ECE 341 - Solutions to Homework #5

(change) Geometric, Pascal. Due May 27th

Please make the subject "ECE 341 HW#5" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

1) Let

- A be the number of times you roll a 6-sided die until you roll a 1
- B be the result of rolling a six-sided die.

What is the pdf of A + B? (hint: use colvolution)

This is the colvolution of a geometric distribution and a uniform distribution

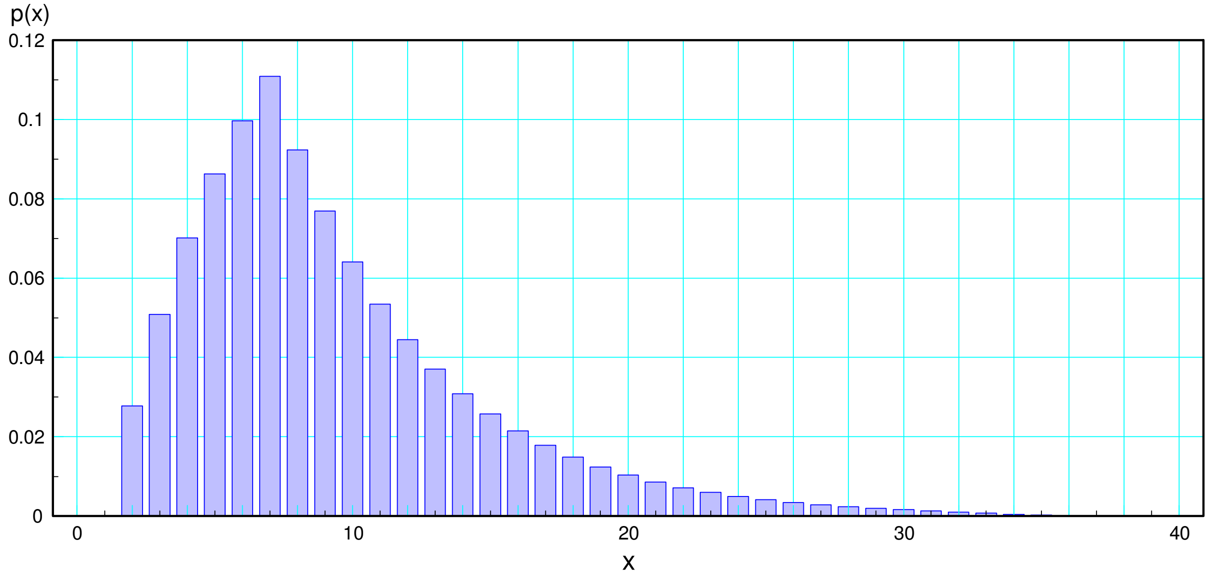
$$A(x) = \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{x-1} u(x-1)$$

$$B(x) = \left\{0, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$$

In matlab: In theory, x should go out to infinity. Go far enough so that the probability is approximately zero. After 30 terms, A(30) = 0.0007, which is close enough to zero to ignore all subsequent terms...

```
x = [0:30]';
A = (1/6) * (5/6).^(x-1) .* (x>=1);
B = [0,1,1,1,1,1,1]/6;
Y = conv(A,B);

0         0
1         0
2    0.0278     check:  should get first non-zero term at x=2
3    0.0509
4    0.0702
5    0.0863
```



pdf for Y = A + B

2) Let

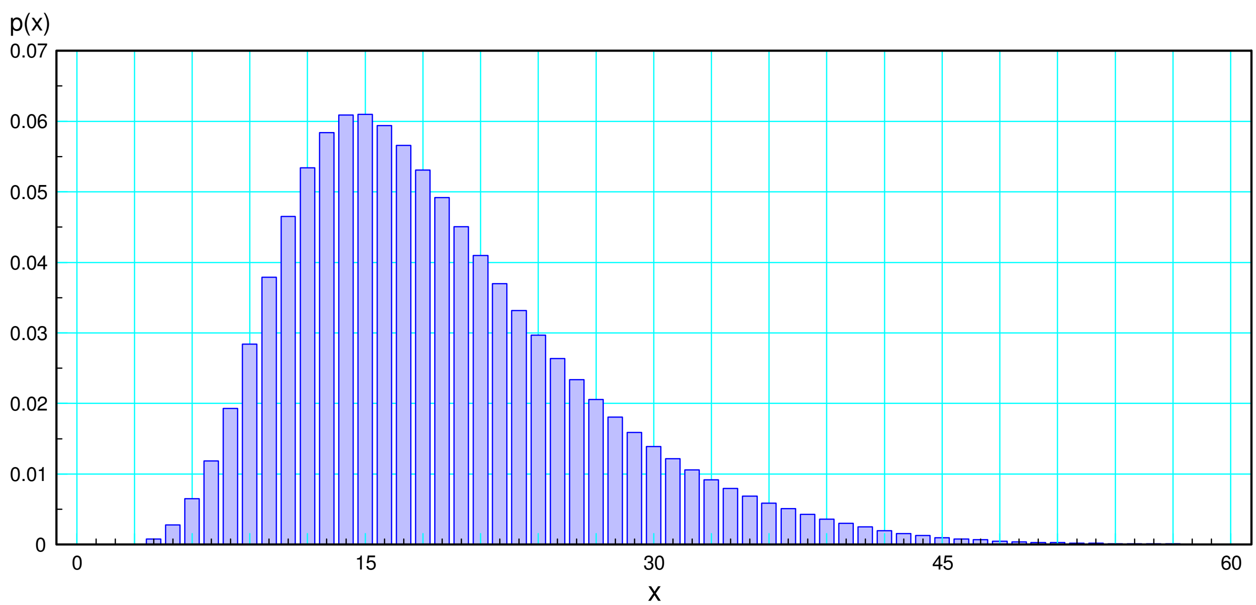
- A be the the number of times you roll a 6 sided die until you roll a 1 two times
- B be the sum of two 6-sided dice

What is the pdf A + B? (hint: convolution again)

Use convolution in Matlab

```
A2 = conv(A,A);
B2 = conv(B,B);
Y = conv(A2,B2);

0         0
1         0
2         0
3         0
4    0.0008            check:  should get first non-zero term at x=4
5    0.0028
6    0.0065
```



pdf for Y = A + B

3) Let

- A be the number of times you roll a 6-sided die until you roll a 1 (p = 1/6).
- B be the number of times you roll a 6-sided die until you get a 1 or 2 (p = 1/3)

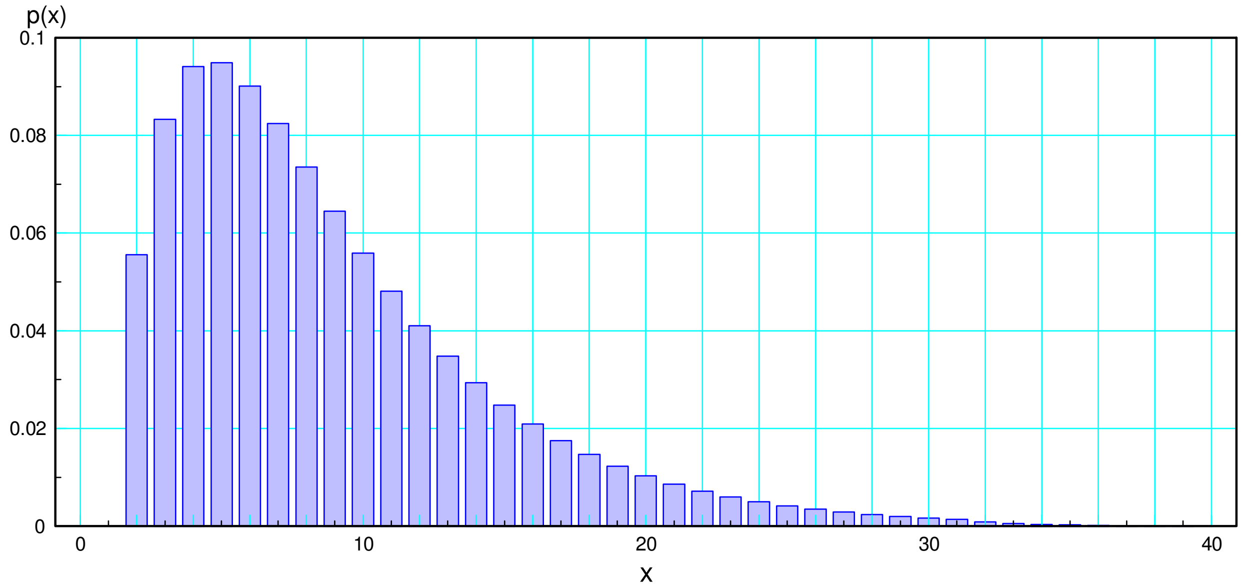
What is the pdf of A+B using convolution?

$$A(x) = \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{x-1} u(x-1)$$

$$B(x) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{x-1} u(x-1)$$

```
x = [0:30]';
A = (1/6) * (5/6).^(x-1) .* (x>=1);
B = (1/3) * (2/3).^(x-1) .* (x>=1);
Y = conv(A,B)

0          0
1          0
2     0.0556          check:  should get first non-zero term at x=2
3     0.0833
4     0.0941
5     0.0949
6     0.0901
7     0.0824
```



pdf for Y = A + b

4) Let

- A be the number of times you roll a 6-sided die until you roll a 1 (p = 1/6).
- B be the number of times you roll a 6-sided die until you get a 1 or 2 (p = 1/3)

What is the pdf of A+B using z-transforms?

$$A(z) = \left(\frac{1/6}{z-5/6}\right)$$

$$B(z) = \left(\frac{1/3}{z-2/3}\right)$$

$$Y(z) = A(z)\ B(z)$$

$$Y(z) = \left(\frac{1/6}{z-5/6}\right)\left(\frac{1/3}{z-2/3}\right)$$

Use partial fractions

$$Y(z) = \left(\frac{0.3333}{z-5/6}\right) + \left(\frac{-0.3333}{z-2/3}\right)$$

To match terms in my table of z-transforms, multiply by z

$$zY = \left(\frac{0.3333z}{z-5/6}\right) + \left(\frac{-0.3333z}{z-2/3}\right)$$

Take the inverse z-transform

$$z\ y(x) = \left(0.3333\left(\frac{5}{6}\right)^{x} - 0.3333\left(\frac{2}{3}\right)^{x}\right)\ u(x)$$

Divide by z (delay one)

$$y(x) = \left(0.3333\left(\frac{5}{6}\right)^{x-1} - 0.3333\left(\frac{2}{3}\right)^{x-1}\right)\ u(x-1)$$

This matches the results from problem #3

```
Y4 = ( 0.33333 * (5/6).^(x-1) - 0.33333 * (2/3) .^ (x-1) ) .* (x >= 1);

[x(1:10), Y(1:10), Y4(1:10)]
```

x	prob 3	prob 4
0	0	0
1.0000	0	0
2.0000	0.0556	0.0556
3.0000	0.0833	0.0833
4.0000	0.0941	0.0941
5.0000	0.0949	0.0949
6.0000	0.0901	0.0901
7.0000	0.0824	0.0824
8.0000	0.0735	0.0735
9.0000	0.0645	0.0645