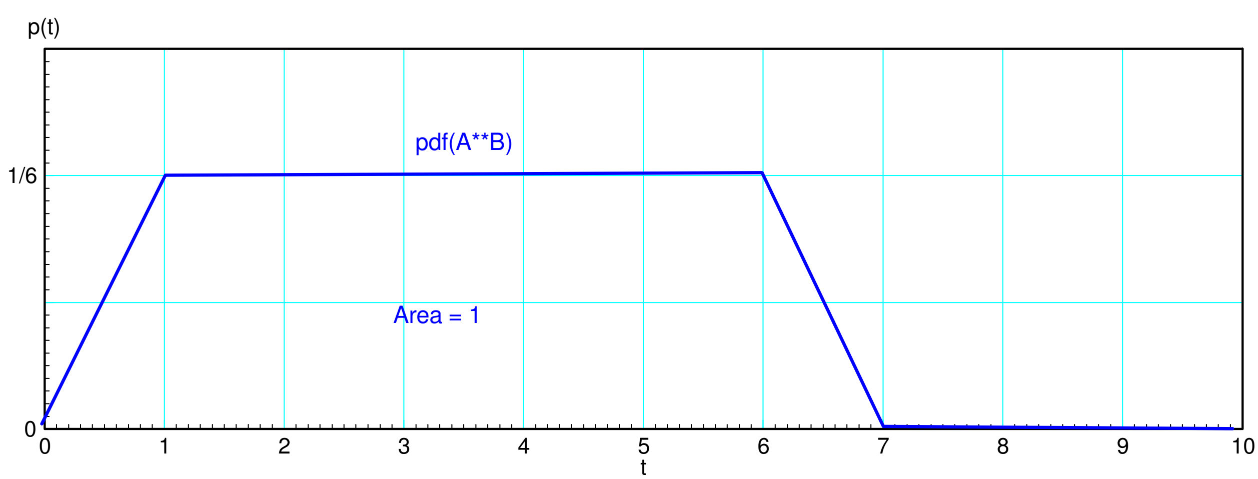
$$Y = \left(\frac{1}{6s^2}\right)(1 - e^{-s})(1 - e^{-6s})$$

$$Y = \left(\frac{1}{6s^2}\right)(1 - e^{-s} - e^{-6s} + e^{-7s})$$

Take the inverse LaPlace trasform

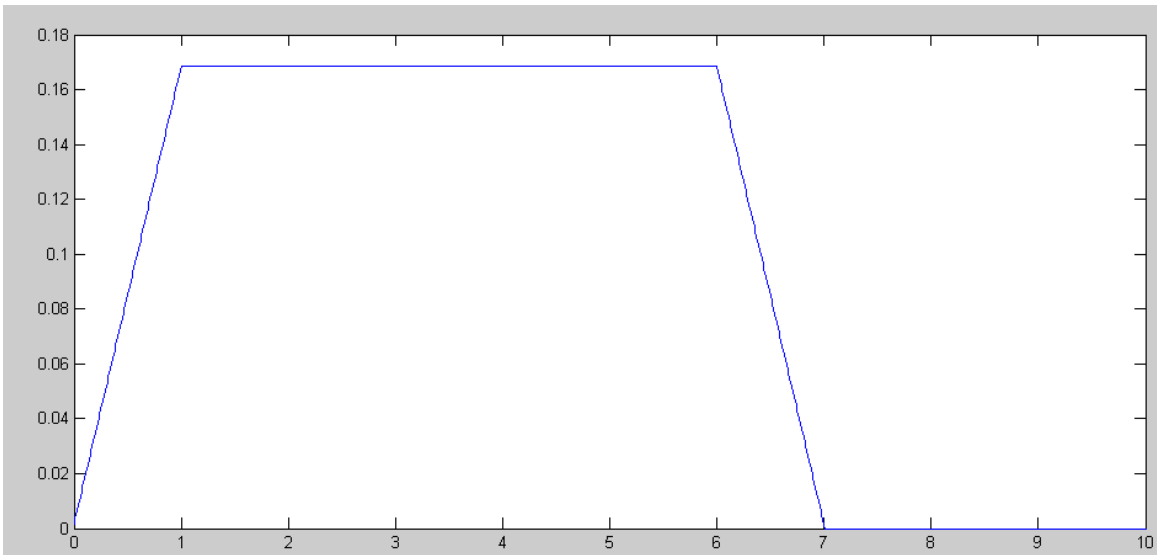
$$y(t) = \left(\frac{1}{6}\right)(tu(t) - (t-1)u(t-1) - (t-6)u(t-6) + (t-7)u(t-7))$$

This is a trapezoid:



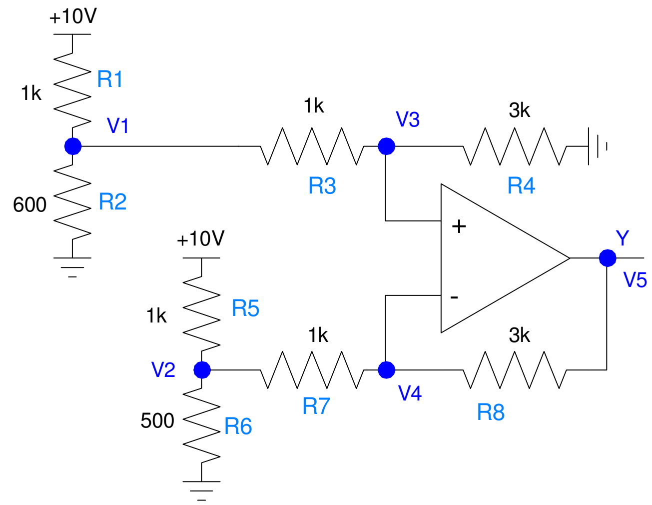
2) Determine the pdf for a + b using convolution (by hand or Matlab)

```
t = [0:0.01:10]';
A = 1 * (t<=1);
B = 1/6 * (t <= 6);
dt = 0.01;
Y = conv(A,B)*dt;
ty = [0:length(Y)-1]' * dt;
plot(ty,Y)
xlim([0,10])
```



which is the same as we got in problem #1

3) Assume each resistor has a tolerance of 5% (i.e. a uniform distribution over the range of (0.95, 1.05) of the nominal value. Determine the mean and standard deviation for the voltage at Y for the following circuit.



Write the node equations

$$V_3 = V_4$$

$$\left(\frac{V_1-10}{R_1}\right) + \left(\frac{V_1}{R_2}\right) + \left(\frac{V_1-V_3}{R_3}\right) = 0$$

$$\left(\frac{V_2-10}{R_5}\right) + \left(\frac{V_2}{R_6}\right) + \left(\frac{V_2-V_4}{R_7}\right) = 0$$

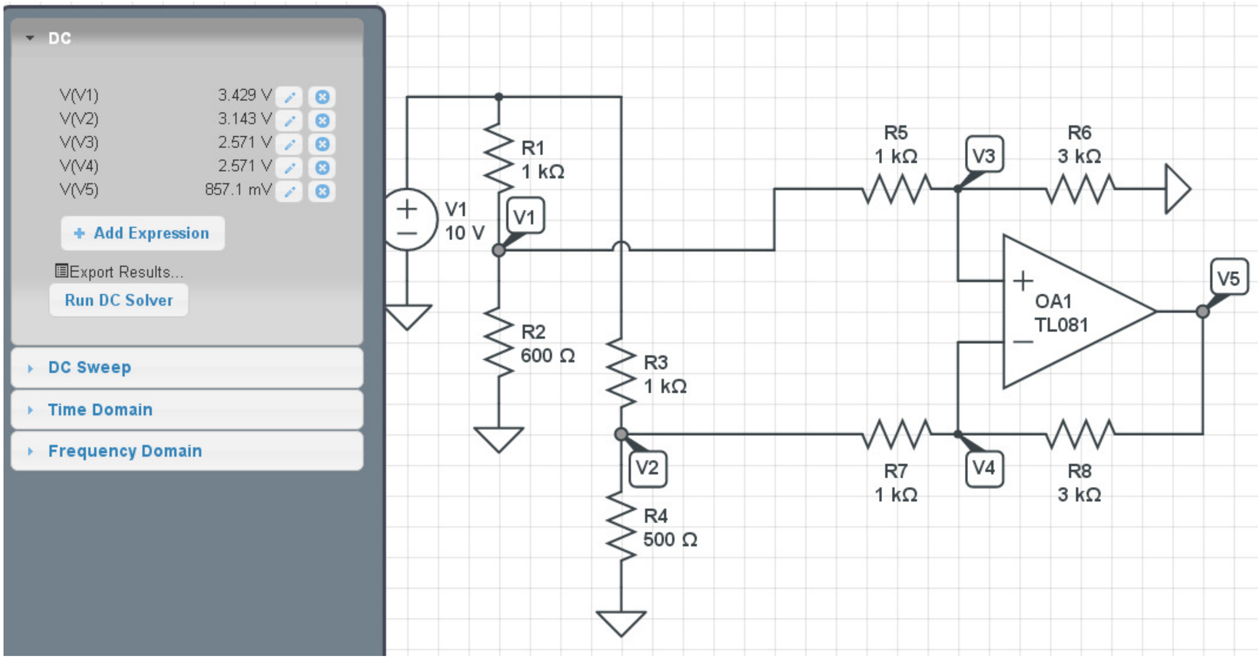
$$\left(\frac{V_3-V_1}{R_3}\right) + \left(\frac{V_3}{R_4}\right) = 0$$

$$\left(\frac{V_4-V_2}{R_7}\right) + \left(\frac{V_4-V_5}{R_8}\right) = 0$$

Group terms and place in matrix form

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) & 0 & \left(\frac{-1}{R_3}\right) & 0 & 0 \\ 0 & \left(\frac{1}{R_5}+\frac{1}{R_6}+\frac{1}{R_7}\right) & 0 & \left(\frac{-1}{R_7}\right) & 0 \\ \left(\frac{-1}{R_3}\right) & 0 & \left(\frac{1}{R_3}+\frac{1}{R_4}\right) & 0 & 0 \\ 0 & \left(\frac{-1}{R_7}\right) & 0 & \left(\frac{1}{R_7}+\frac{1}{R_8}\right) & \left(\frac{-1}{R_8}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ \left(\frac{10}{R_1}\right) \\ \left(\frac{10}{R_5}\right) \\ 0 \\ 0 \end{bmatrix}$$

Pick random values for R and solve for V5

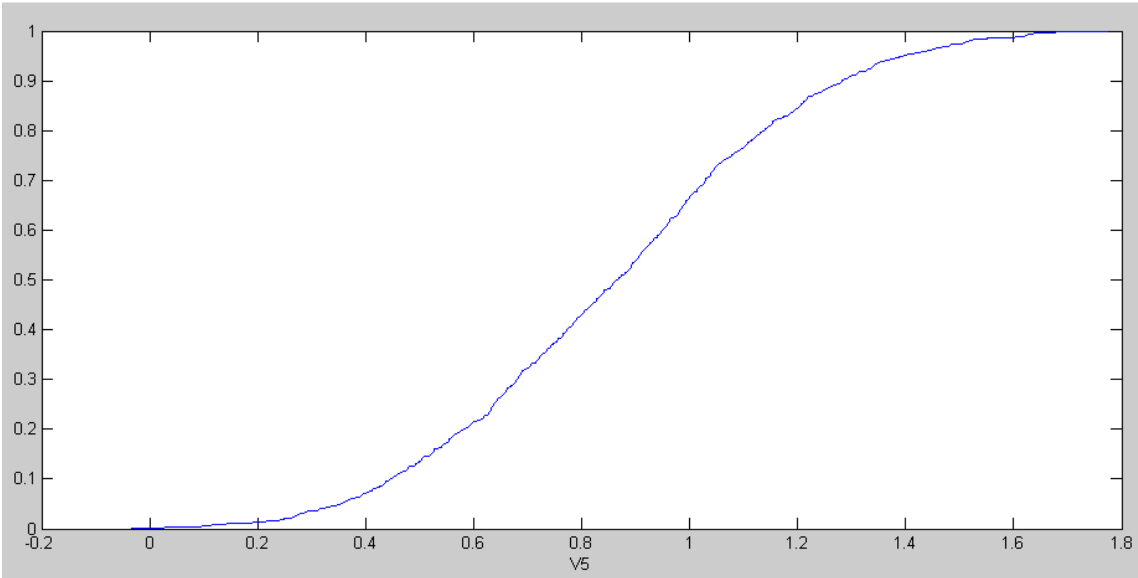


Use the nominal values and check agains CircuitLab

```
% Homework #7 problem #3

R1 = 1000 * (1 + 0.0*(2*rand-1));
R2 =  600 * (1 + 0.0*(2*rand-1));
R3 = 1000 * (1 + 0.0*(2*rand-1));
R4 = 3000 * (1 + 0.0*(2*rand-1));
R5 = 1000 * (1 + 0.0*(2*rand-1));
R6 =  500 * (1 + 0.0*(2*rand-1));
R7 = 1000 * (1 + 0.0*(2*rand-1));
R8 = 3000 * (1 + 0.0*(2*rand-1));

a1 = [0,0,1,-1,0];
a2 = [1/R1+1/R2+1/R3,0,-1/R3,0,0];
a3 = [0,1/R5+1/R6+1/R7,0,-1/R7,0];
a4 = [-1/R3,0,1/R3+1/R4,0,0];
a5 = [0,-1/R7,0,1/R7+1/R8,-1/R8];
A = [a1;a2;a3;a4;a5];
B = [0;10/R1;10/R5;0;0];
V = inv(A)*B

    3.4286
    3.1429
    2.5714
    2.5714
    0.8571
```

This matches CircuitLab, so it looks like the equations are correct. Change the percentages to 5% and run 1000 times

```
p = [1:1000]' / 1000;
plot(V5,p)
xlabel('V5');
```



cdf for V5

This gives the cdf. To determine the pdf, you could use a Weibull distribution (see homework #9)

The mean and standard deviation are

```
mean(V5)

ans =     0.8637

std(V5)

ans =     0.3188
```

Exponential Distributions

Let

- d be a sample from D, an exponential distribution with a mean of 5
- e be a sample from E, an exponential distribution with a mean of 10
- f be a sample from F, an exponential distribution with a mean of 15

4) Use moment generating functions to determine the pdf for d + d + d (i.e. the time for three events to be observed in D)

$$d(t) = 0.2\ e^{-0.2t}\ u(t)$$

$$D(s) = \left(\frac{0.2}{s+0.2}\right)$$

$$y(t) = d(t) ** d(t) ** d(t)$$

$$Y(s) = D(s) \cdot D(s) \cdot D(s)$$

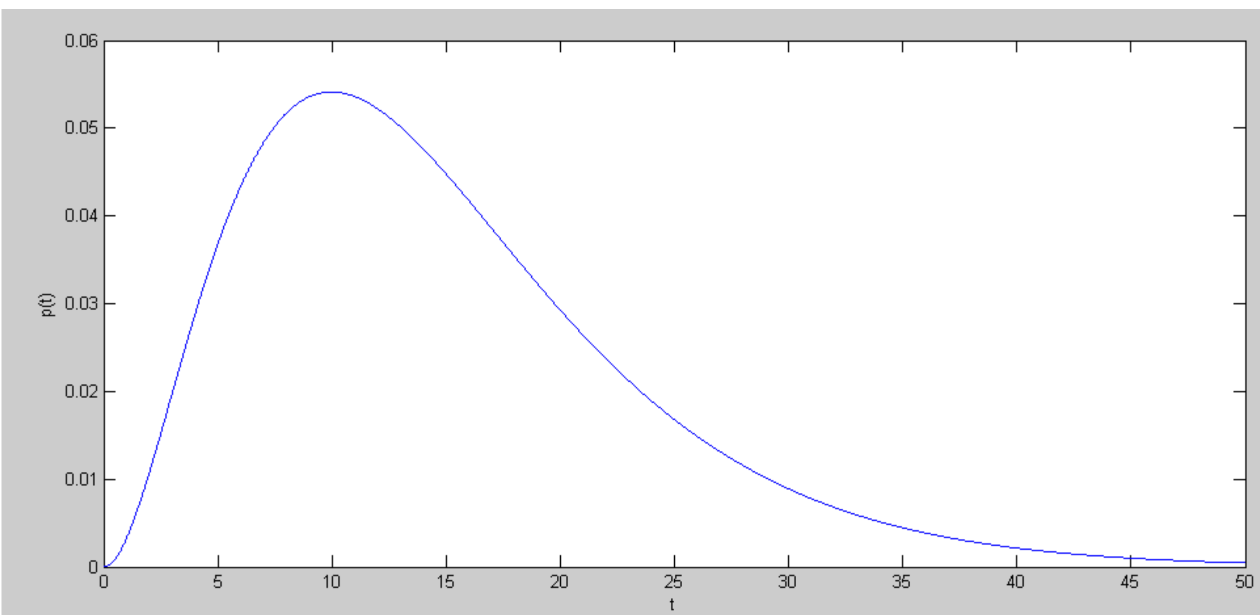
$$Y(s) = \left(\frac{0.2^3}{(s+0.2)^3}\right)$$

From the table of LaPlace transforms

$$\left(\frac{2}{(s+b)^3}\right) \rightarrow t^2\ e^{-bt}\ u(t)$$

$$Y(s) = \left(\frac{0.2^3}{2}\right)\left(\frac{2}{(s+0.2)^3}\right)$$

$$y(t) = \left(\frac{0.2^3}{2}\right)\ t^2\ e^{-0.2t}\ u(t)$$



5) Use moment generating functions to determine the pdf for the sum: d + e + f (i.e. the time for one event from D, E, and F)

$$D(s) = \left(\frac{0.2}{s+0.2}\right)$$

$$E(s) = \left(\frac{0.1}{s+0.1}\right)$$

$$F(s) = \left(\frac{0.0667}{s+0.0667}\right)$$

$$Y = \left(\frac{0.2}{s+0.2}\right)\left(\frac{0.1}{s+0.1}\right)\left(\frac{0.0667}{s+0.0667}\right)$$

Expand using partial fractions

$$Y = \left(\frac{0.1}{s+0.2}\right) + \left(\frac{-0.4}{s+0.1}\right) + \left(\frac{0.3}{s+0.0667}\right)$$

Take the inverse LaPlace transform

$$y(t) = (0.1\ e^{-0.2t} - 0.4\ e^{-0.1t} + 0.3\ e^{-0.0667t})u(t)$$