ECE 341 - Homework #3

Dice Games and z-Transform. Due Friday, May 22nd

Please make the subject "ECE 341 HW#3" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Farkle

1) Compute the odds or rolling a 3 of a kind, 3 of a kind (two triplets) in Farkle

dice = xxx yyy

The number of ways you can roll 6 dice is

$$N = 6^6 = 46,656$$

The number of ways you can get two triplets is

M = (6 numbers choose 2) (6 spots for x, choose 3)(3 remaining spots for y, choose 3)

$$M = \binom{6}{2}\binom{6}{3}\binom{3}{3} = 300$$

The odds of getting two triplets is

$$p = \left(\frac{M}{N}\right) = \left(\frac{300}{46,656}\right) = 0.00643$$

With a value of 2500 points, this adds to the expected value or rolling all six dice

$$2500\,p = 16.075$$

Running a Monte-Carlo simulation in Matlab results in 647 / 100,000 cases of two triplets

p = 0.006470 (experimental)

2) Compute the odds of rolling 3 of a kind in Farkle.

dice = xxx abc

M = (6 numbers choose 1 for x)(6 spots for x, choose 3)(5 other numbers pick 1 for a)

(5 other numbers for b pick 1)(5 other numbers for c pick 1)

minus the case where a=b=c (300 ways from problem #1)

$$M = \binom{6}{1}\binom{6}{3}\binom{5}{1}\binom{5}{1}\binom{5}{1} - 300$$

$$M = 15,000 - 300 = 14,700$$

The probabity of getting three of a kind is thus

$$p = \left(\frac{14,700}{46,656}\right) = 0.315$$

Using a Monte-Carlo simulation of rolling 6 dice, the chance of getting 3 of a kind is

p = 30,780 / 100,000 = 0.30780

Reasonably close to what we calculated.

z-Transforms

Assume X and Y have the following z-transforms

$$X = \left(\frac{1}{2}\right)\left(\frac{z+1}{z}\right)$$ *bernoulli trial (coin toss)*

$$Y = \left(\frac{1}{3}\right)\left(\frac{z^2+z+1}{z^2}\right)$$ *uniform distribution (3-sided die)*

3) Determine the z-transform and inverse z-transform for XX

XX is

$$XX = \left(\frac{1}{2}\right)\left(\frac{z+1}{z}\right) \cdot \left(\frac{1}{2}\right)\left(\frac{z+1}{z}\right)$$

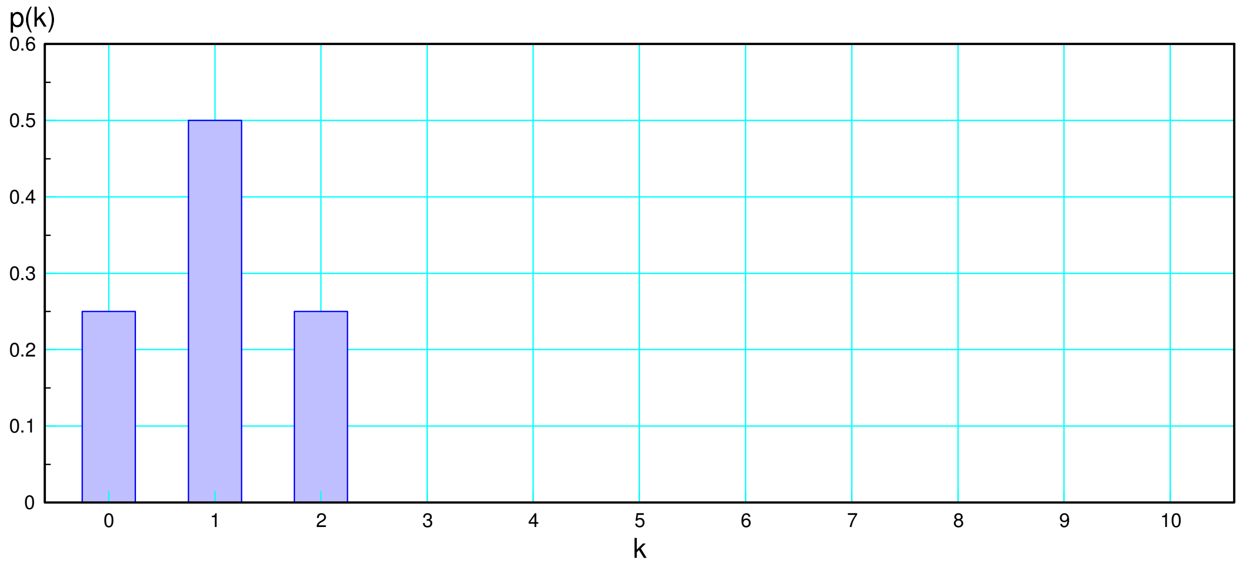
$$XX = \left(\frac{1}{4}\right)\left(\frac{z^2+2z+1}{z^2}\right)$$ *binomial distribution*

You can also write this as

$$XX = \left(\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)z^{-1} + \left(\frac{1}{4}\right)z^{-2}\right)$$

pdf: Apply the definition of z-transform

$$xx(k) = \left(\frac{1}{4}\right)\delta(k) + \left(\frac{1}{2}\right)\delta(k-1) + \left(\frac{1}{4}\right)\delta(k-2)$$



4) Determine the z-transform and inverse z-transform for XY

$$X = \left(\frac{1}{2}\right)\left(\frac{z+1}{z}\right)$$ *bernoulli trial (coin toss)*

$$Y = \left(\frac{1}{3}\right)\left(\frac{z^2+z+1}{z^2}\right)$$ *uniform distribution (3 sided die)*

z-transform of XY (also known as the moment generating function)

$$XY = \left(\frac{1}{6}\right)\left(\frac{z+1}{z}\right)\left(\frac{z^2+z+1}{z^2}\right)$$

$$XY = \left(\frac{1}{6}\right)\left(\frac{z^3+2z^2+2z+1}{z^3}\right)$$

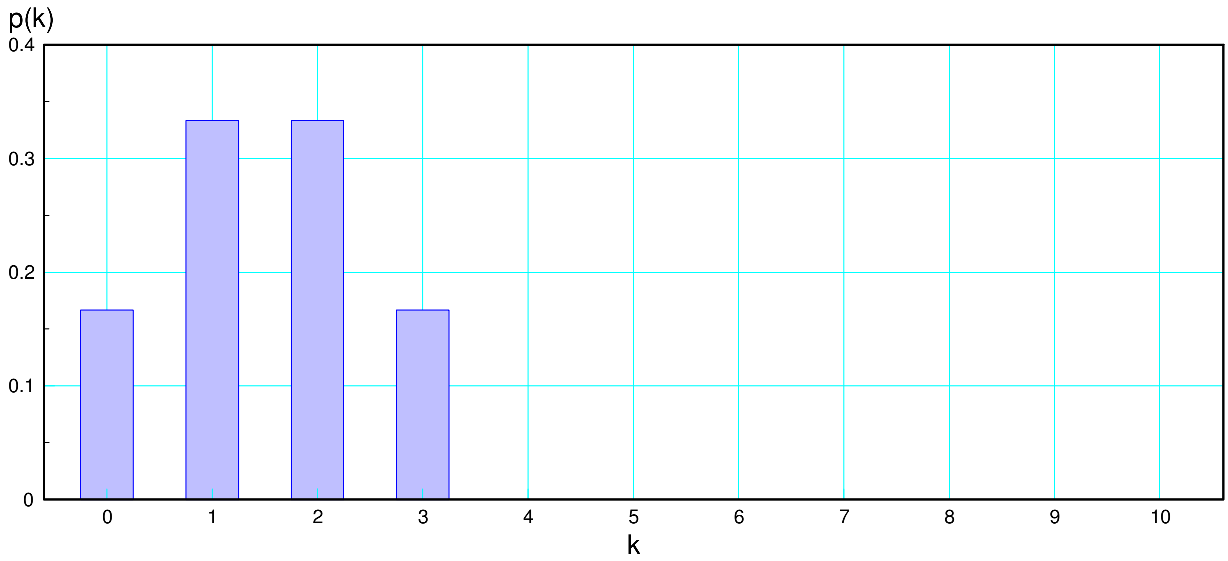
To take the inverse z-transform, simply apply the definition of z-transform

moment generating funciton:

$$XY = \left(\left(\frac{1}{6}\right) + \left(\frac{2}{6}\right)z^{-1} + \left(\frac{2}{6}\right)z^{-2} + \left(\frac{1}{6}\right)z^{-3}\right)$$

pdf

$$xy(k) = \left(\frac{1}{6}\right)\delta(k) + \left(\frac{2}{6}\right)\delta(k-1) + \left(\frac{2}{6}\right)\delta(k-2) + \left(\frac{1}{6}\right)\delta(k-3)$$



pdf for XY

5) Determine the z-transform and inverse z-transform of XY

$$X = 0.2\left(\frac{z}{z-0.8}\right)$$ *geometric distribution*

$$Y = 0.5\left(\frac{z}{z-0.5}\right)$$ *geometric distribution*

Solution: (moment generating function):

$$XY = 0.1\left(\frac{z}{z-0.8}\right)\left(\frac{z}{z-0.5}\right)$$ *Pascal distribution*

Inverse z-transform (pdf)

$$XY = \left(\frac{0.1z}{(z-0.8)(z-0.5)}\right)z$$

$$XY = \left(\left(\frac{0.2667}{z-0.8}\right) + \left(\frac{-0.1667}{z-0.5}\right)\right)z$$

$$XY = \left(\frac{0.2667z}{z-0.8}\right) + \left(\frac{-0.1667z}{z-0.5}\right)$$

$$xy(k) = \left(0.26667(0.8)^k - 0.1667(0.5)^k\right)u(k)$$